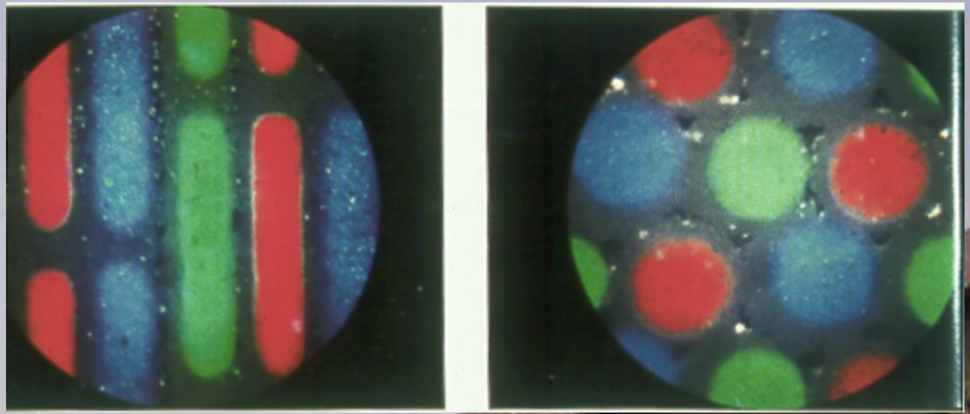
Principles of color matching with displays

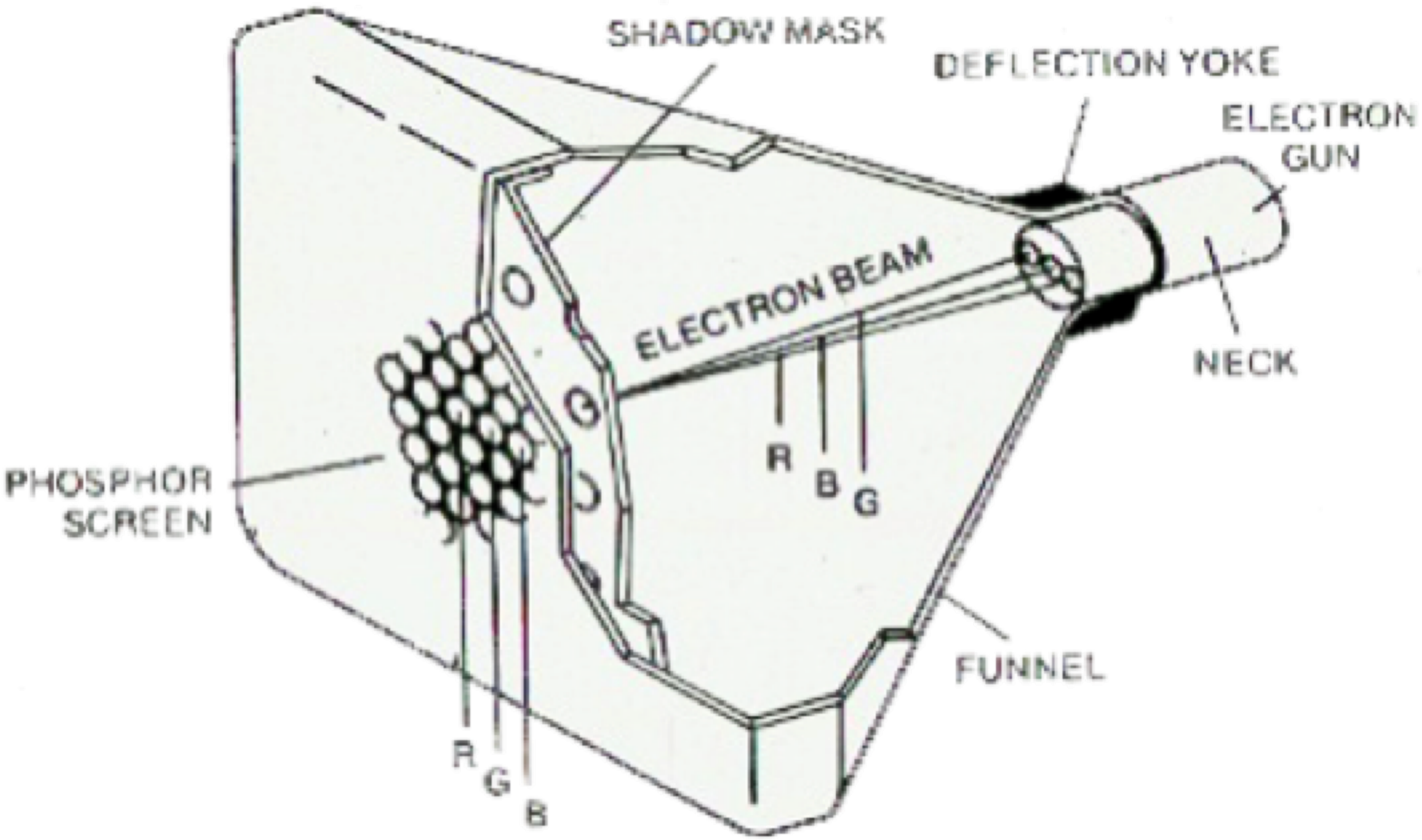
- CRT displays
- Linear equations for matching
- The nonlinearity (gamma) curve



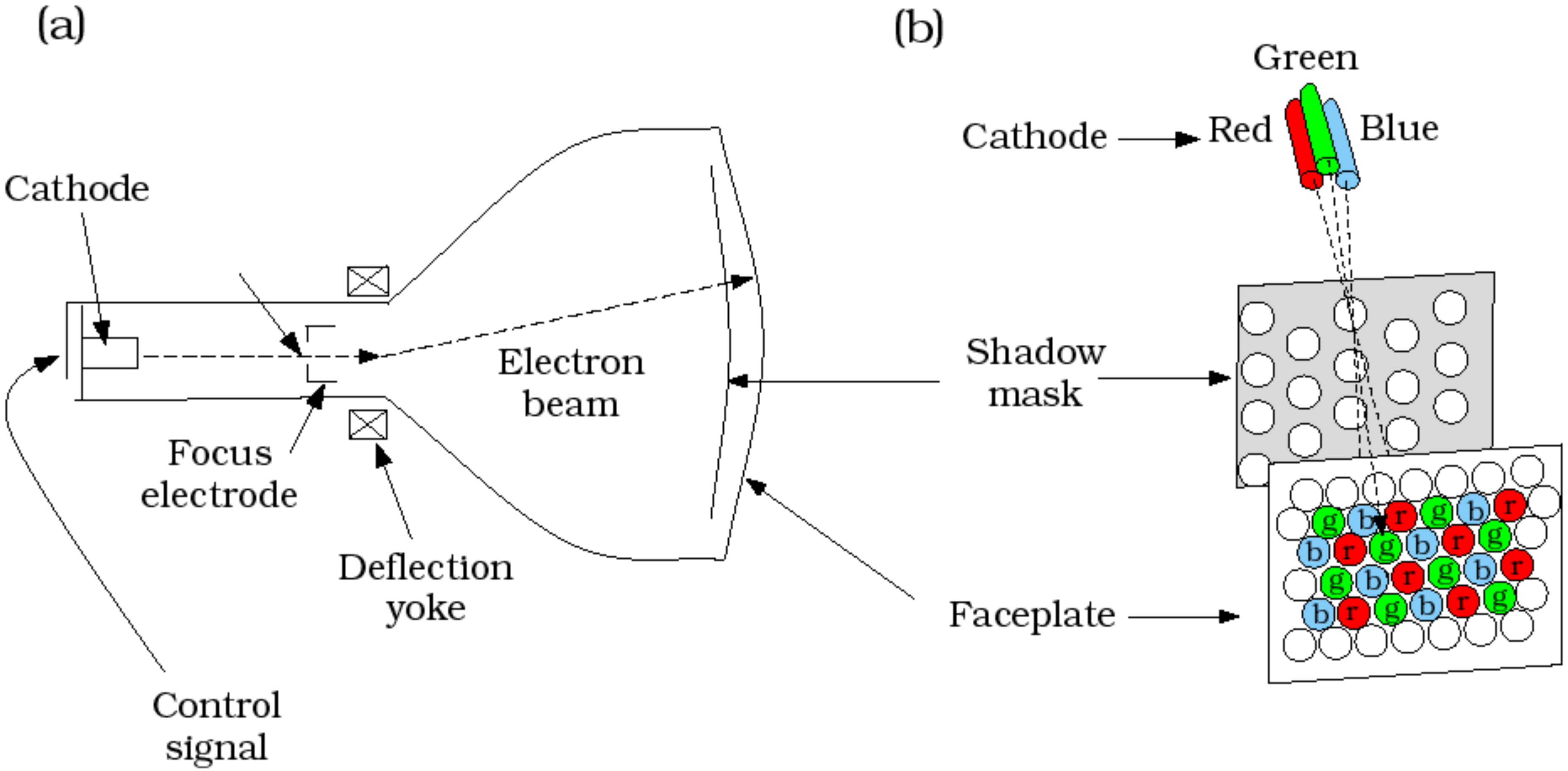
Color matching with displays

CRT's use the logic of the color matching experiment on every scan

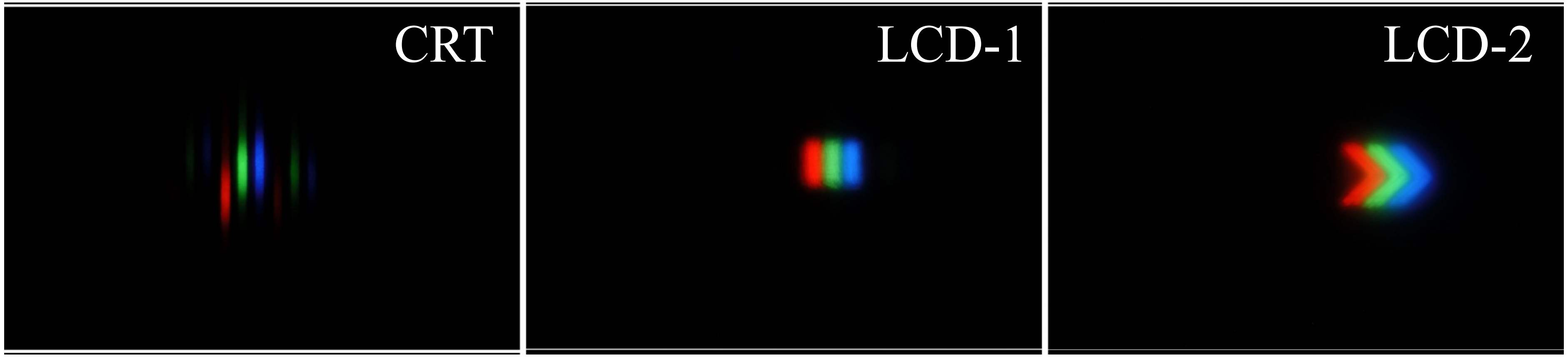
Each pixel is like a miniature color-matching display

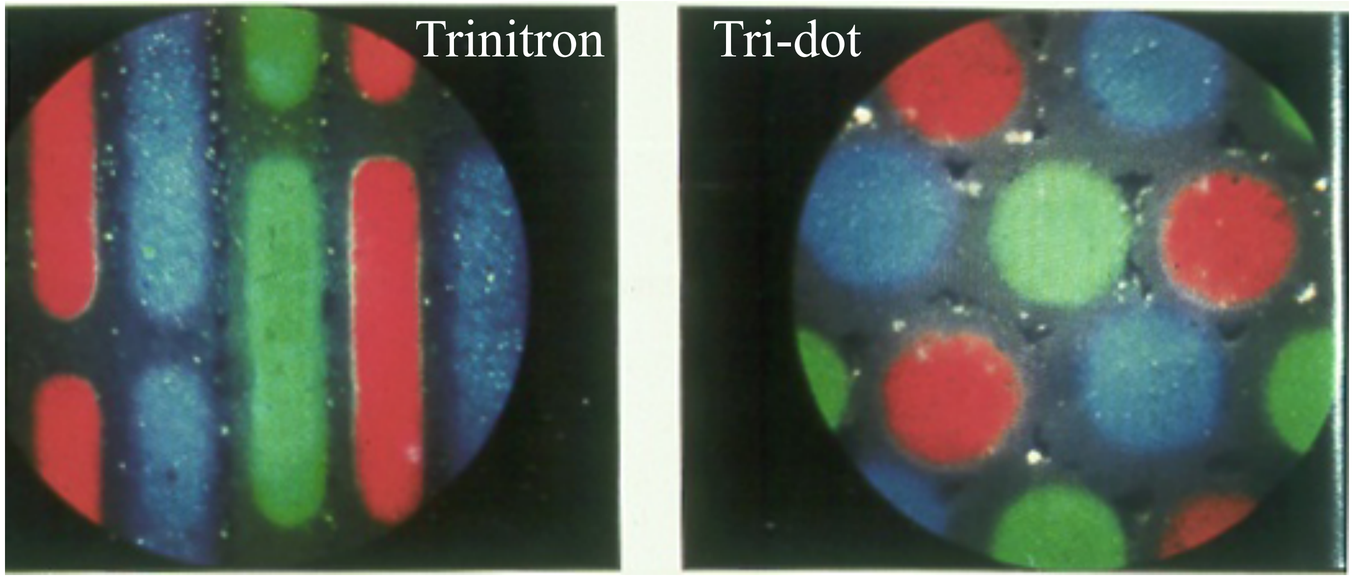


CRT signal path

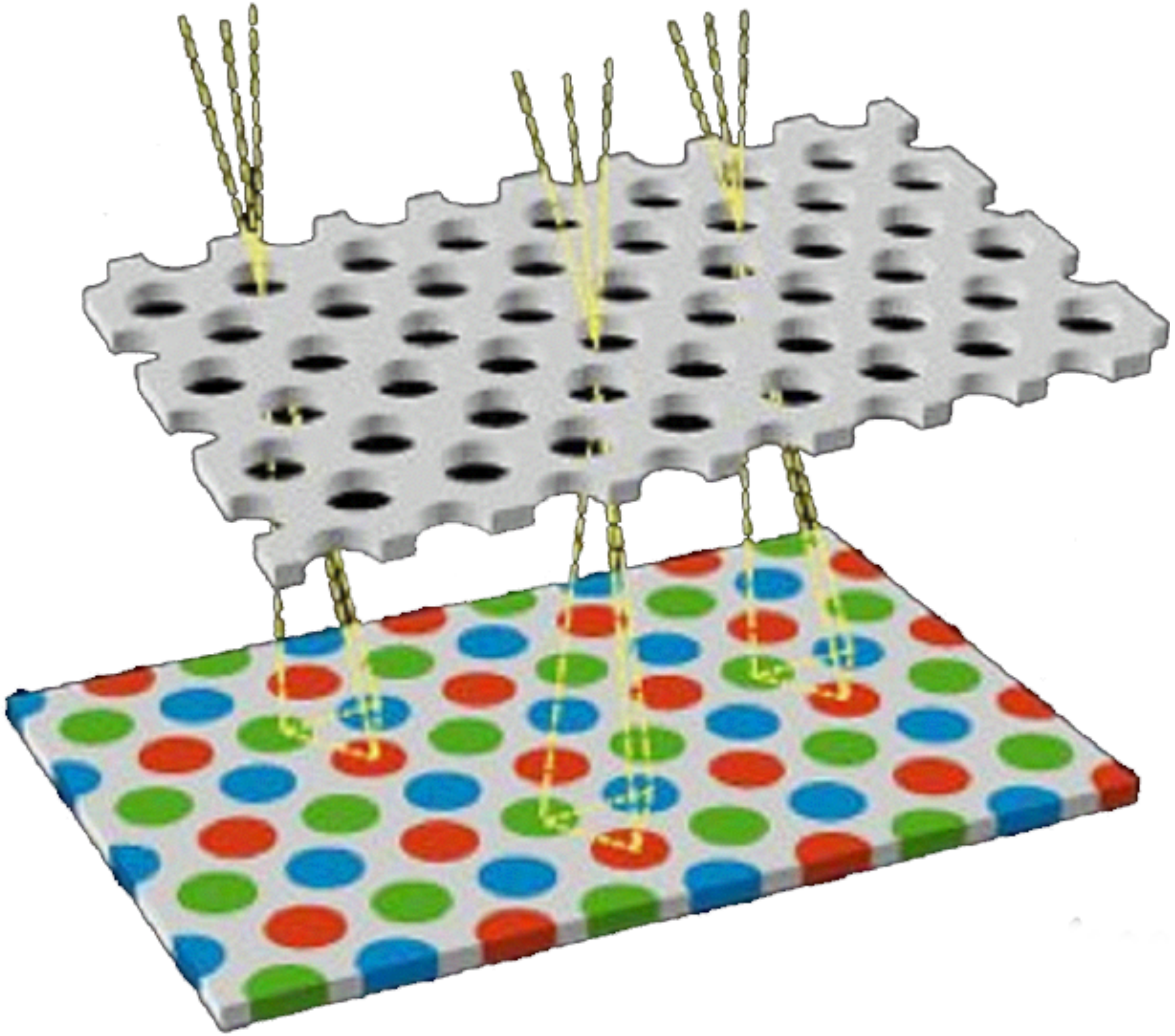


Displays mimic the color-matching experiment at each point



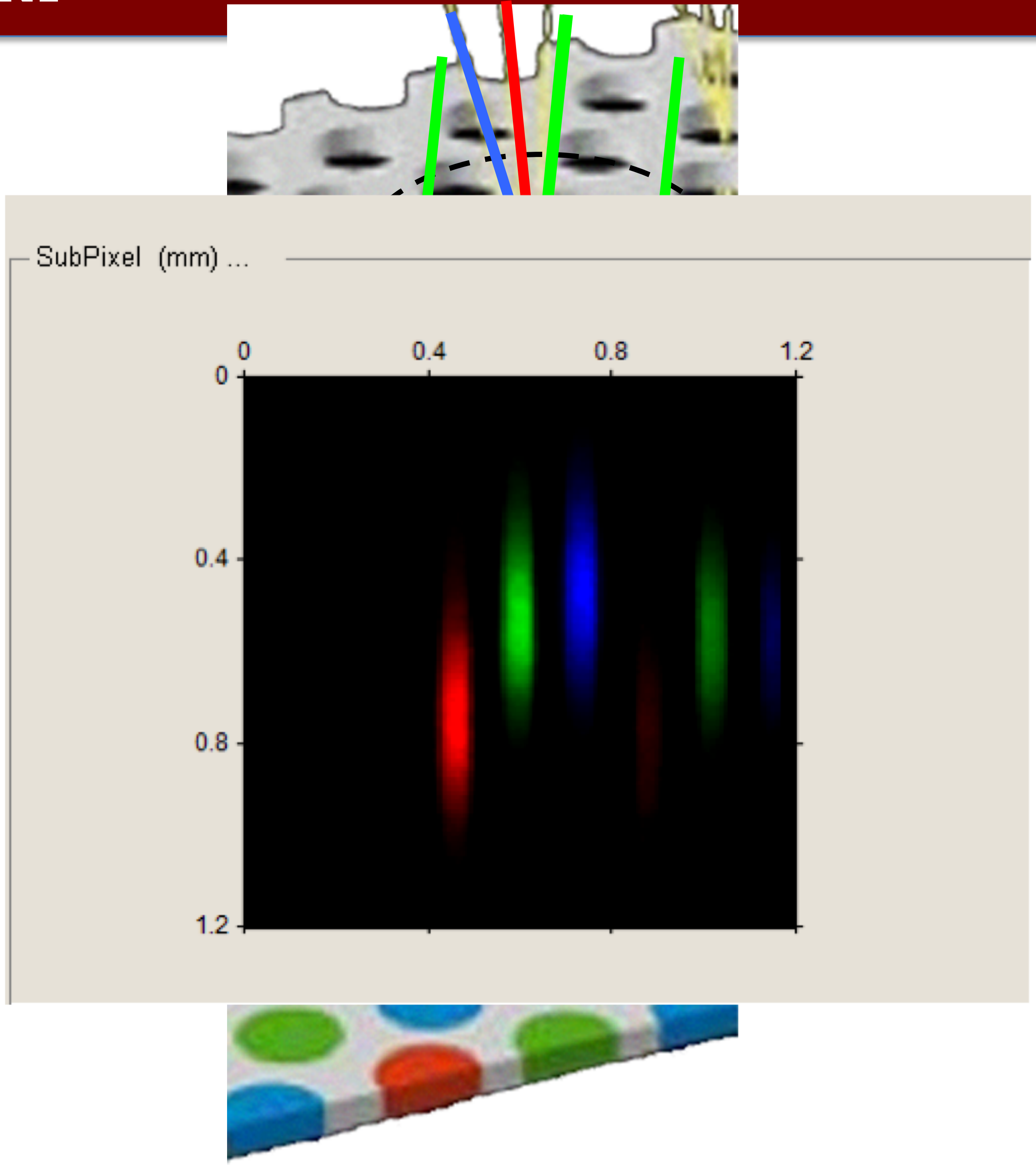


Shadow mask geometry associates each neam with a single phosphor



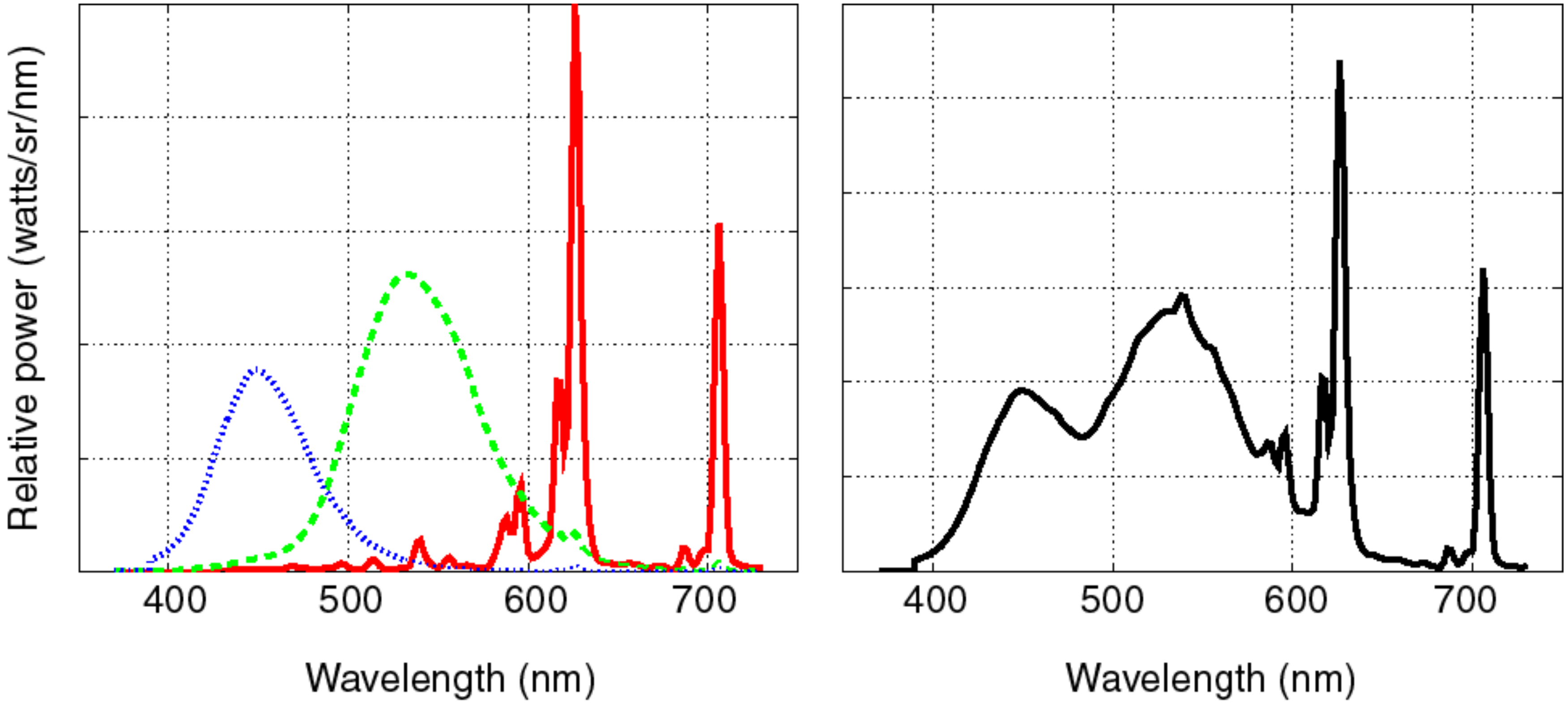
The shadow mask directs the electrons to one of the phosphors

The electron beam spreads across a few locations, but the local rays are nearly parallel and preserve color association



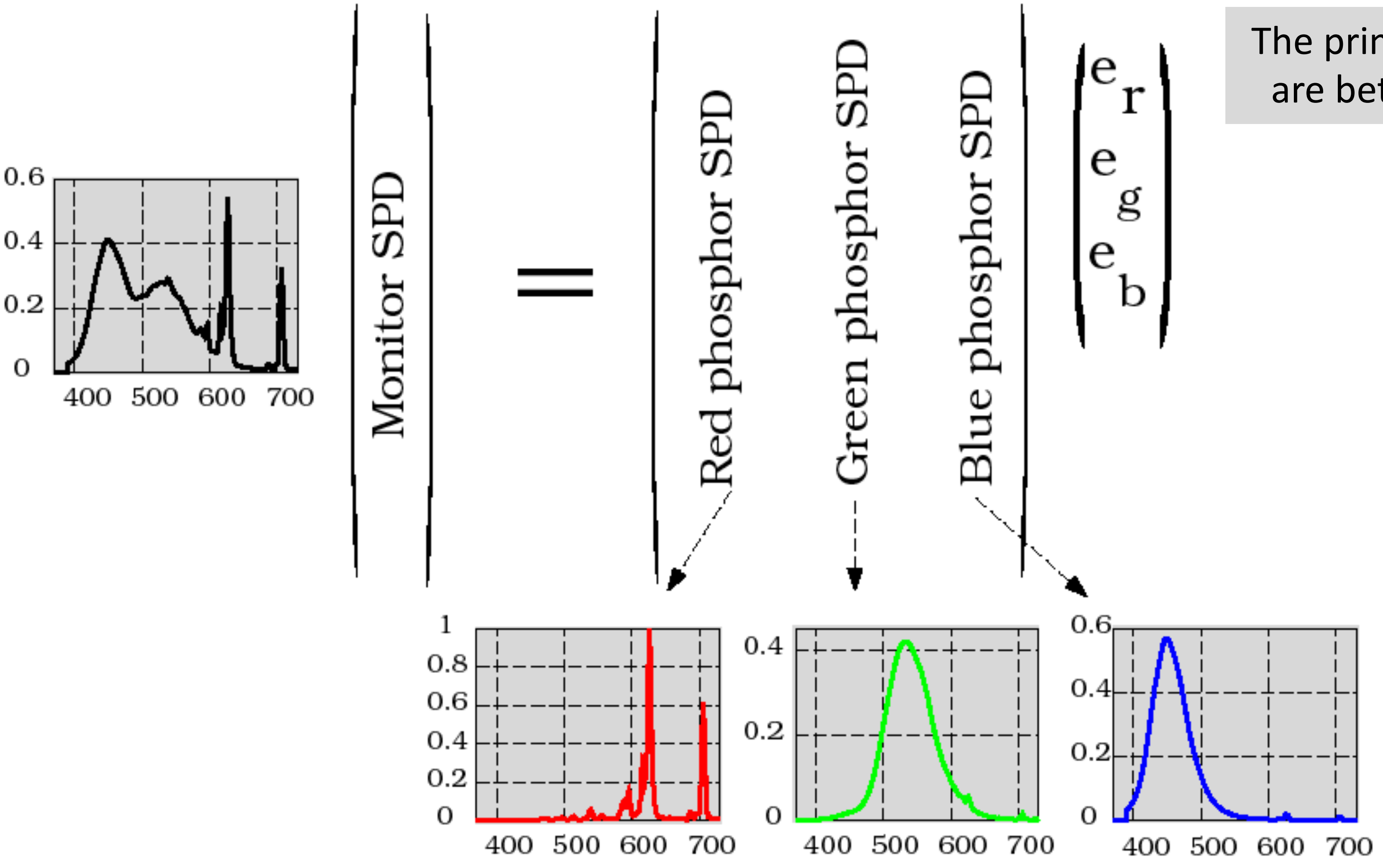
Superposition of phosphor SPDs

Remember additivity of light and
Newton's wall



CRT emissions in matrix tableau form

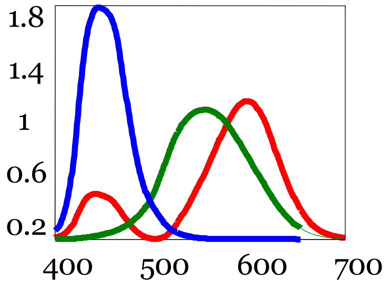
The columns are the maximum emission from each primary



The primary intensities are between 0 and 1

Setting the display output to a particular (X,Y,Z) value

Solve for the linear intensities

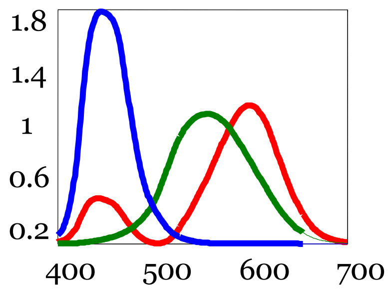


$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \text{XYZ functions} \end{pmatrix} \begin{pmatrix} \text{Monitor primaries} \end{pmatrix} \begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix}$$

Monitor primary intensities e_r, e_g, e_b

Setting the display output to a particular (X,Y,Z) value

Solve for the primary intensities that match the camera measurements



$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \text{XYZ functions} \end{pmatrix} \begin{pmatrix} \text{Monitor primaries} \end{pmatrix} \begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix}$$

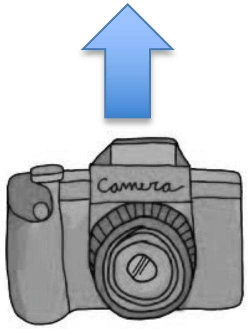
Monitor primary intensities e_r, e_g, e_b

Camera measures XYZ

Setting the display output to a particular (X,Y,Z) value

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} & & \\ & \text{XYZ * Display} & \\ & & \end{pmatrix}_{3x3} \begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix}$$

Monitor primary intensities e_r, e_g, e_b



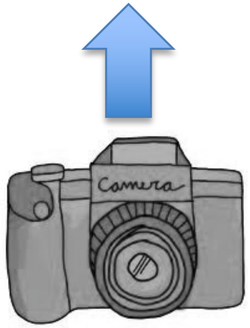
Camera measures XYZ

Create a metamer

Primary intensities

Image contains a target value

$$\begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix} = \left((\mathrm{XYZ} * \mathrm{Display})^{-1} \right)_{3\mathrm{x}3} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

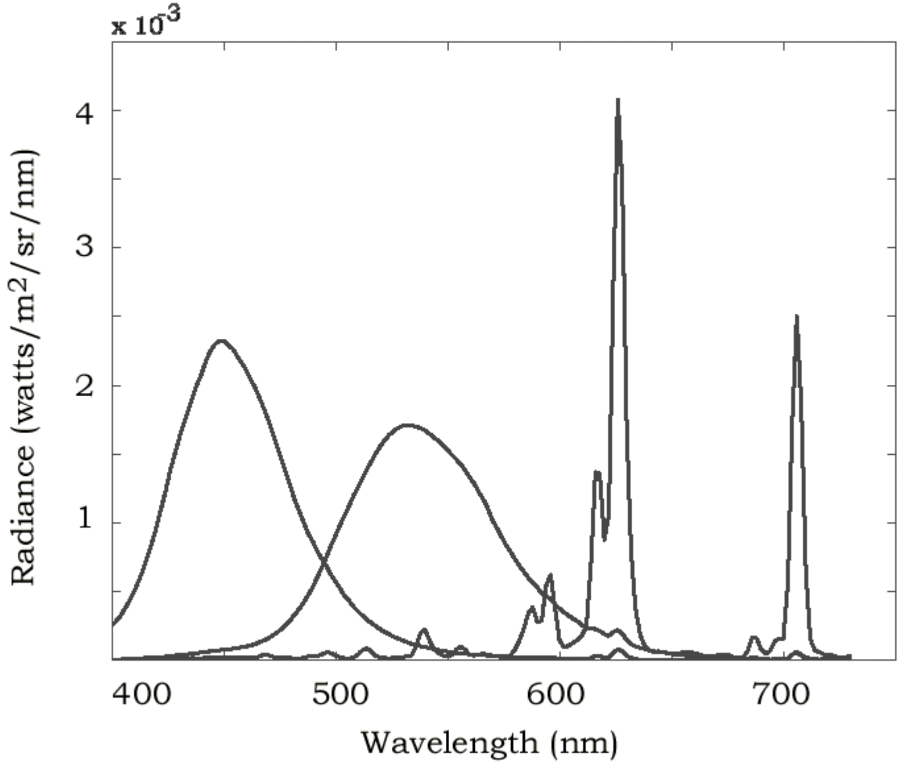


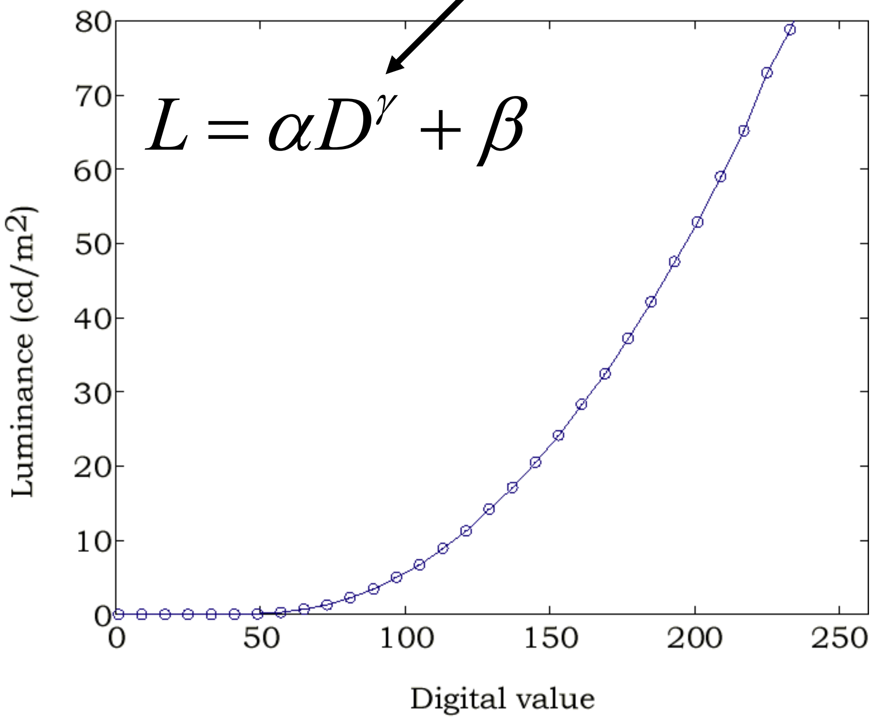
Camera measures XYZ

A word about the display transduction function

Often called the display gamma

$$L = \alpha D^{\gamma} + \beta$$

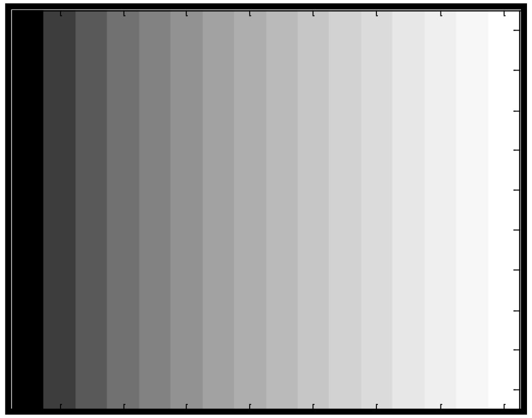




Some Comments About Display Gamma Levels of 2.0

Linear spacing

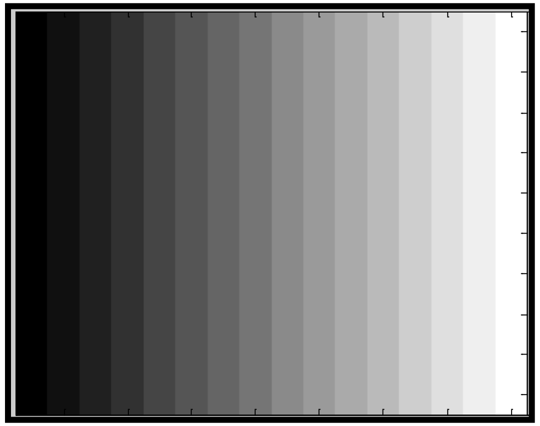
Steps are too big in dark regions



Bunched appearance in bright region

Uniform appearance

Equal L spacing*



Board work to explain relationship to transduction

Camera-Display Matching

Camera measures RGB and we convert to a value related to CIE-XYZ

→

Estimate monitor linear intensities that match CIE-XYZ related value

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \text{XYZ functions} \end{pmatrix} \begin{pmatrix} \text{Monitor primaries} \end{pmatrix} \begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} XM \end{pmatrix} \begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix}$$

Camera-Display Matching

Camera measures something that we convert to a CIE-XYZ value

Estimate monitor linear intensities that match CIE-XYZ related value

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} & & \\ & XM & \\ & & \end{pmatrix} \begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix}$$

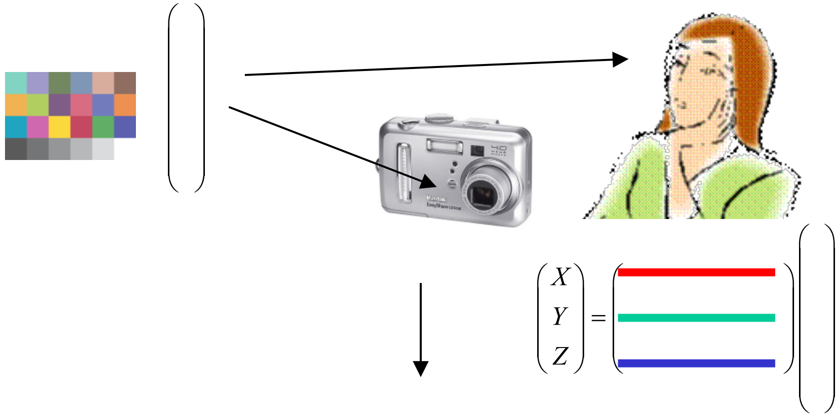
$$\begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

Correct for display gamma

$$\begin{pmatrix} e_r \\ e_g \\ e_b \end{pmatrix} = \begin{pmatrix} & & \\ & (XM)^{-1} & \\ & & \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Image processing pipeline

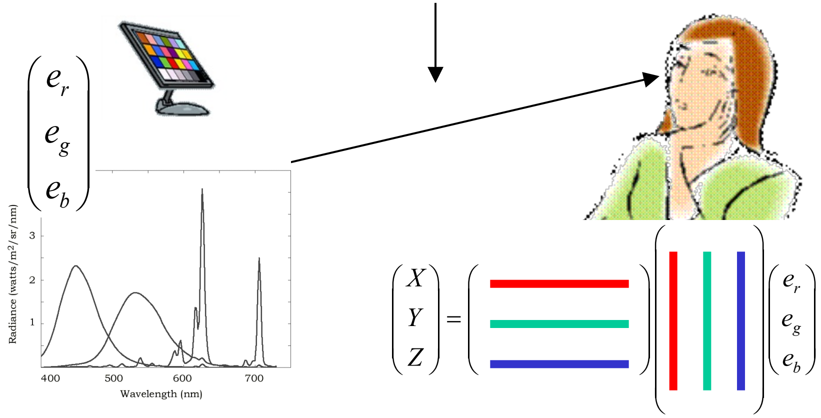
Camera estimates calibrated values



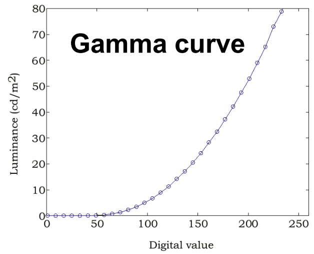
Encoded for transmission or storage

NTSC, PAL, JPEG, etc.

Produce (linear RGB) color matches



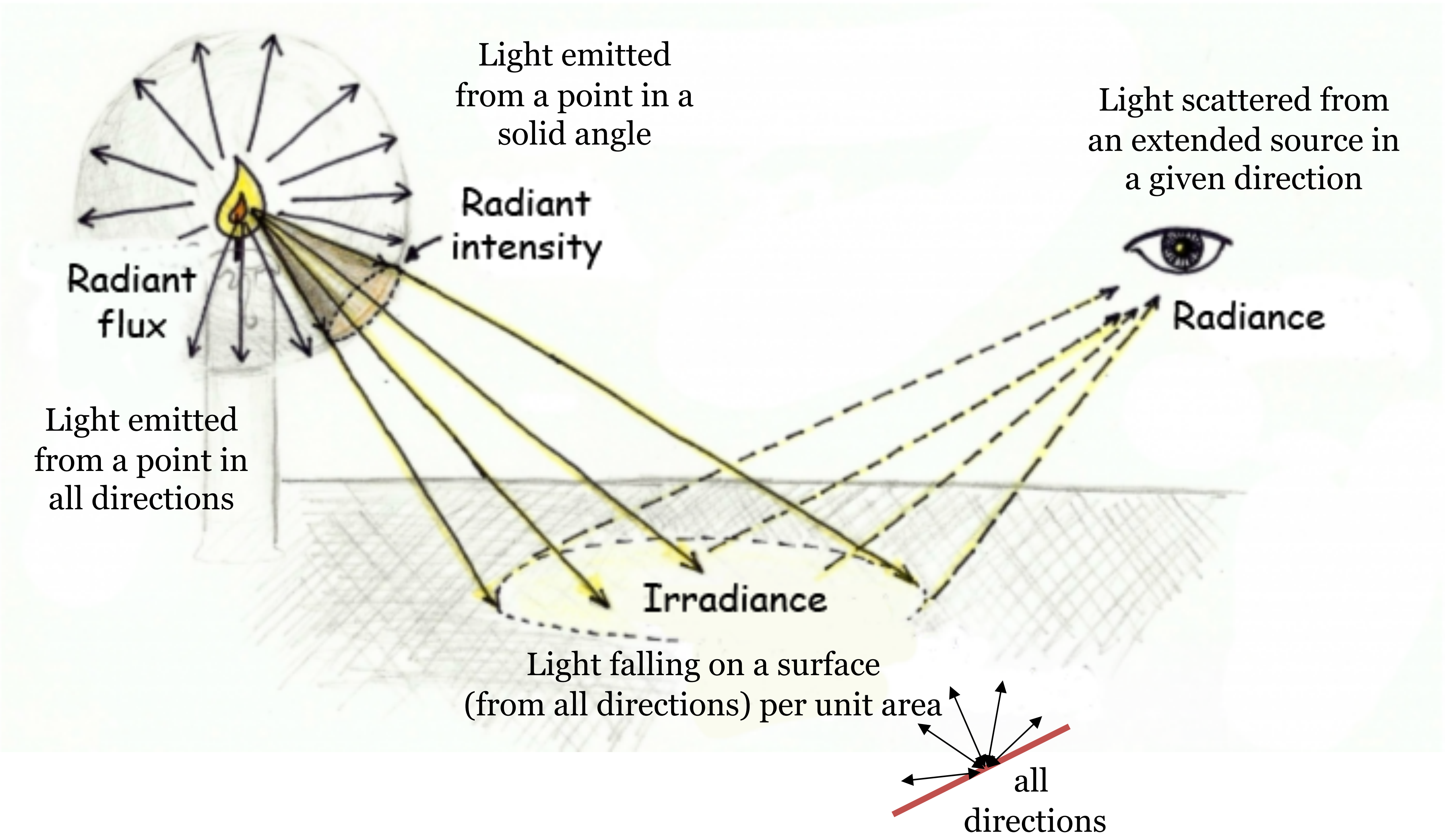
Encode for device RGB gamma



Photometry: Radiance and Luminance

- **Video** tutorial for spectral radiance
- Luminance
- Flicker photometry

Radiometry

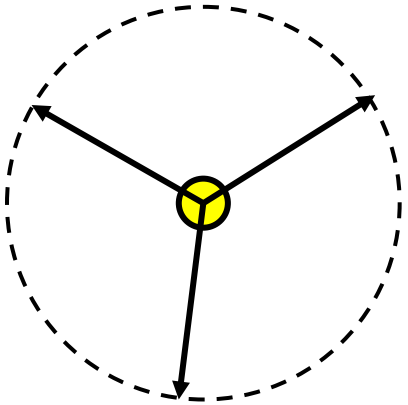

Light emitted
from a point in a
solid angle
Light scattered from
an extended source in
a given direction
Radiant
intensity
Radiant
flux
Radiance
Light emitted
from a point in
all directions
Irradiance
Light falling on a surface
(from all directions) per unit area
all
directions

Radiance geometry

Functions of wavelength

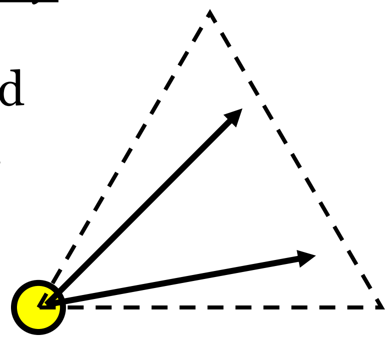
Radiant flux

Light emitted from a point



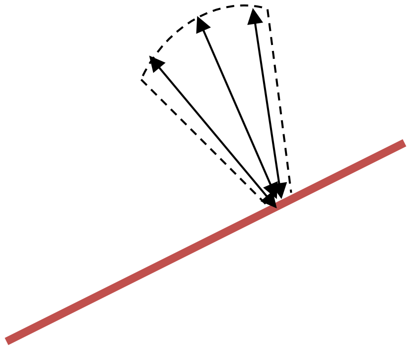
Radiant intensity

Light emitted from a point in a solid angle



Radiance

Light scattered from an extended source in a given direction

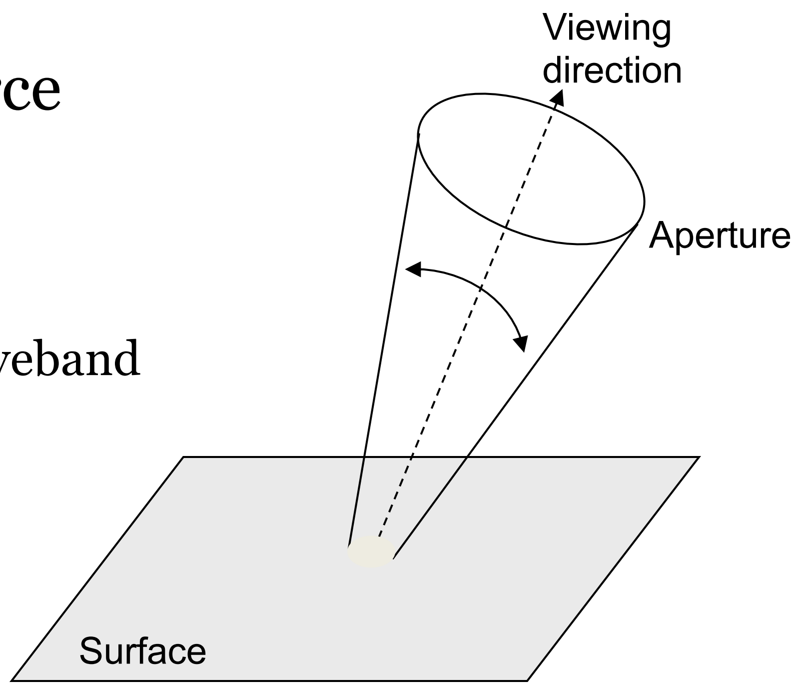


Spectral radiance units

Definition: Light emitted or reflected from an extended source in a given direction. The light is specified in units of

energy/second/steradian/surface area/waveband

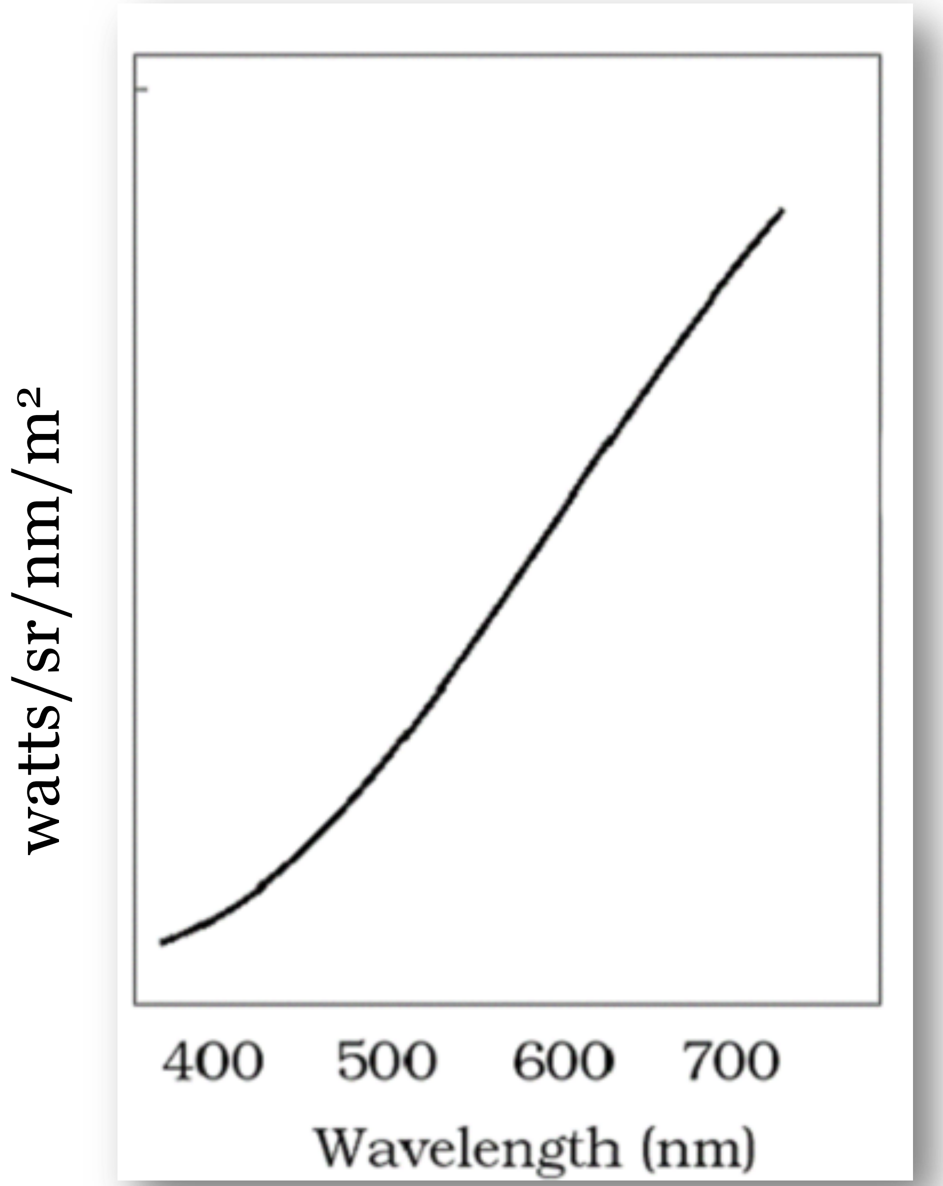
watts/steradian/surface area/waveband



The surface area is foreshortened according to the viewing direction

Spectral radiance function

- Source ***Radiance***
  - *Symbol: L*
  - *Units: W/(sr nm m²)*



Spectral radiance

$$L(\lambda) = \frac{E(\lambda)}{\underbrace{\omega}\, A_s \cos(\theta)\; t\; nm}$$

• Solid angle of a cone (steradians): the surface area on a unit sphere intersected by the cone whose apex is at the center of the sphere

• A sphere is 4π steradians

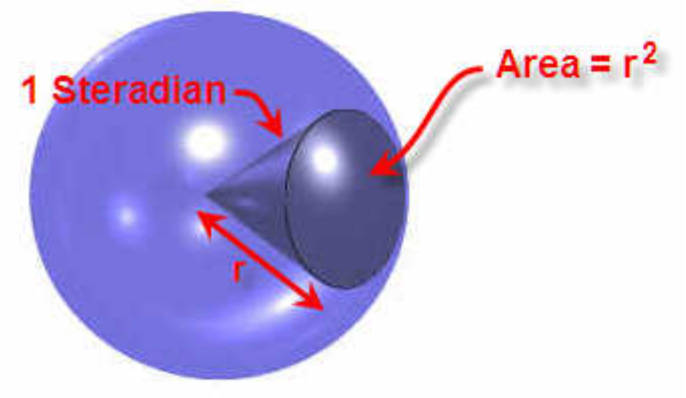
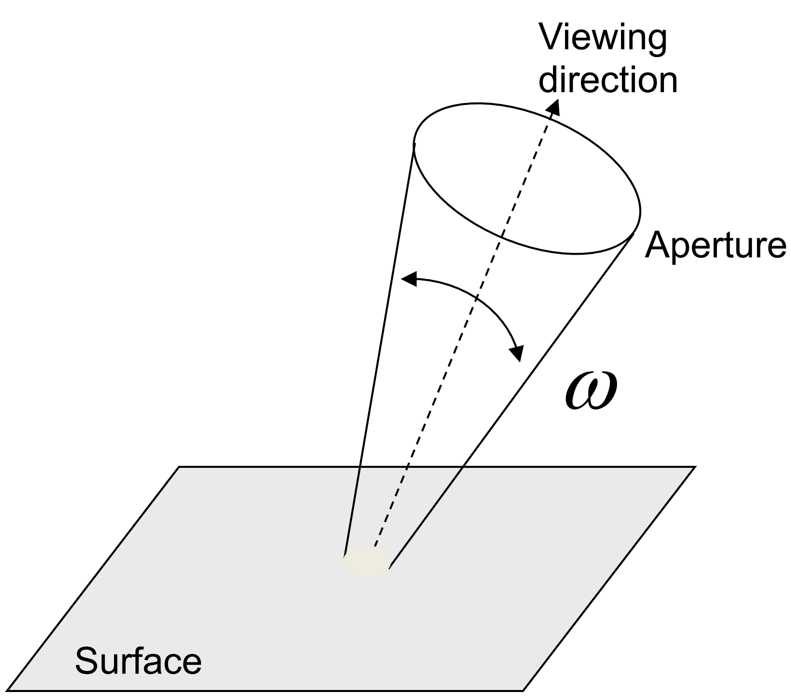


Image from http://www.mathsisfun.com/geometry/steradian.html

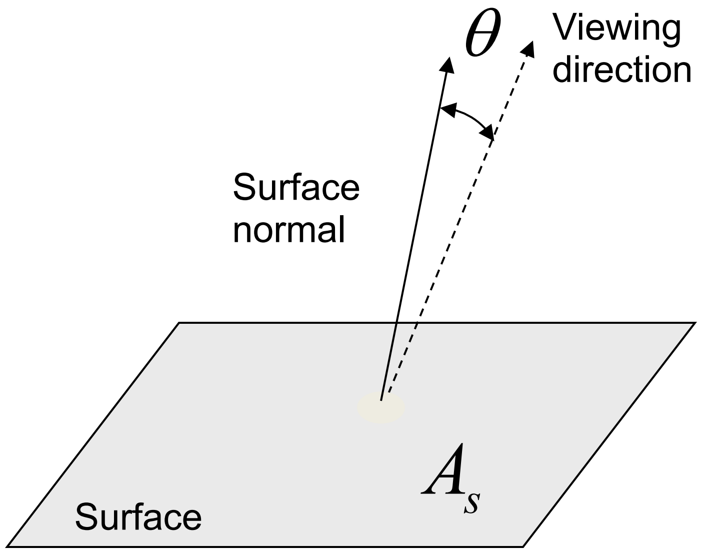
Solid angle



Spectral radiance

$$L(\lambda) = \frac{E(\lambda)}{\omega \underbrace{A_s \cos(\theta)} t\ nm}$$

Foreshortened surface area

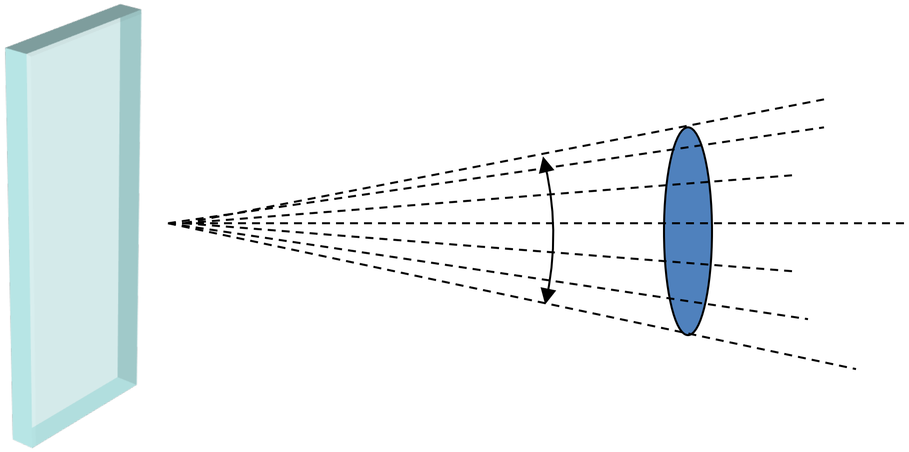


- The surface area is foreshortened and consistent with the area as seen from the viewing direction.

- As the angle approaches 90 deg, the area becomes zero and the energy becomes zero.

Radiance geometry

Spectroradiometer uses this principle to measure luminance without measuring distance



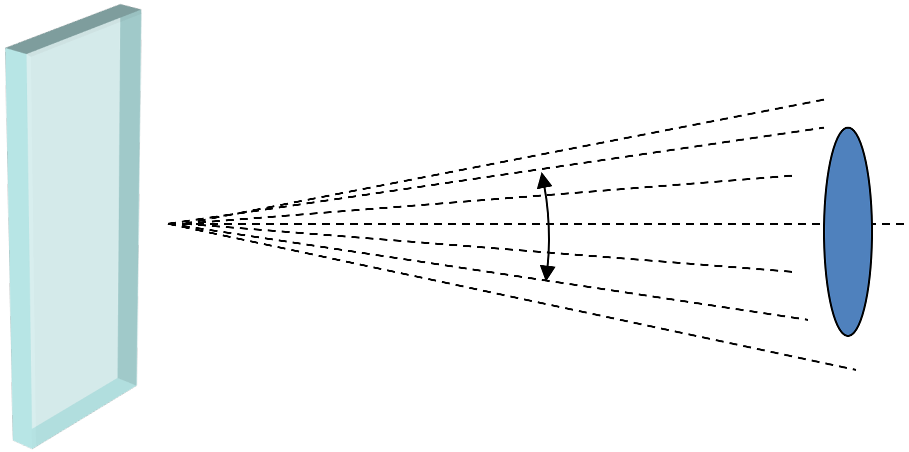
Closer, acquires ***smaller surface area*** *than farther*

Parameter	Distance
Solid angle	
Surface area	

Radiance geometry

Spectroradiometer uses this principle to measure luminance without measuring distance

Closer, acquires ***smaller surface area*** *than farther*

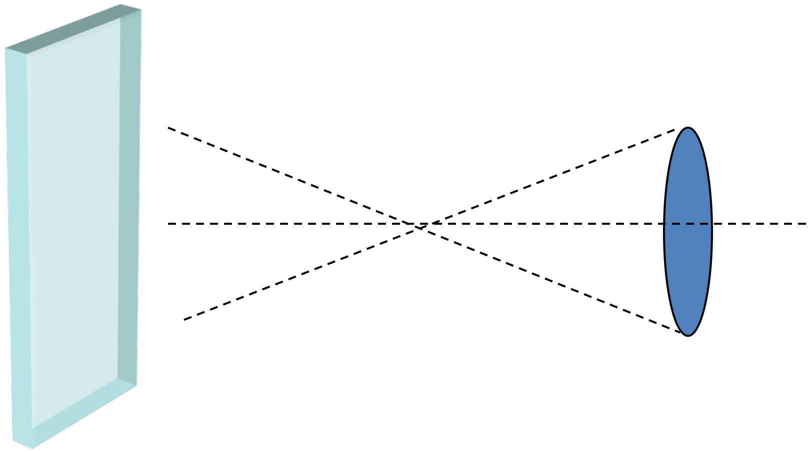


Parameter	**Distance**
Solid angle	Decreases
Surface area	

Radiance geometry

Spectroradiometer uses this principle to measure luminance without measuring distance

Closer, acquires ***smaller surface area*** *than farther*

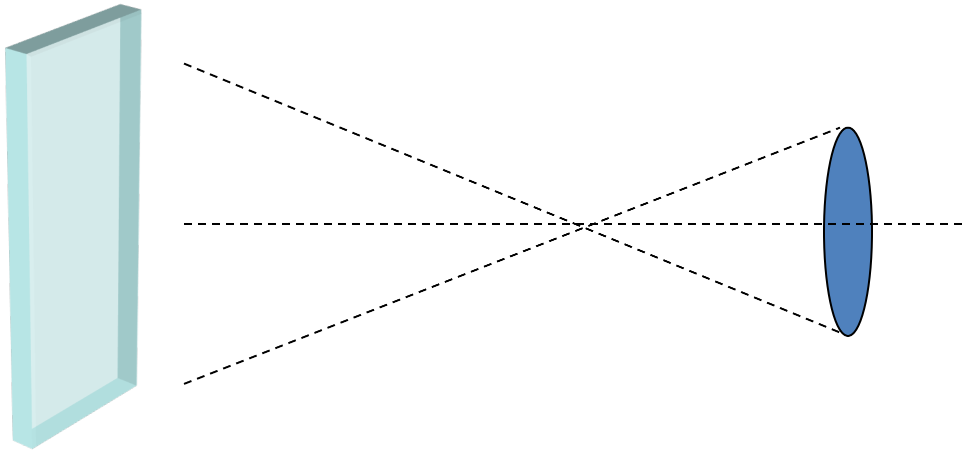


Parameter	Distance
Solid angle	Decreases
Surface area	

Radiance geometry

Spectroradiometer uses this principle to measure luminance without measuring distance

Closer, acquires ***smaller surface area*** *than farther*

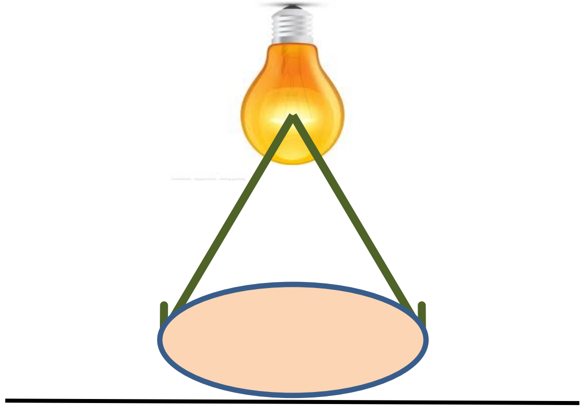


Parameter	**Distance**
Solid angle	Decreases
Surface area	Increases

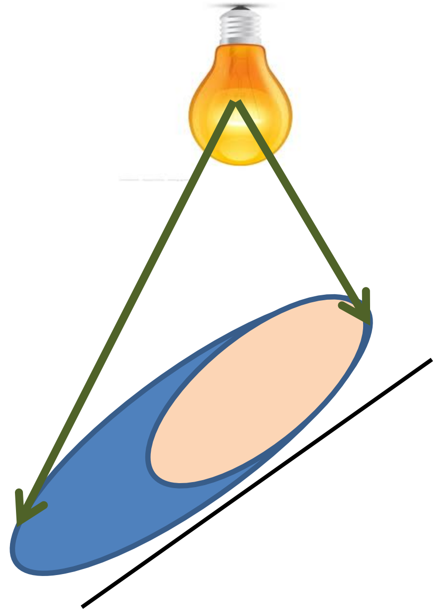
Irradiance (E)

$$J\,s^{-1}\,m^{-2} = w\,m^{-2}$$

- The **irradiance** at a surface depends on the angle of the surface with respect to the light



- As surface tilts away from the light, the same amount of light covers a larger surface area and thus less irradiance (light per square meter)



Basic conversion of radiance to irradiance

Irradiance formula

$$E = \frac{\pi}{1+4(f\#)^2(1+|m|)^2}TL$$

E = *irradiance at the sensor*

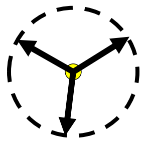
L = *radiance in the scene*

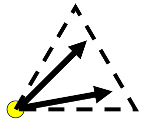
T = *lens and other media transmittance*

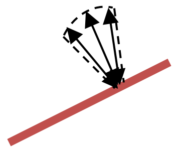
$f\#$ = *f-number of the lens*

m = *magnification of the lens*

Spectral radiance and irradiance units







Term	Defining Equation	Application	SI Unit
Radiant Flux	$F = \dfrac{\Delta Q}{\Delta t}$	Light emitted from a point	watt
Radiant Intensity	$I = \dfrac{\Delta F}{\Delta \omega}$	Light emitted from a point in a given solid angle	watt / sr
Radiance	$L = \dfrac{\Delta I}{\Delta A_s \cos(\theta)}$	Light from an extended source in a given direction	watt / sr / m^2
Irradiance	$E = \dfrac{\Delta F}{\Delta A_r}$	Light density incident on a plane	watt / m^2

Q = energy (joules)

F = Q/sec (watt)

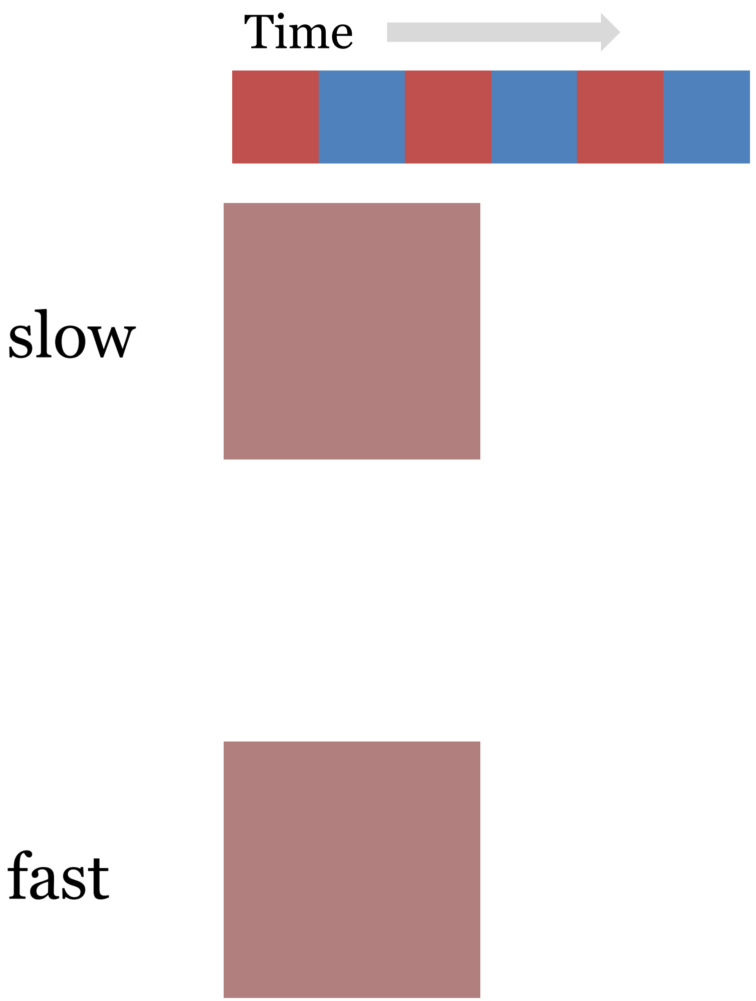
t = time (sec)

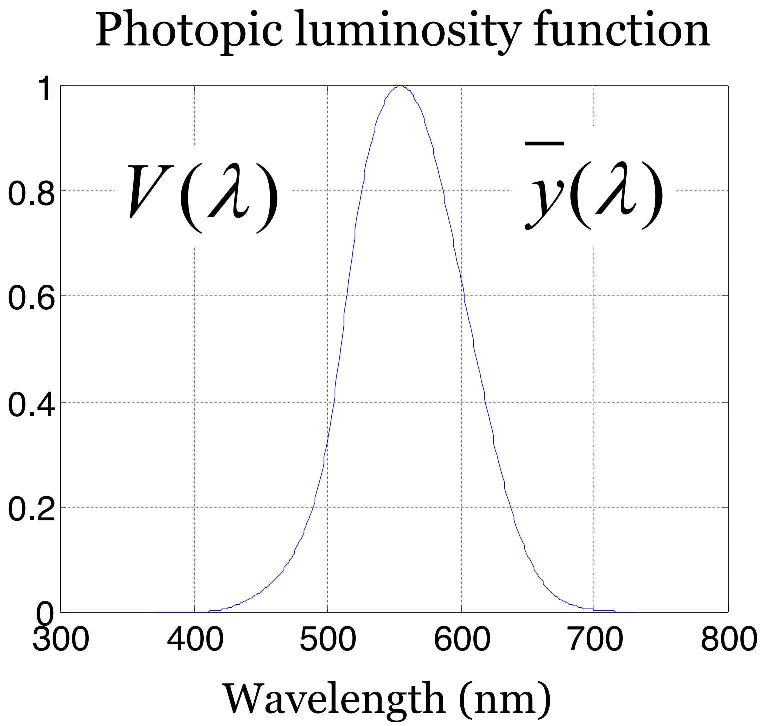
ω = solid angle (steradian)

A = area ($meter^2$)

θ = angle incident to plane

The empirical basis of the luminance curve

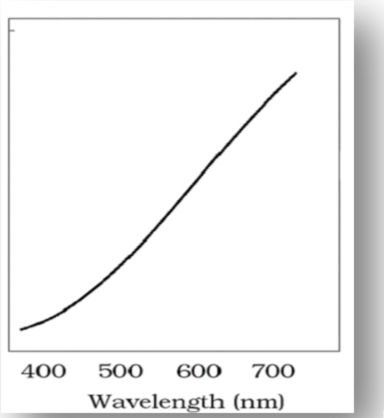




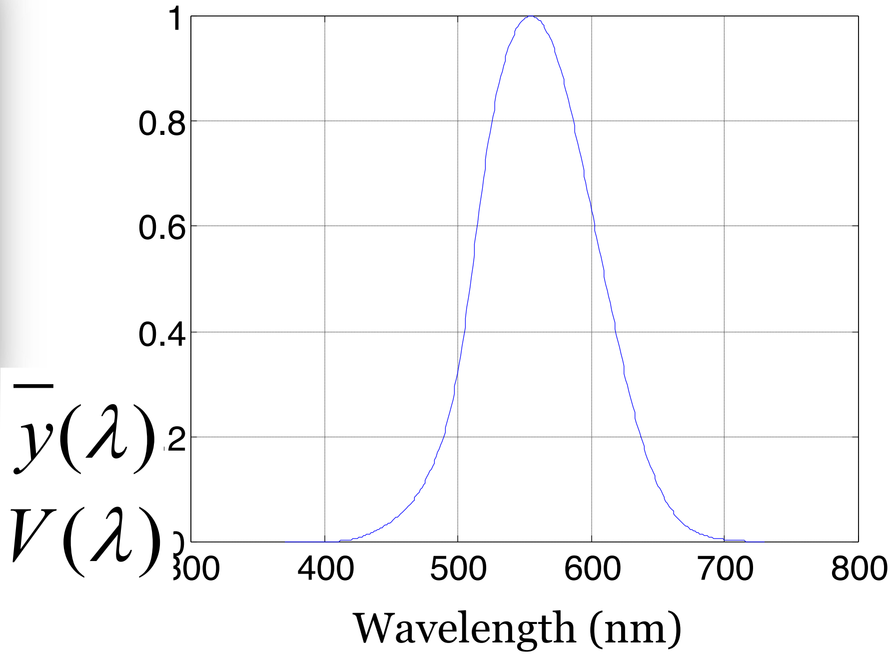
Luminance from spectral radiance

- Source ***Radiance***
  - *Symbol: L*
  - *Units: W/(sr nm m²)*

- Source ***Luminance***
  - *Symbol: L_v*
  - *Units: candelas/m²*



Photopic luminosity function



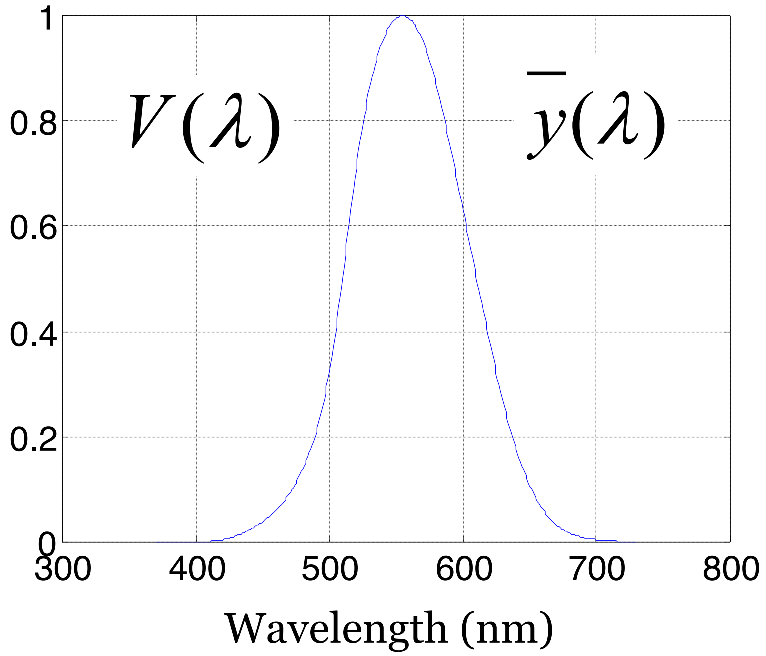
127.4 cd/m²

Luminance from spectral radiance

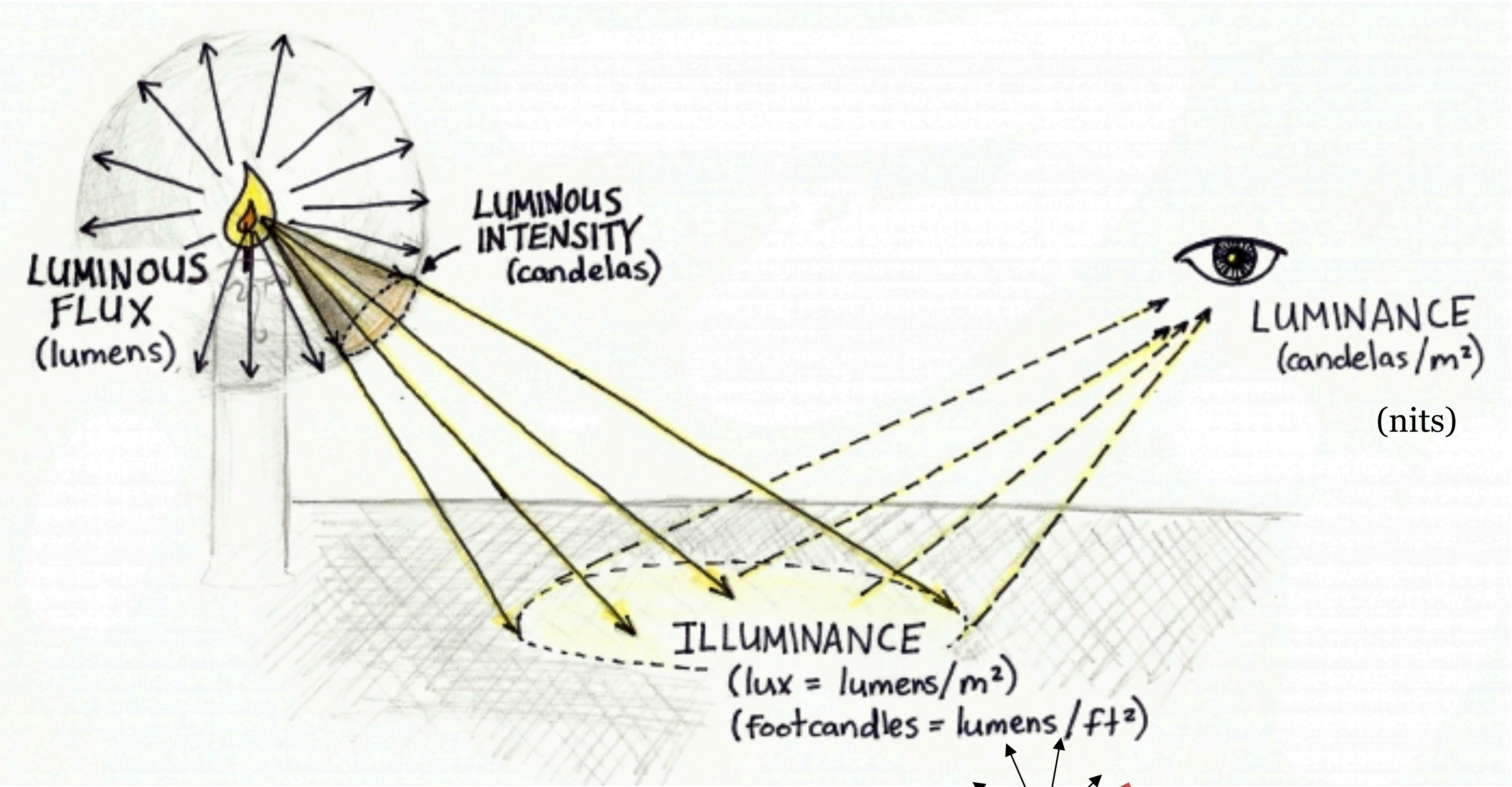
$$L_v = K_m \int V(\lambda) \cdot L(\lambda)\, d\lambda$$

- Conversion
  - Peak luminosity: unity at 555 nm
  - K_m = 683 lm/W

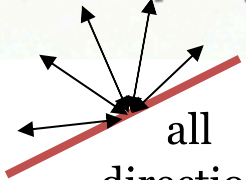
Photopic luminosity function



Photometric units



(nits)



all directions

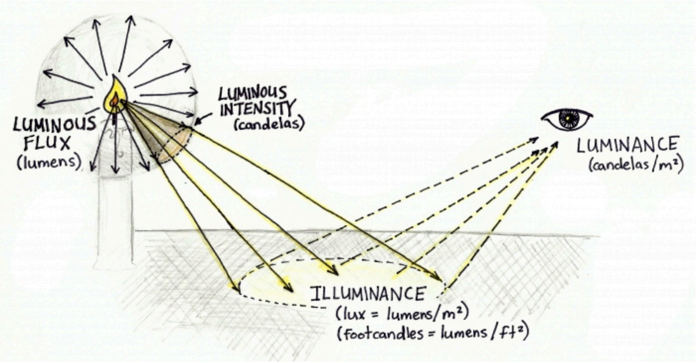
Typical luminance levels

Outdoors (cd/m²)

Sun	$6x10^8$
Visual saturation	49,000
Just below saturation	25,000
Outdoor building façade	10,000
Blue sky (morning)	4,600
Concrete sidewalk	
in sun	3,200
in shadow	570
in deep shadow	290

Indoors (cd/m²)

Interior room (fluorescent lighting)	
floor/walls	90
in shadow	10
Interior room (no lighting)	
floor/walls	30
in shadow	5
in closet door	1



Corresponding photometric units

Term	Defining Equation	Application	SI Unit
Luminous Flux	$F_v = K_m \int F_e(\lambda)V(\lambda)d\lambda$	Light emitted from a point	lumen
Luminous Intensity	$I_v = \frac{\Delta F_v}{\Delta\omega}$	Light emitted from a point in a given solid angle	candela (cd)
Luminance	$L_v = \frac{\Delta I}{\Delta A_s \cos(\theta)}$	Light from an extended source in a given direction	lumens/m^2 (lux)
Illuminance	$E_v = \frac{\Delta L_v}{\Delta A_r}$	Light density incident on a plane	cd/m^2

Q = energy (joules)
F = Q/sec (watt)
t = time (sec)
ω = solid angle (steradian)
A = area (meter^2)
θ = angle incident to plane

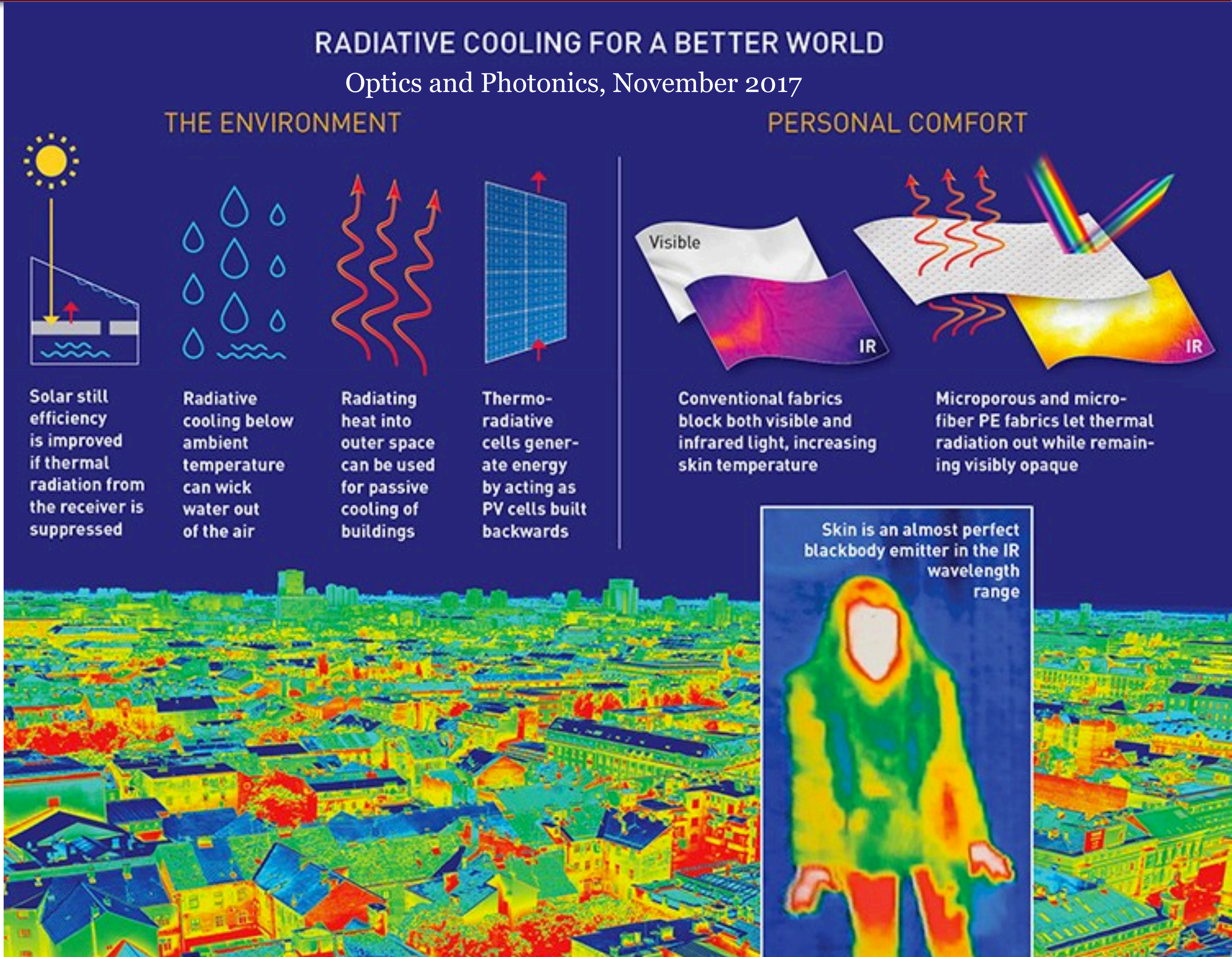
Measuring spectral radiance

Photoresearch PR-655

Spectral radiance
[photons/(s sr nm m^2)]

Understanding radiation and heat transfer is important

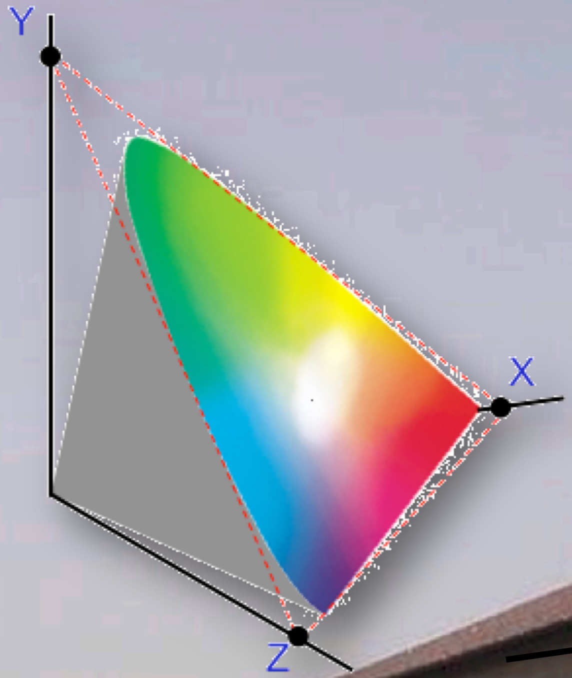
- Atmosphere is transparent to photons between 8 and 15 microns
- Such photons are coupled directly to the vast reaches of outer space (talk about a big heat sink!)
- Converting energy into this waveband has amazing opportunities from cooling to clothing



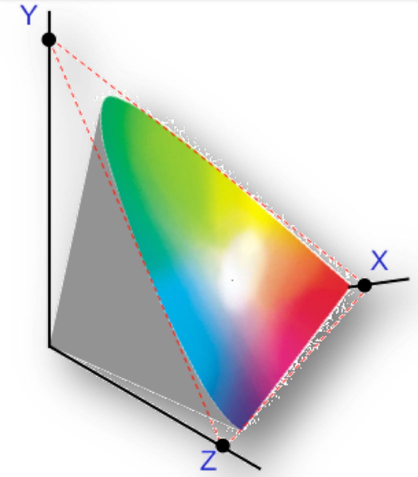
CIE chromaticity coordinates (xy)

Geometry of chromaticity coordinates

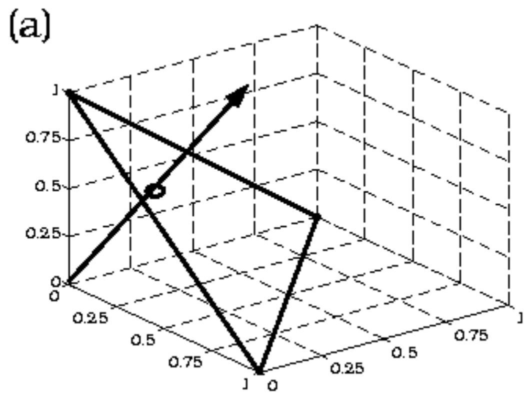
Applications



Chromaticity coordinates



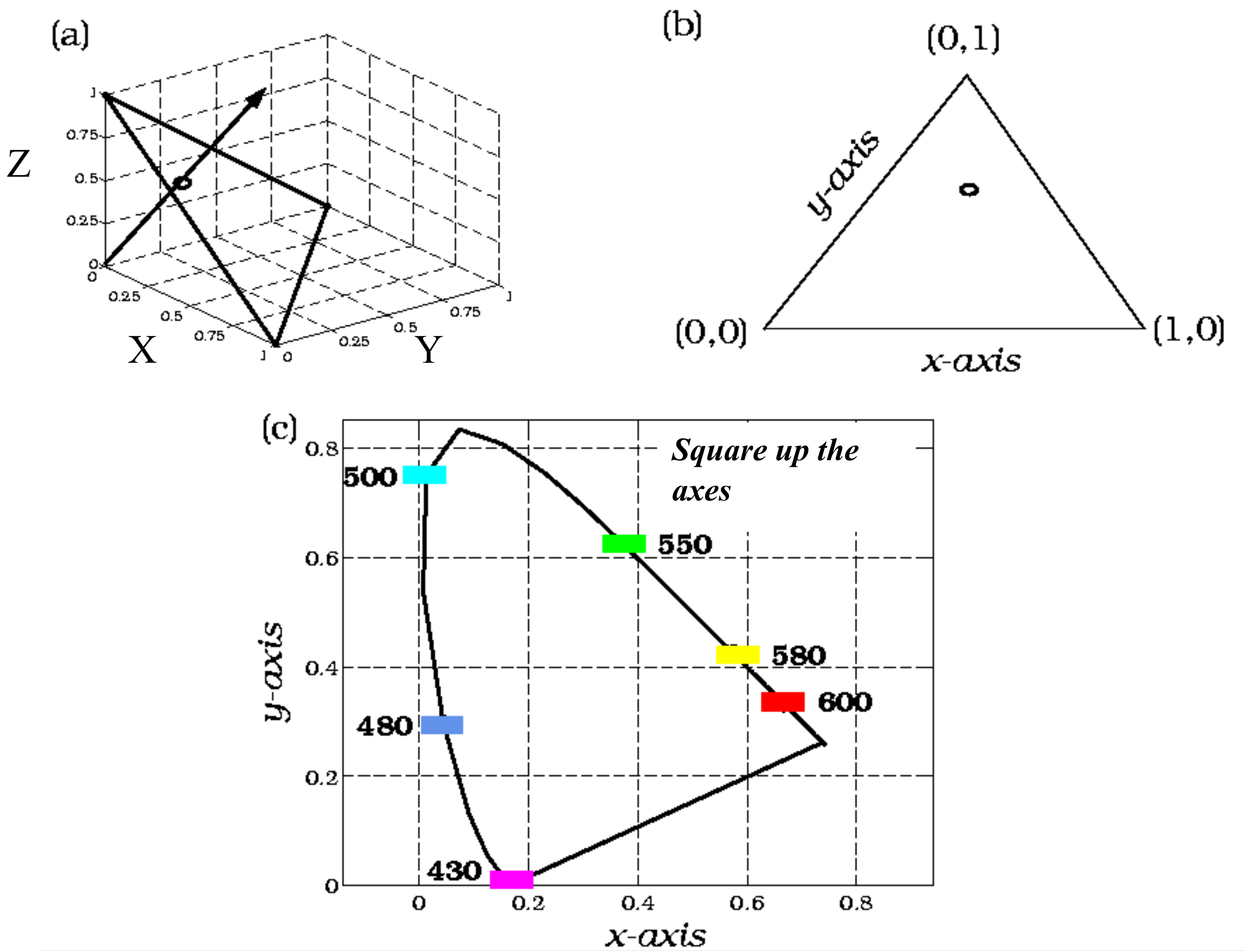
Projective transform

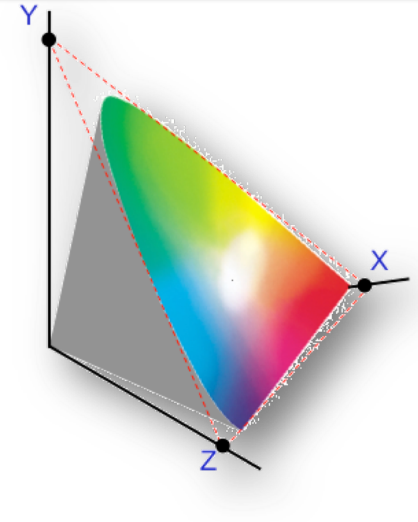


$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z}$$

When two lights differ in intensity, say
K*(X,Y,Z) and (X,Y,Z),
their chromaticity coordinates are the same

Chromaticity diagram



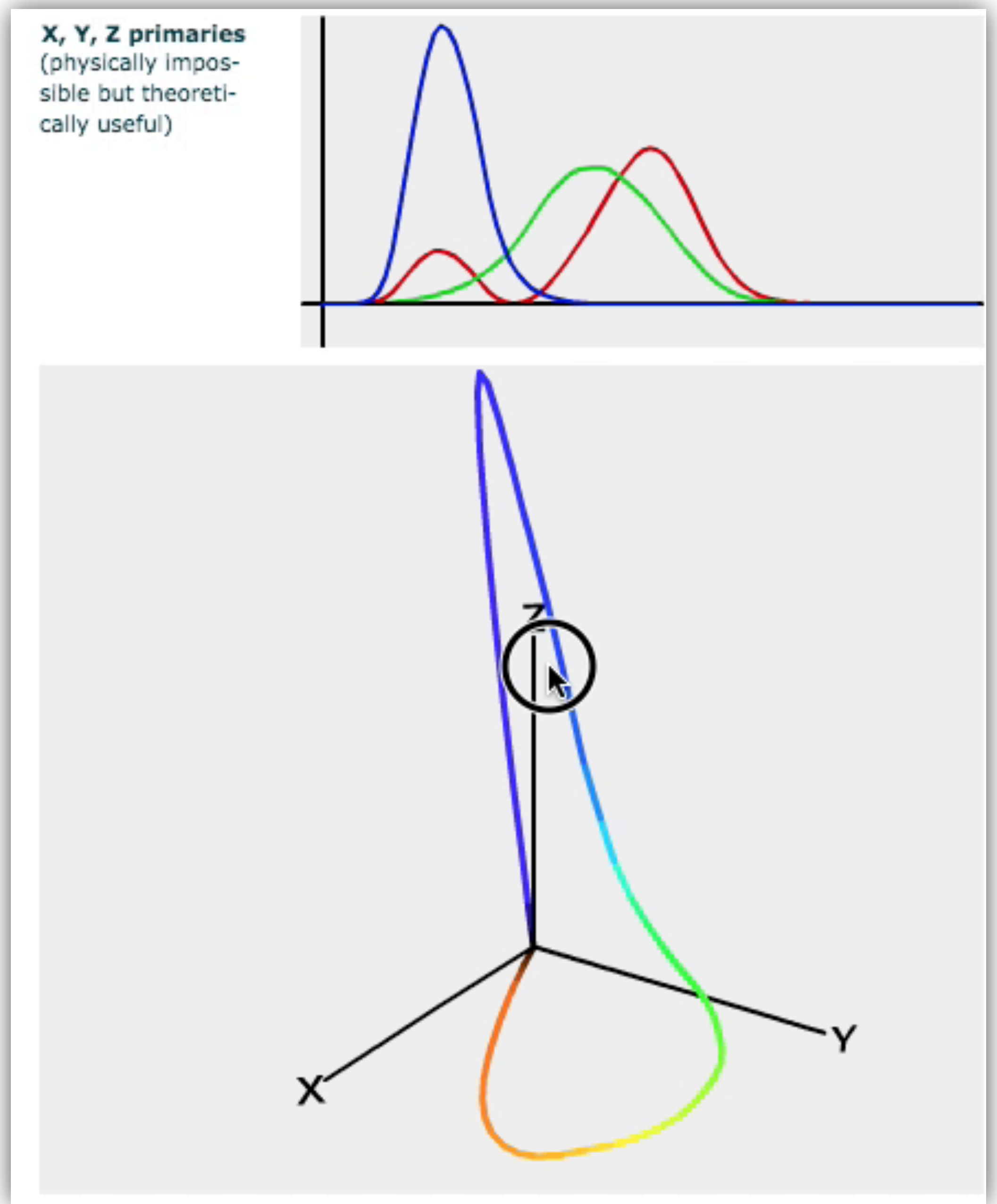


Looking down at the triangle

CIE chromaticity (xy) geometry

Chromaticity diagrams

Applet: Katie Dektar
Text: Marc Levoy
Technical assistance: Andrew Adams

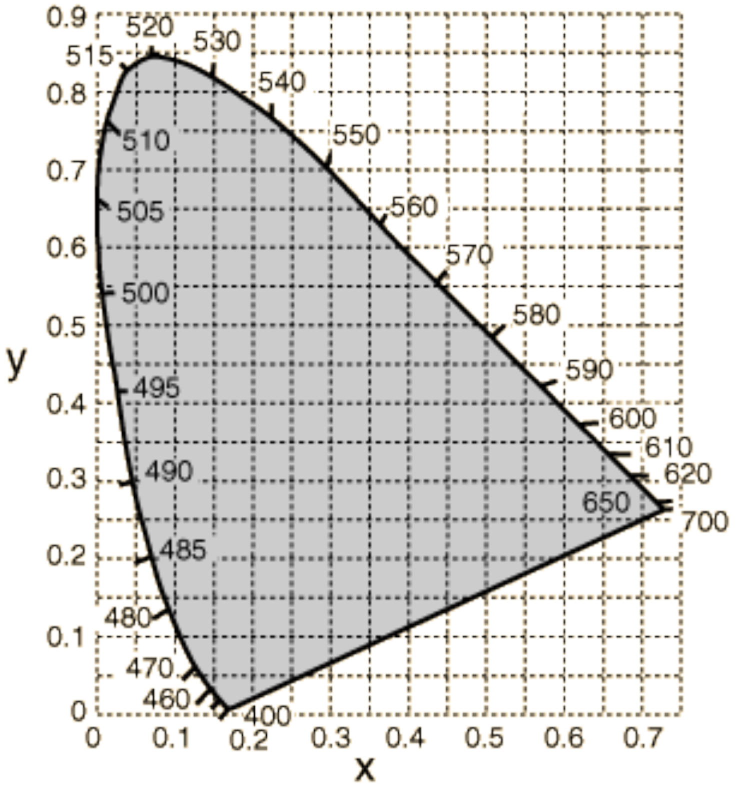


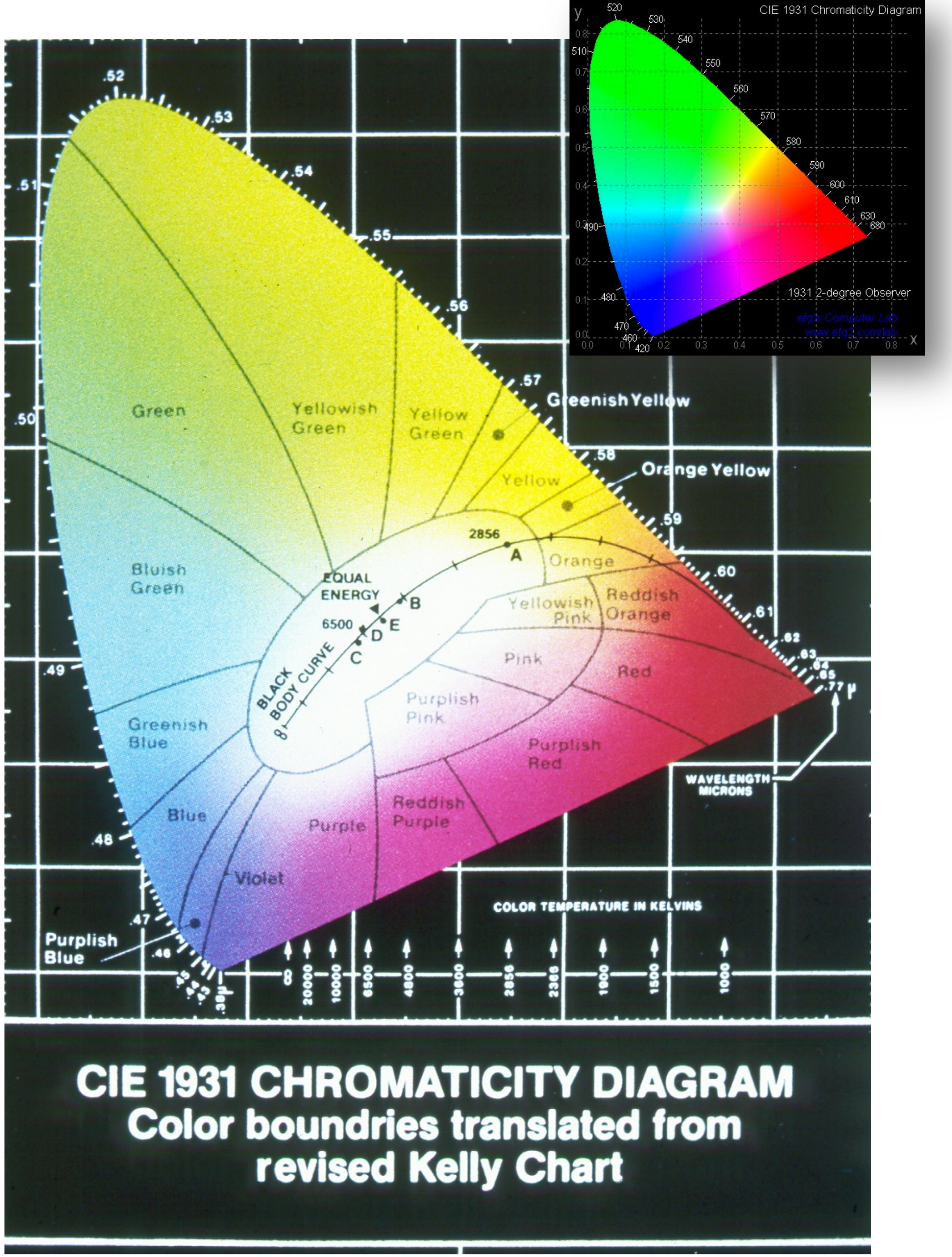
Chromaticity Diagram

Chromaticity Gamut Demo (Stanford)

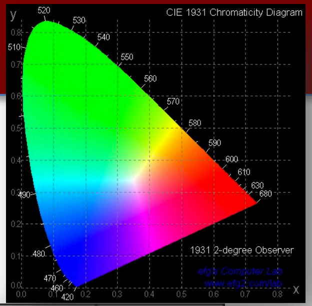
Applet: Katie Dektar
Text: Marc Levoy
Technical assistance: Andrew Adams

https://graphics.stanford.edu/courses/cs178-10/applets/threedgamut.html

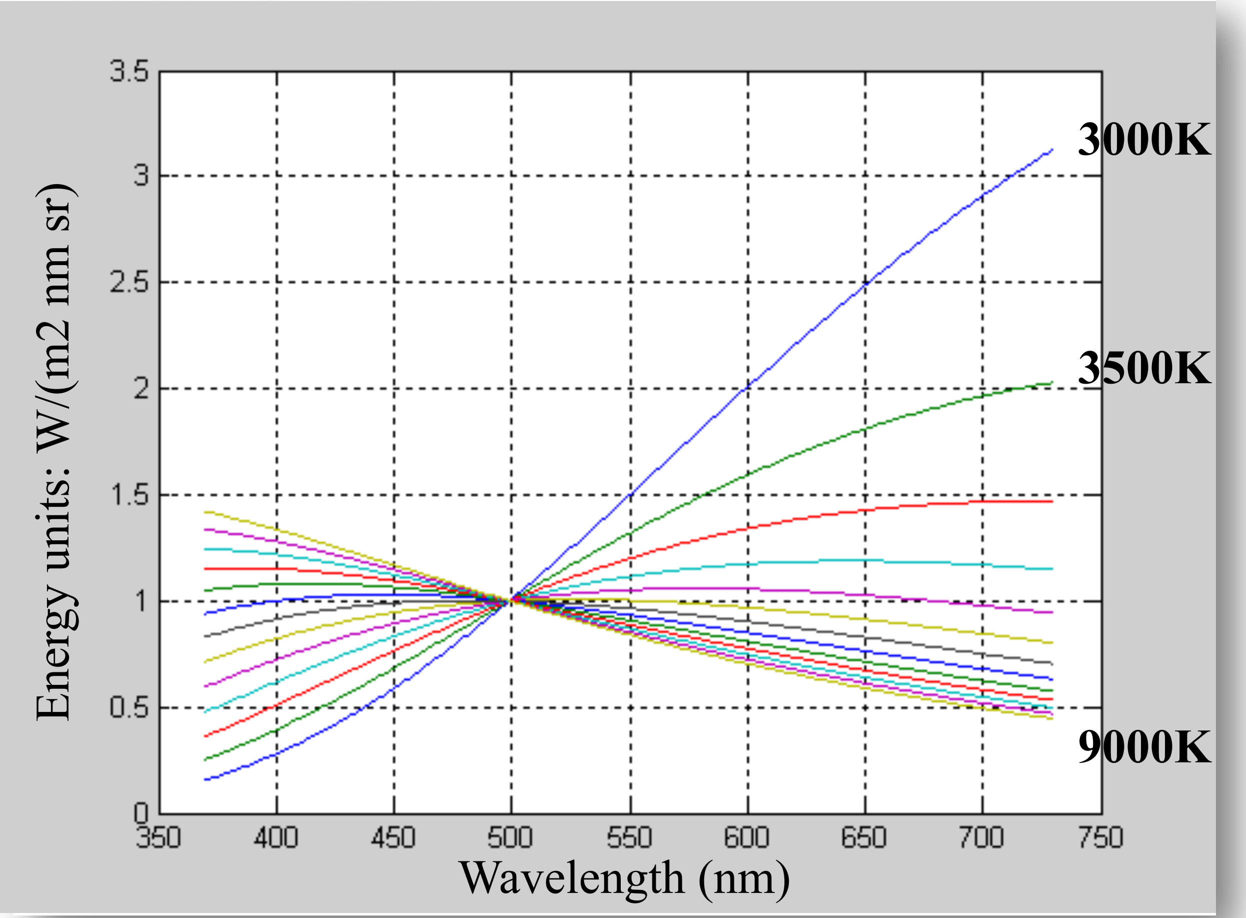


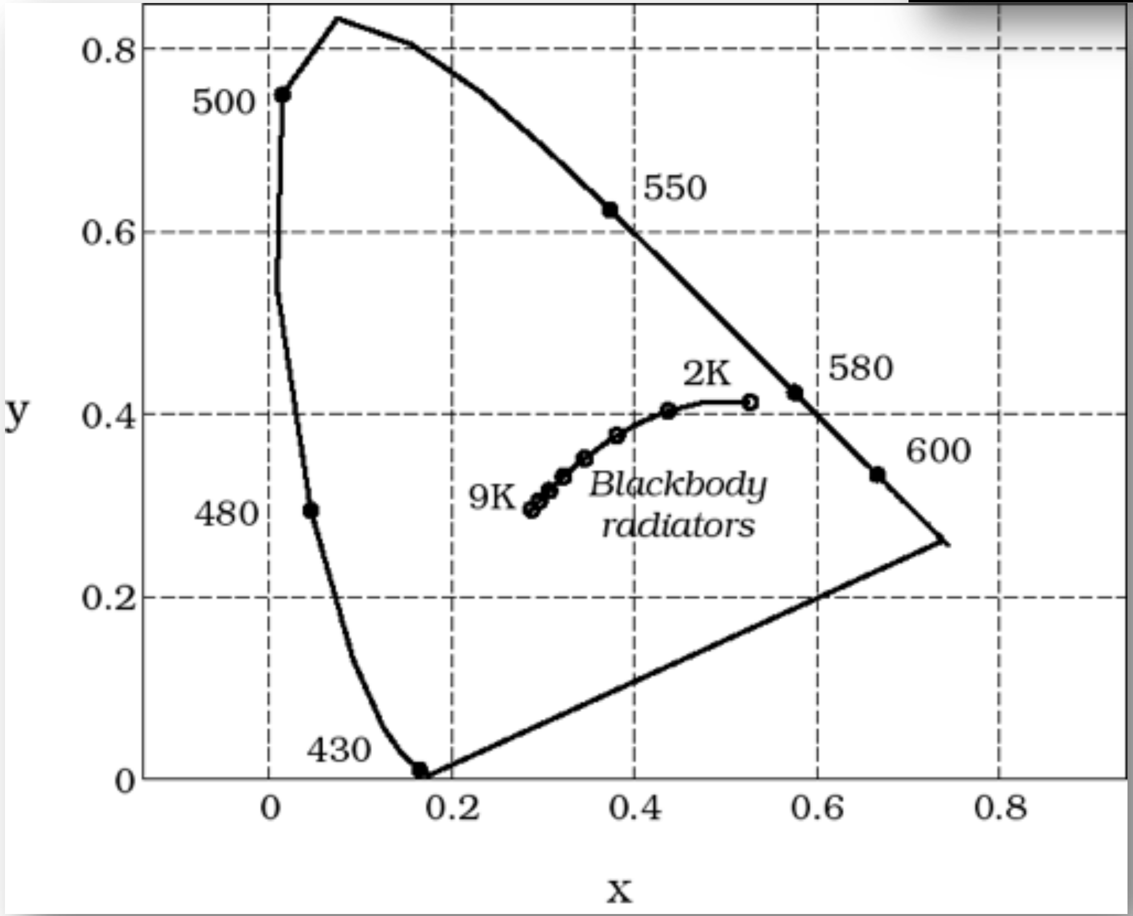


Blackbody radiators

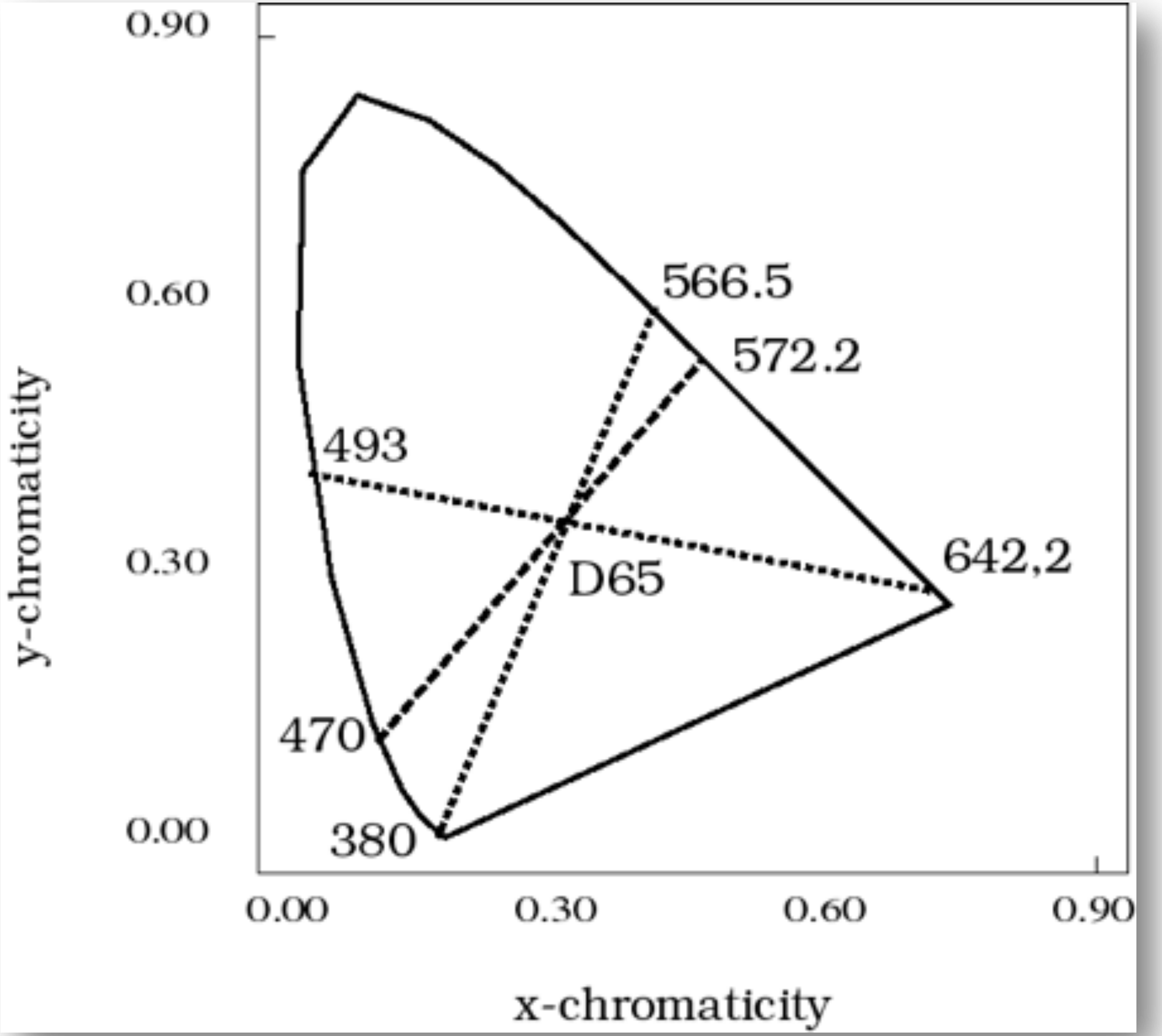


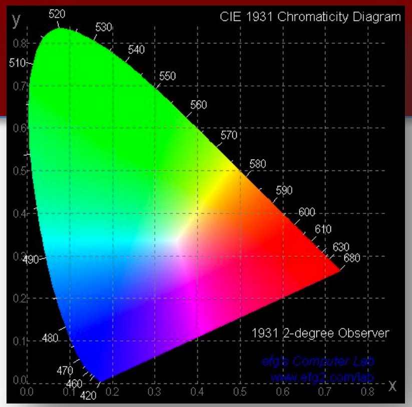
(blackbody.m)



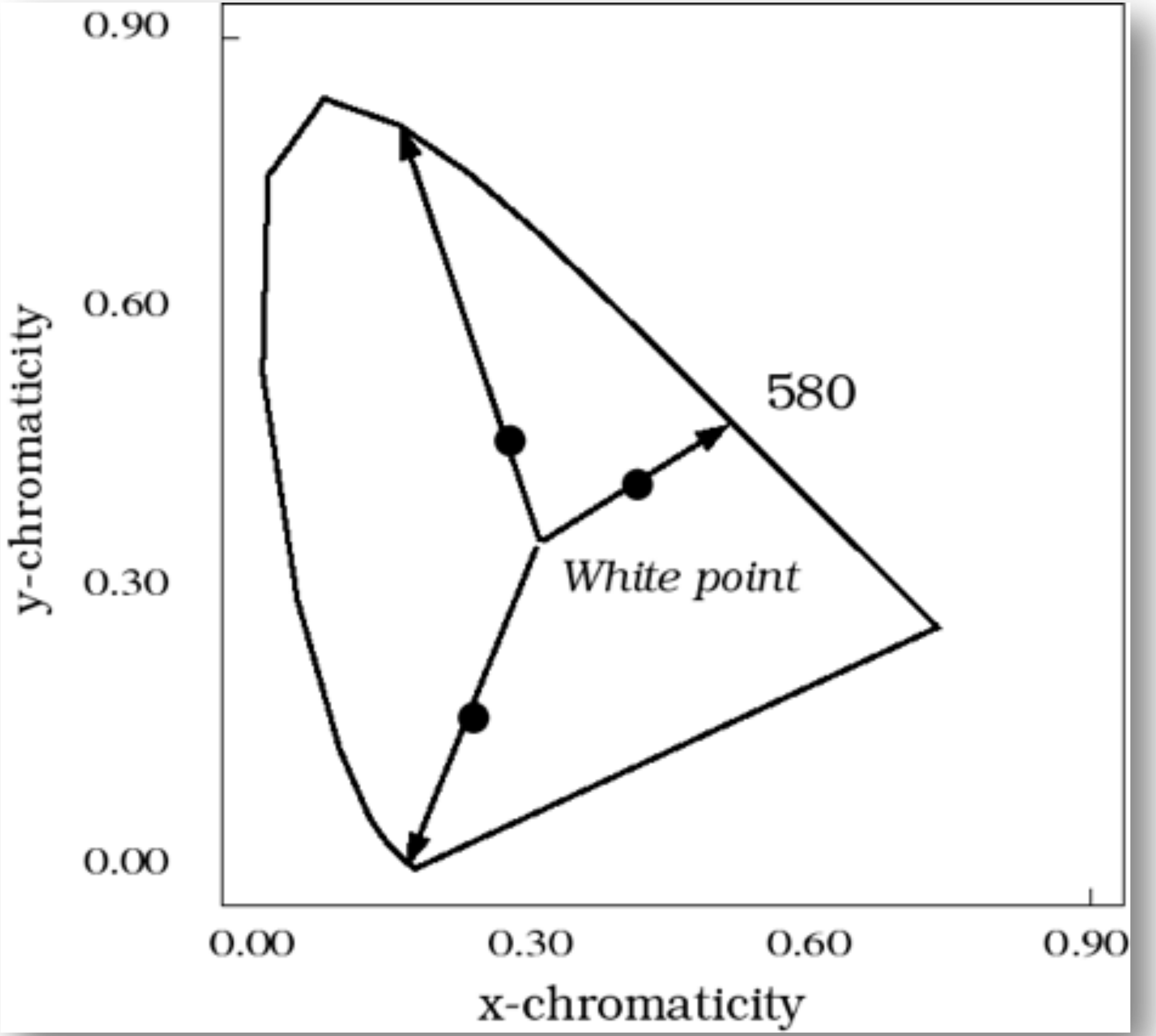


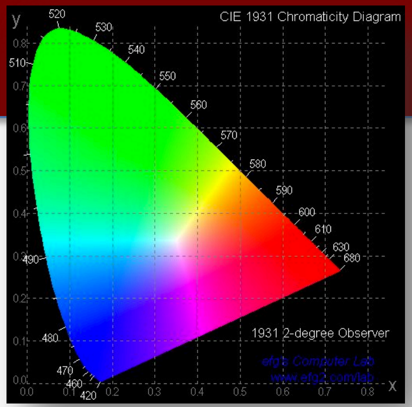
Chromaticity coordinates define complementary colors



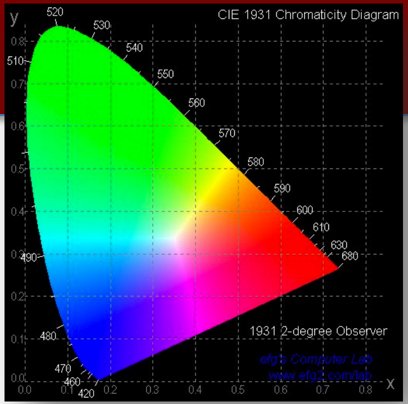


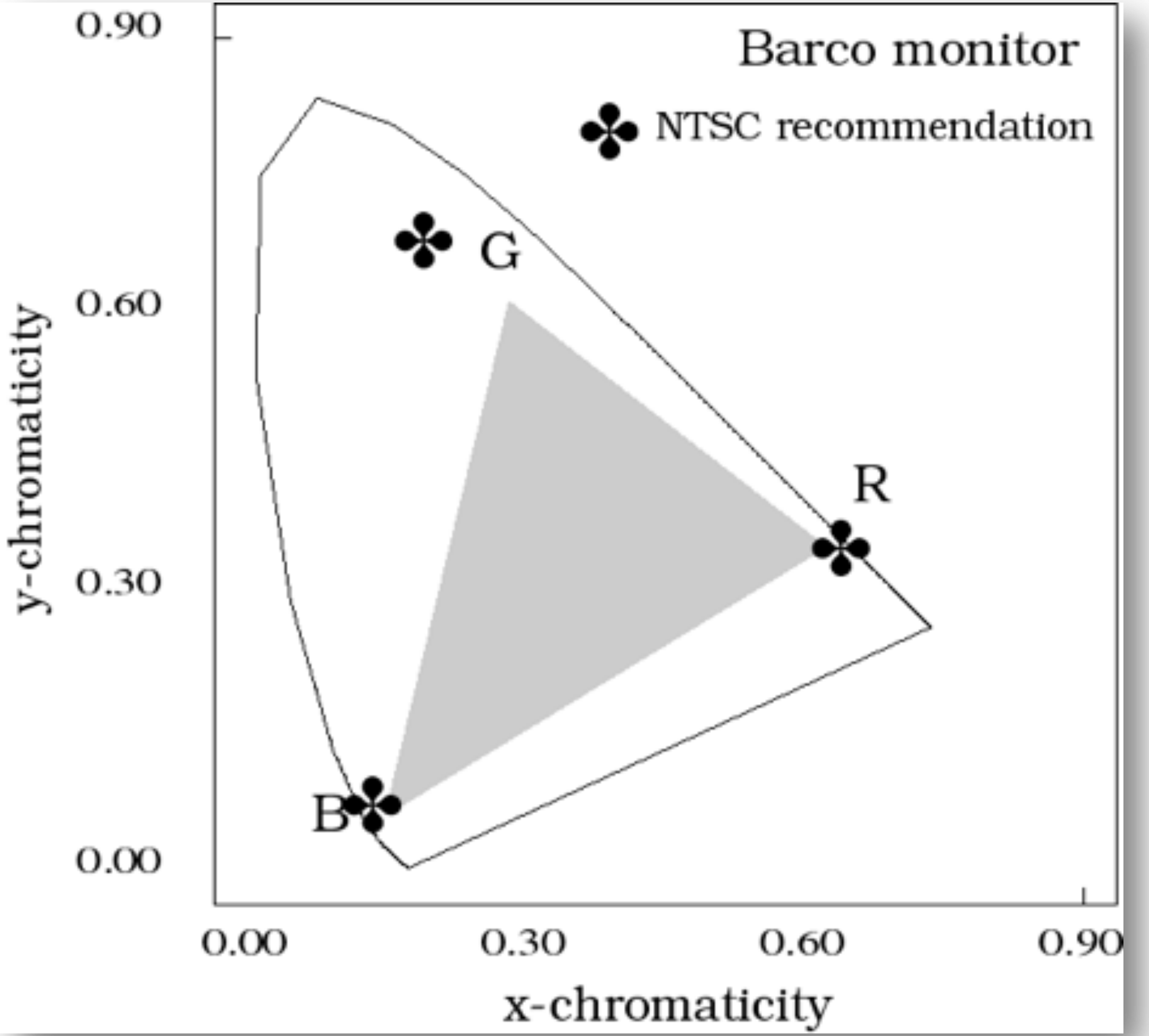
Chromaticity coordinates define dominant wavelength



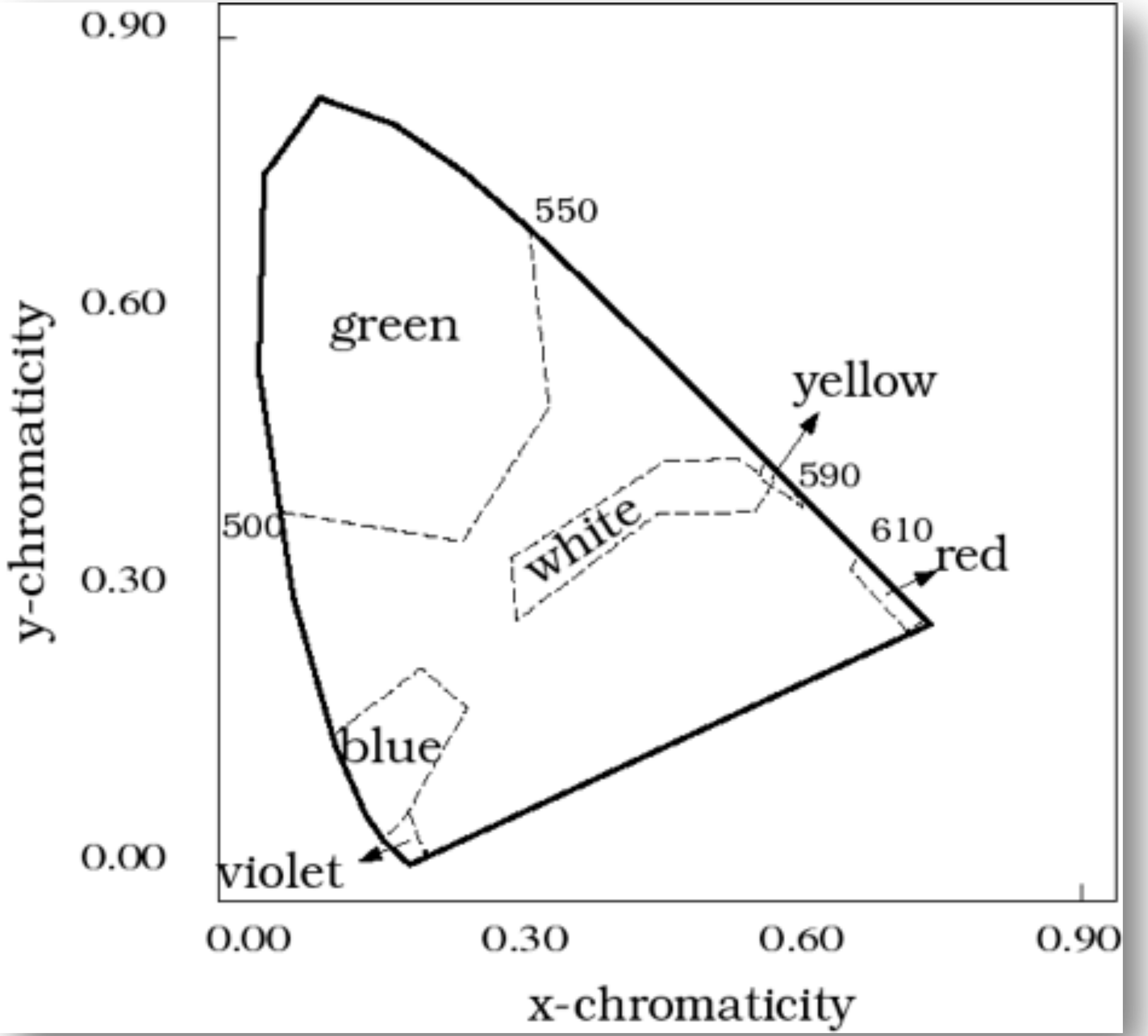


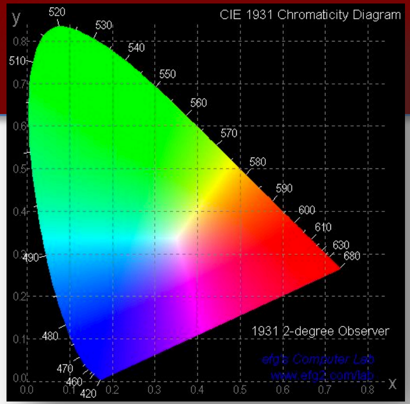
Chromaticity coordinates define display gamuts



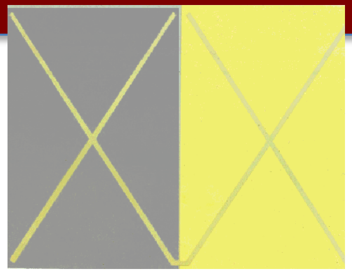


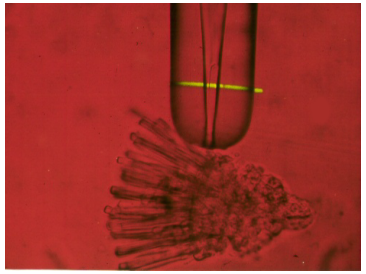
Chromaticity coordinates define signal light colors

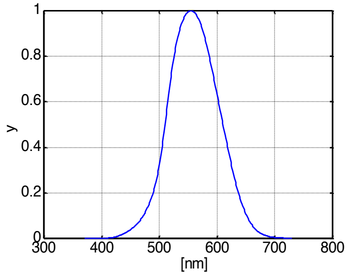


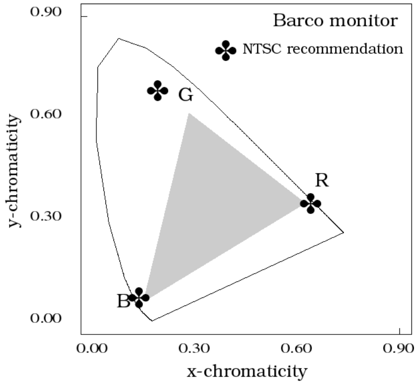


Main points







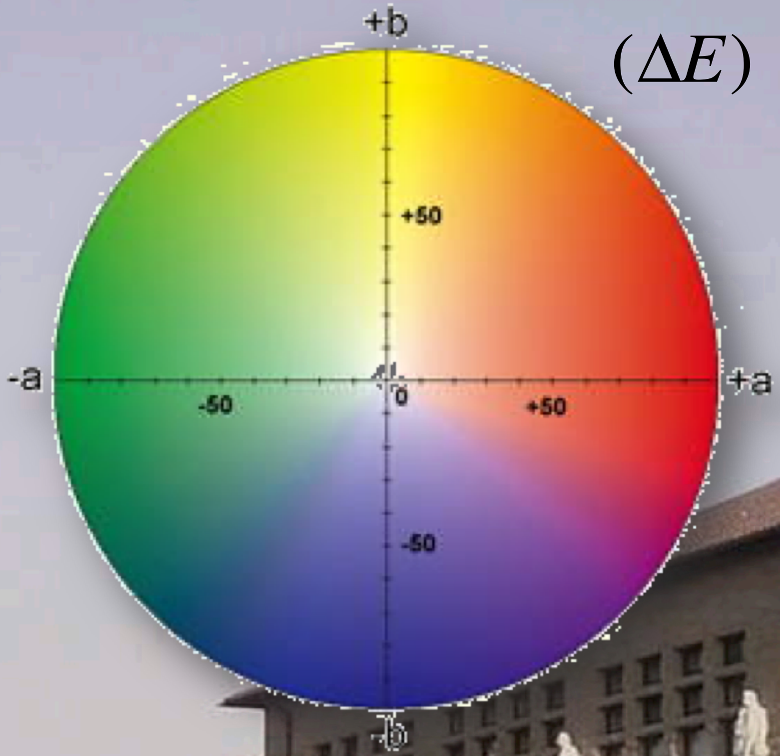


- Matching predicts matches, not appearance
- Color matching is understood profoundly
- CIE standard XYZ are color-matching standards
- Radiance and Luminance universes are connected via the photopic luminosity function

$$V(\lambda), \bar{y}(\lambda)$$

- CIE xy chromaticity coordinates are useful for some color specifications; there are other chromaticity coordinates (uv)

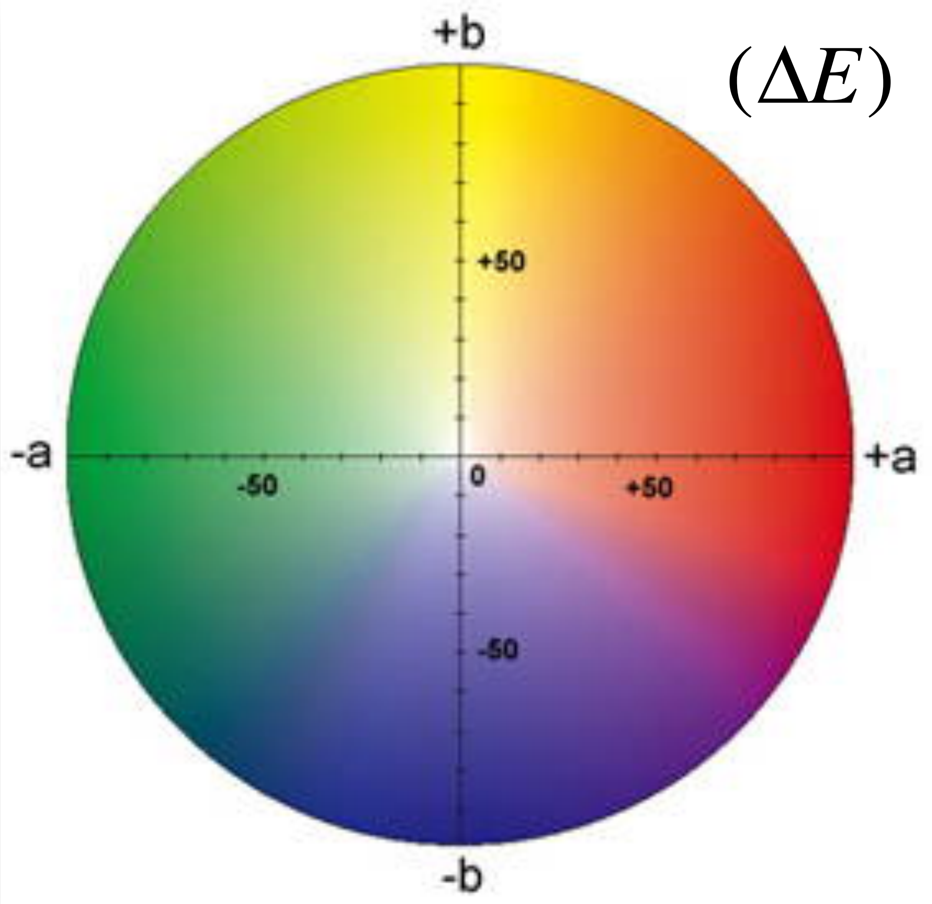
- CIELAB metric



(ΔE)

Human color metric space: CIELAB

- Manufacturers have a great deal of experience with product color (paints, textiles)
  - Consistency over time
  - Tolerance for contracts
- Early experience for uniform patches, not images; digital imaging requires additional thought



Why not use distance in XYZ (or cones) for metrics?

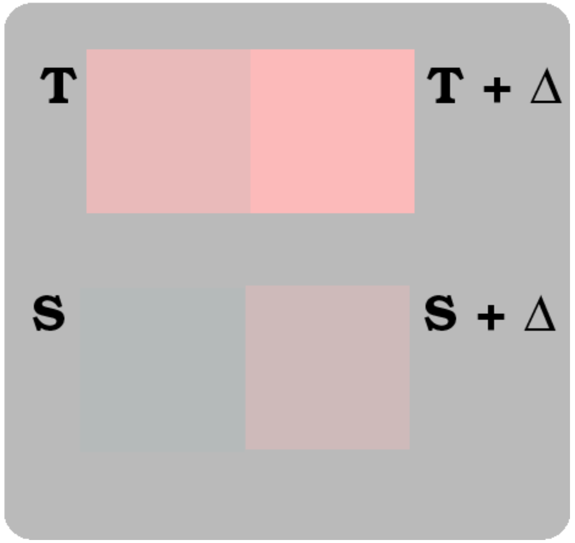
- The distance in any frame that is linear with absorptions is wrong

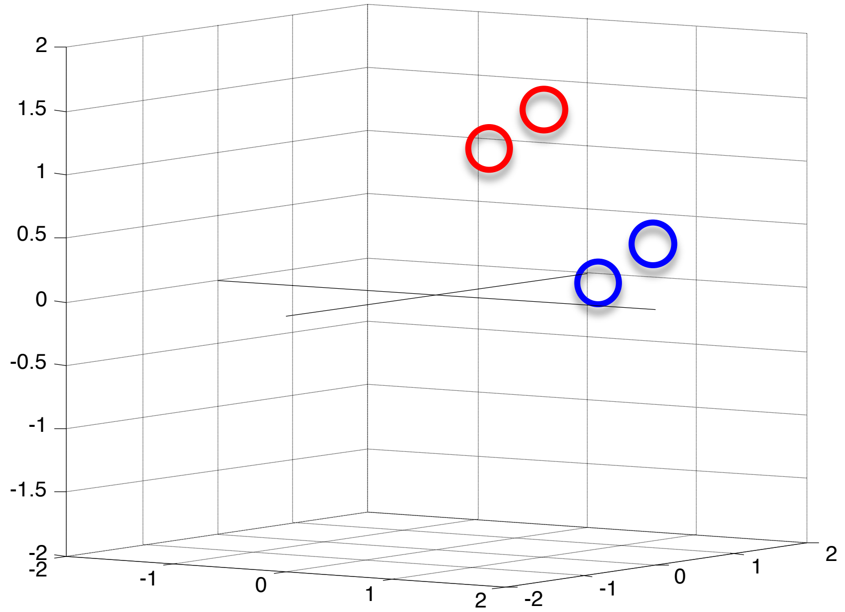
- This claim invariant of linear transformation

- The discriminability of two lights, x and x+Δ depends on x

- The distance in XYZ space depends on only Δ

Example





What factors limit discriminability?

Human wavelength discrimination

The cone absorptions are stochastic, following a Poisson distribution. Hence, the number of absorptions differs from trial-to-trial. When we calculate the information availble to discriminate two lights, we must take into account not just the difference in mean but also the statistical difference between the cone absorptions.

This script calculates the information available in the cone absorptions to discriminate two small, uniform patches at two different wavelengths (520 and 530 nm). We calculate the discriminability as we increase the mean intensity of the two signals.

This script shows the distribution of the L,M and S absorptions caused by two stimuli, and it also shows how the distributions vary as the mean stimulus intensity level increases.

```
ieInit
```

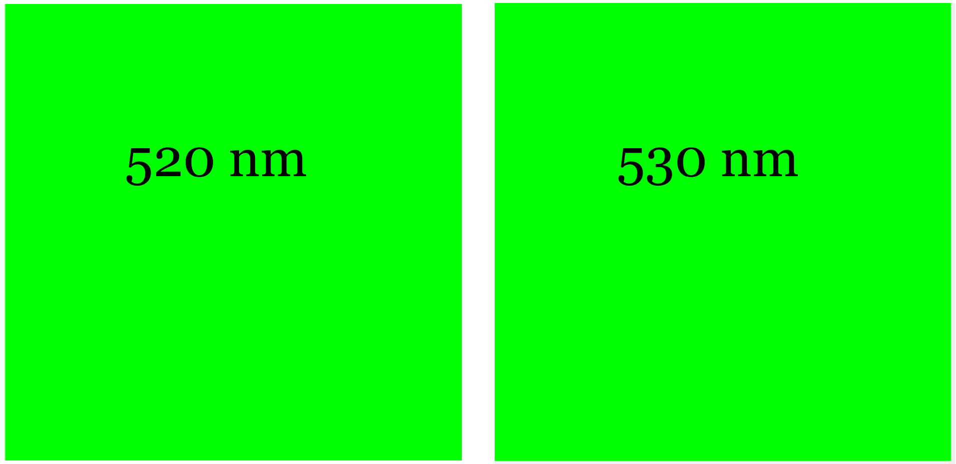
Define the two stimuli

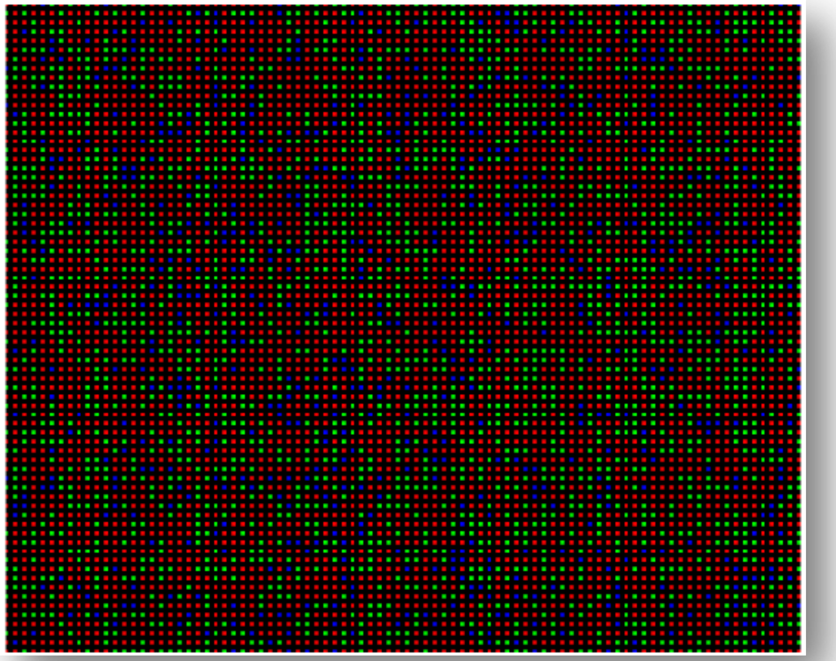
We image uniform fields of monochrome light on a human sensor. These plots are made for two wavelengths and three scene intensity levels

```
wSamples  = [520  530];  nWave = length(wSamples);
luminance = [10 50 200]; nLevels = length(luminance);
sceneSize = 128;

% Make a scene for each of the wavelengths.
scene  = cell(1,nWave);

% We extract the cone absorptions for
% plotting into these variables
L = cell(1,length(wSamples));
M = cell(1,length(wSamples));
S = cell(1,length(wSamples));
```





Stochastic cone absorptions and discriminability

Create human optics and cone mosaic

The lens has an impact on the light transmitted to the retina. The macular pigment and cone photopigments are also important. This cell creates a standard human optics and a typical human cone mosaic.

```
% Human optics
oi = oiCreate;

% Cone mosaic for 100 ms
cMosaic = coneMosaic;
cMosaic.integrationTime = 0.10;

% This is the mosaic
cMosaic.plot('cone mosaic');
```

Calculate and plot the cone absorptions as 3D scatter plot

We make three graphs, one for each intensity level. Notice how the groups of points from the two different wavelengths separate as the scene intensity level increases.

Also, notice how few S-cone absorptions there are compared to the L and M cone absorptions.

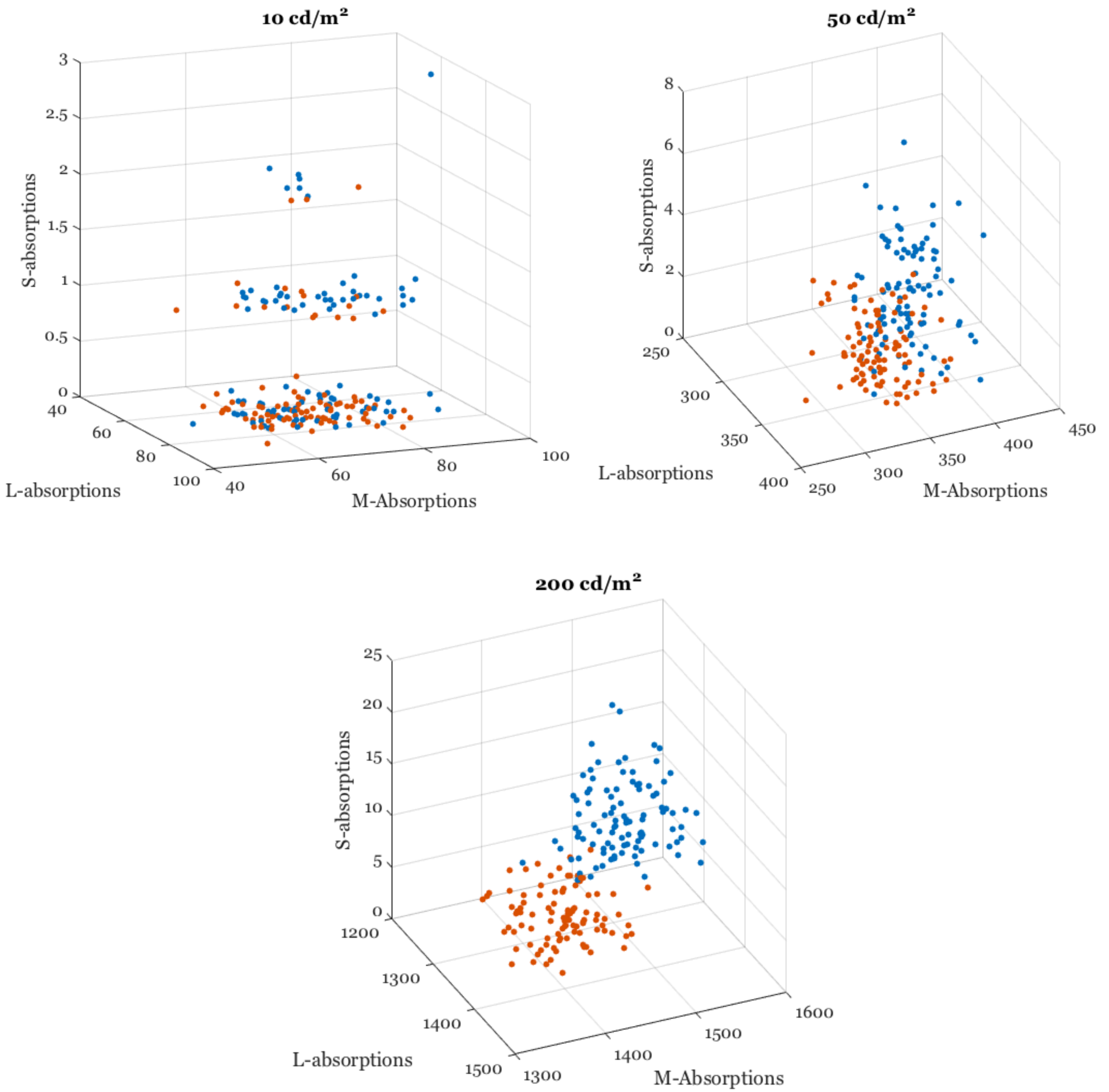
```
for rr = 1:nLevels  % For each luminance level a new graph
    vcNewGraphWin;

    for ww=1:length(wSamples)  % For the two wavelength samples

        % Create a monochromatic scene and set the luminance
        scene{ww} = sceneCreate('uniform monochromatic',wSamples(ww),sceneSize);
        scene{ww} = sceneAdjustLuminance(scene{ww},luminance(rr));

        % Compute the retinal spectral irradiance
        oi = oiCompute(scene{ww},oi);

        % Compute the cone absorptions
        cMosaic.compute(oi);
```

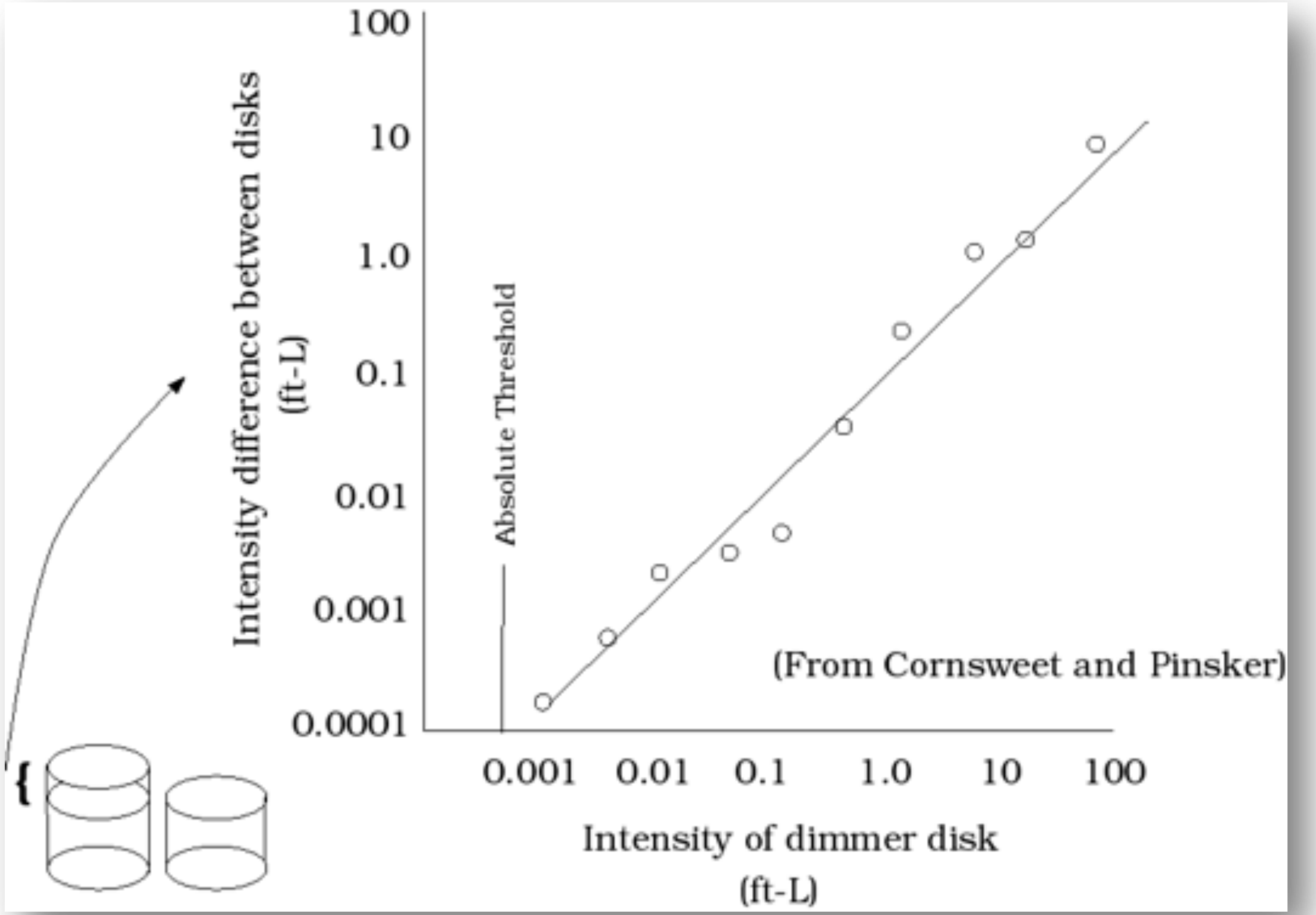


Human color metric space

Weber-Fechner law

• If we could use XYZ units, the threshold line would be a constant, rather than growing in proportion to the background

• The CIE defines a transformation that maps the XYZ units by a roughly logarithmic transformation so that the difference is approximately constant

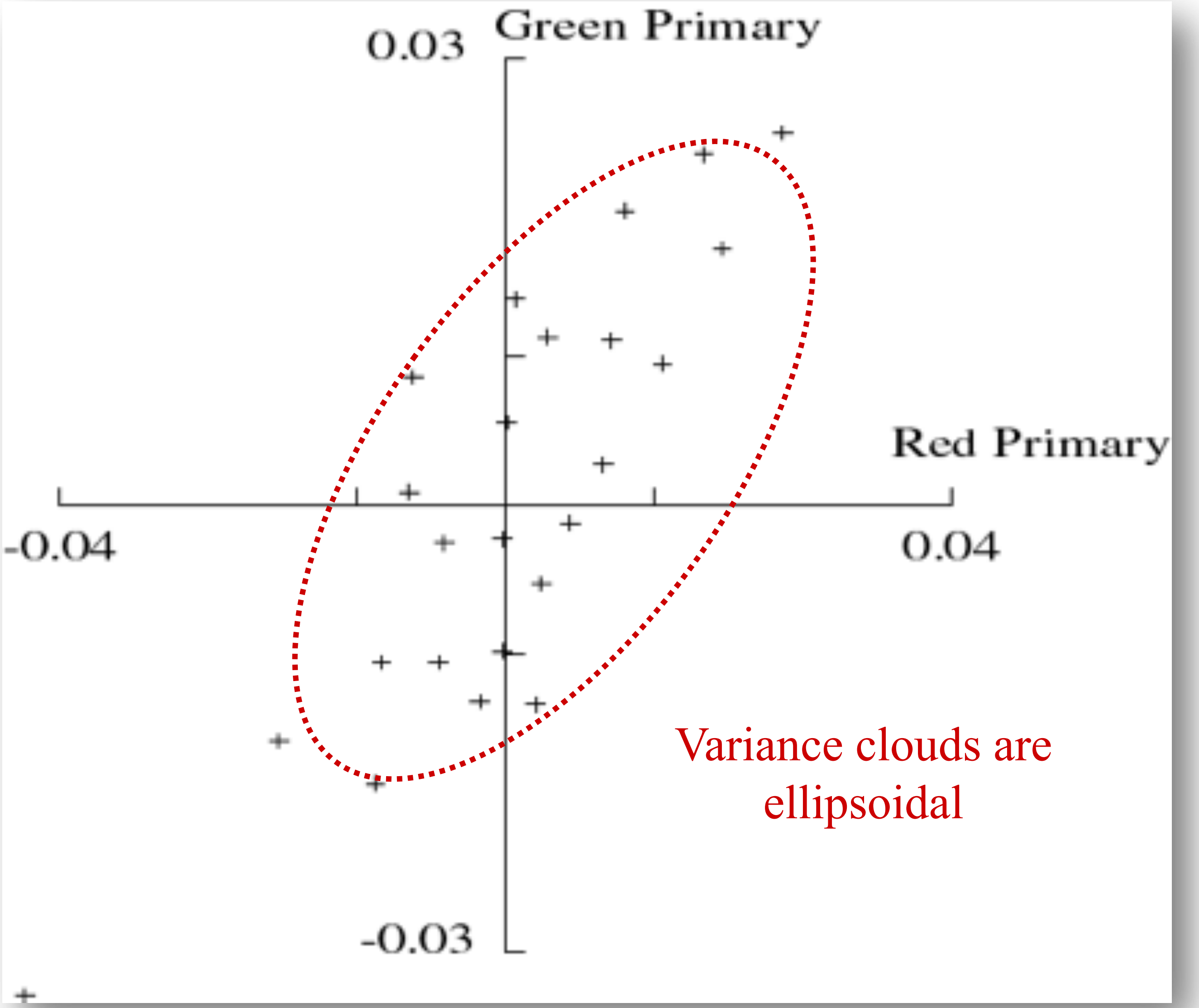


Human color metric space

- Origin is the target color; axes are difference from target color; each point is an additional match by the participant

- **Solution**: In a metric space, these ellipses would be circles.

- The **CIE** defines a nonlinear transform on XYZ into a space where these ellipses are more circular

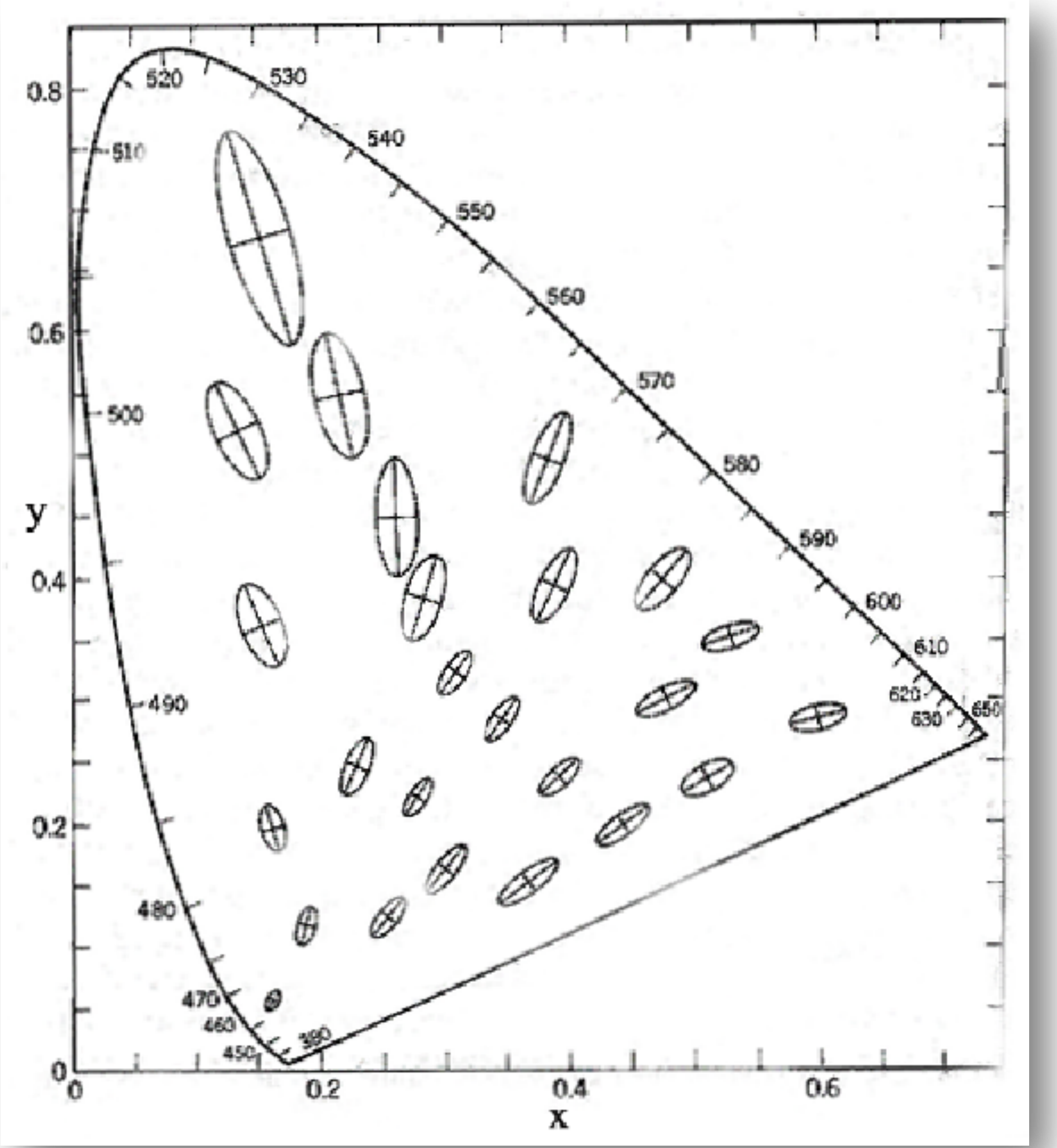


Macadam's Ellipses/Ellipsoids

- Ellipses shown at x10 threshold

- Kodak made 3D measurements

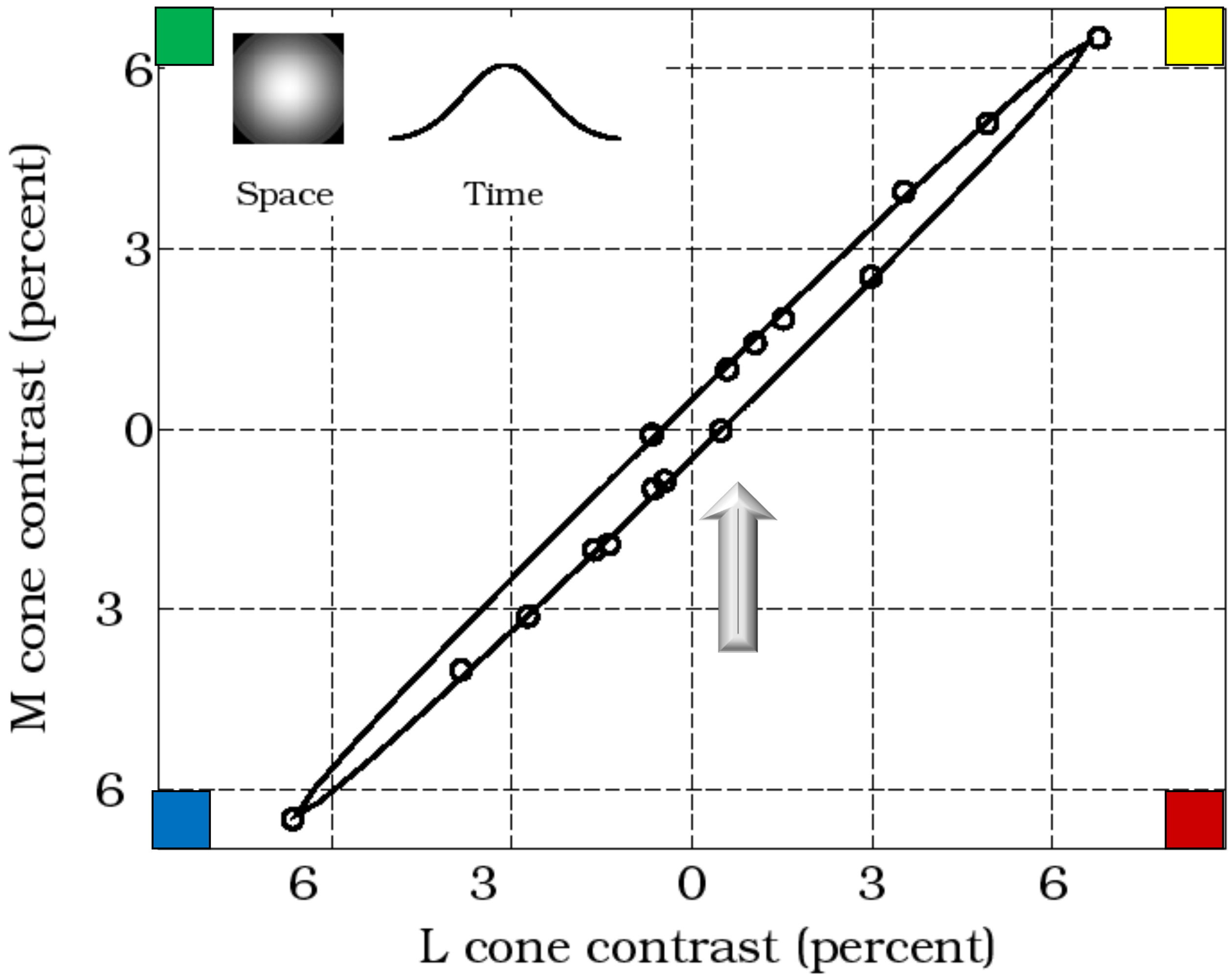
- Circularizing the ellipses is one of several metric constraints



Thresholds – relationship to biology (L- and M-cones)

(Wandell, JOSA, 1986)

- The variance in color-matching settings is consistent with thresholds
- Ellipsoid shape depends on space-time of stimulus
- No single formula for all patterns or mean levels
- **Note** the remarkable cancellation of L and M signals

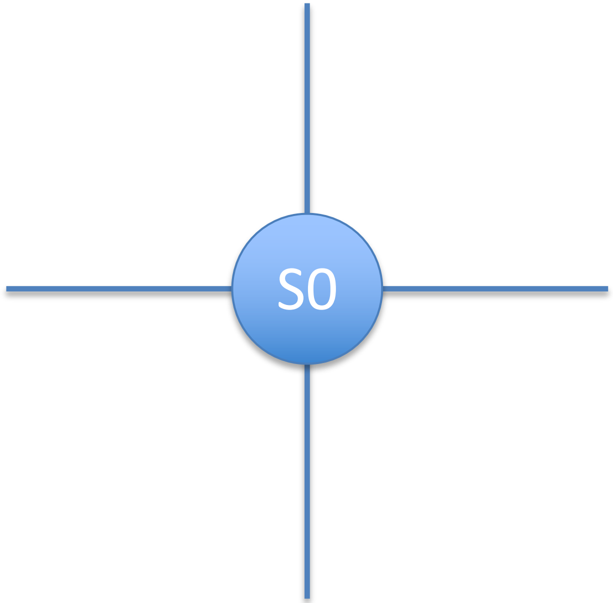


Other behavioral measurements of perceptual distance

Multi-dimensional scaling

- Similarity judgments of color pairs
  - Build a distance matrix
  - Check for consistency
  - Derive a perceptually uniform representation
- Numerical scaling
  - Provide a numerical judgment
  - Steven's Power Law

	S0	**S1**	**S2**
S0	–	D01	D02
S1		–	D12
S2			–



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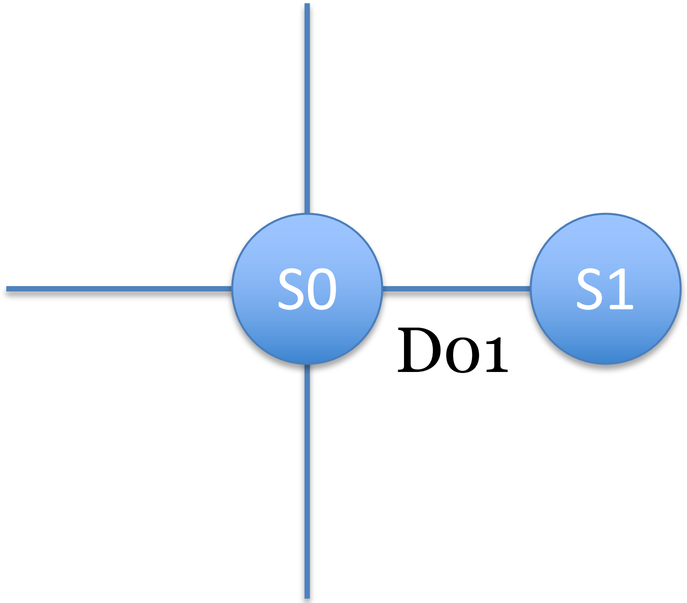

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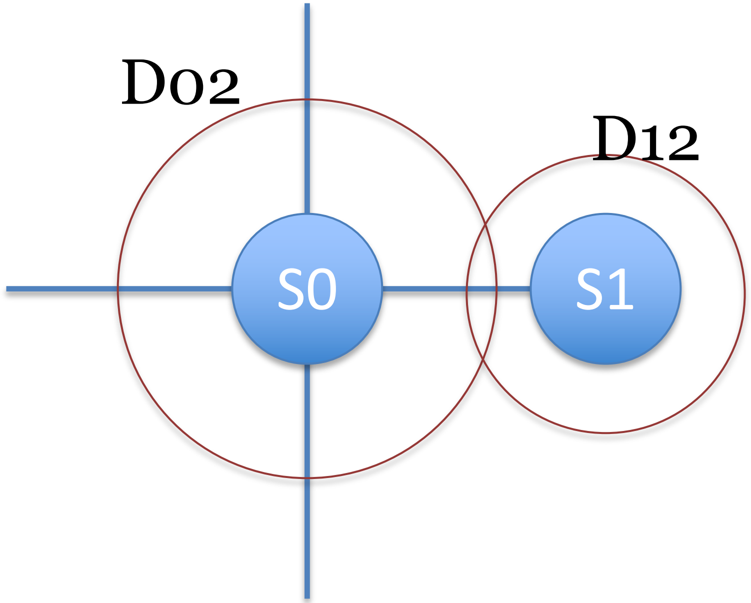

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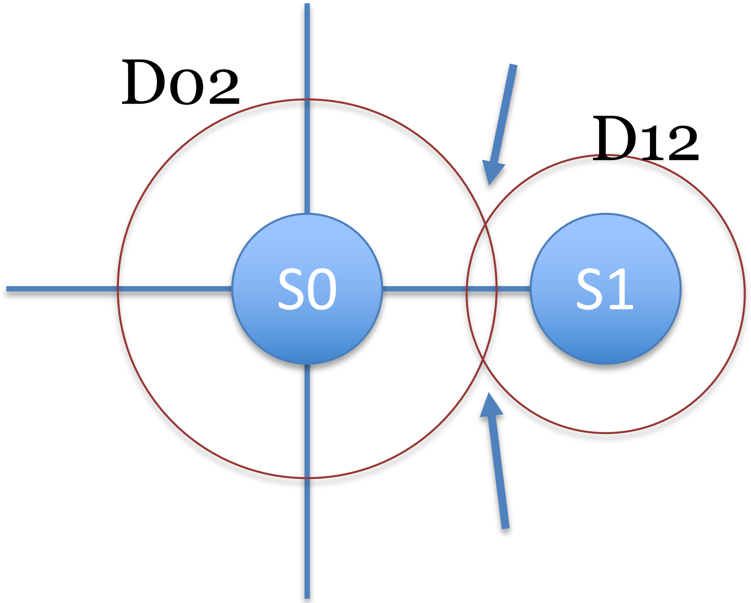
S2			_



CIELAB metric

t_metrics*

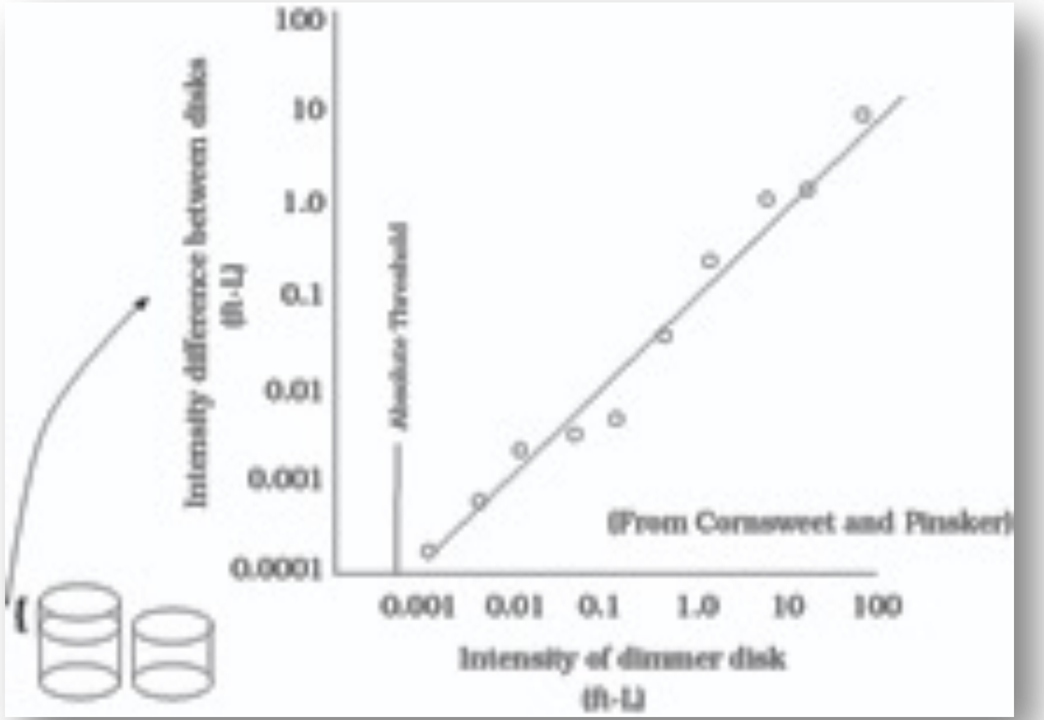
- CIE Lightness (1976)
- CIELAB

CIE Lightness

Cube-root of the **relative** luminance

$$L^* = 116\left(\frac{Y}{Y_w}\right)^{1/3} - 16, \quad if\ \frac{Y}{Y_w} > .00856$$

$$L^* = 903.3\left(\frac{Y}{Y_w}\right), \quad otherwise$$



Y = CIE luminance

Y_n = CIE luminance of a white surface in the same viewing conditions

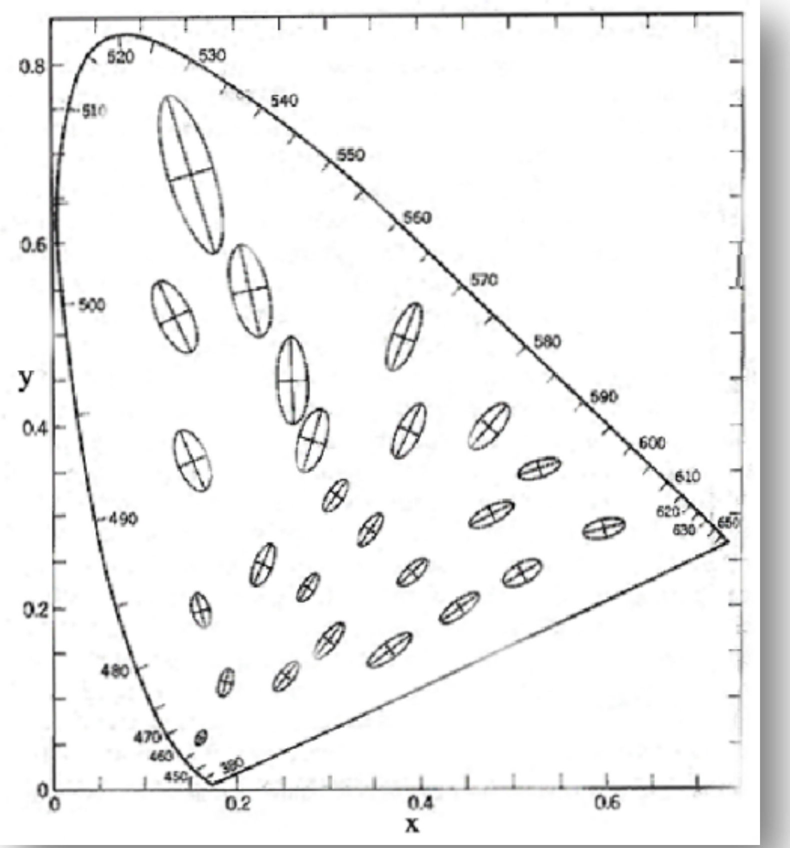
CIELAB chrominance

Cube-root of the **relative** values

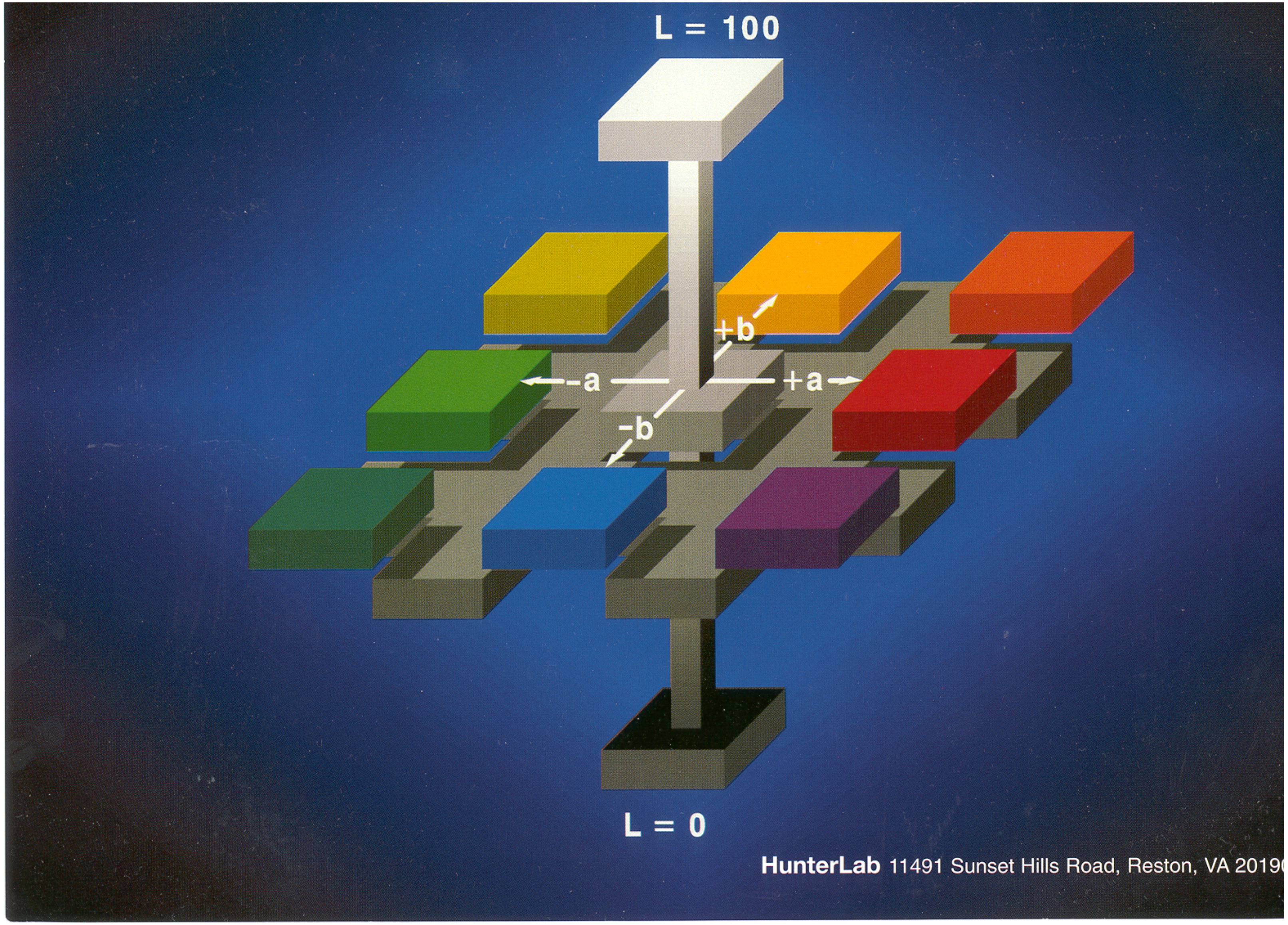
$$a^* = 500\left\{\left(\frac{X}{X_w}\right)^{1/3} - \left(\frac{Y}{Y_w}\right)^{1/3}\right\}$$ (red-green)

$$b^* = 200\left\{\left(\frac{Y}{Y_w}\right)^{1/3} - \left(\frac{Z}{Z_w}\right)^{1/3}\right\}$$ (blue-yellow)

Red-green and blue-yellow chrominance representations



CIELAB color solid

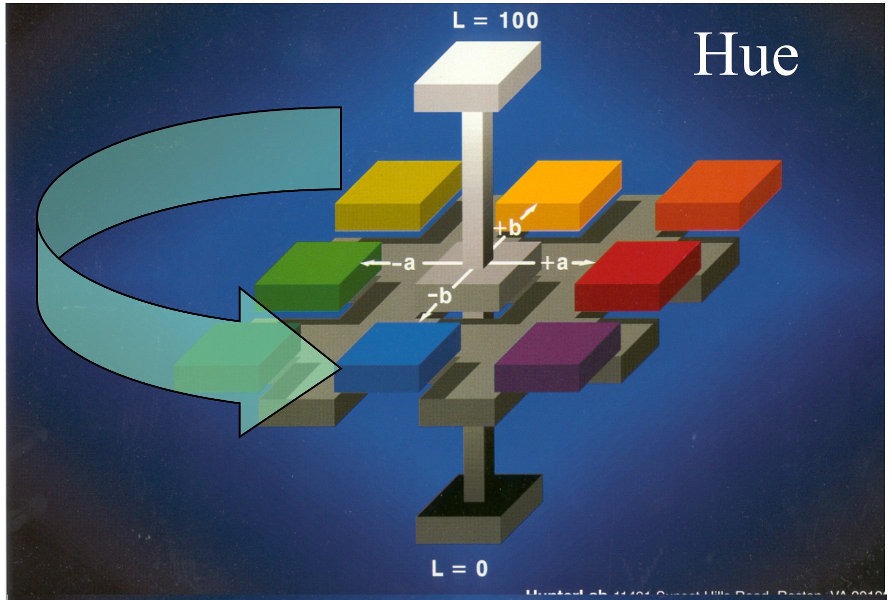


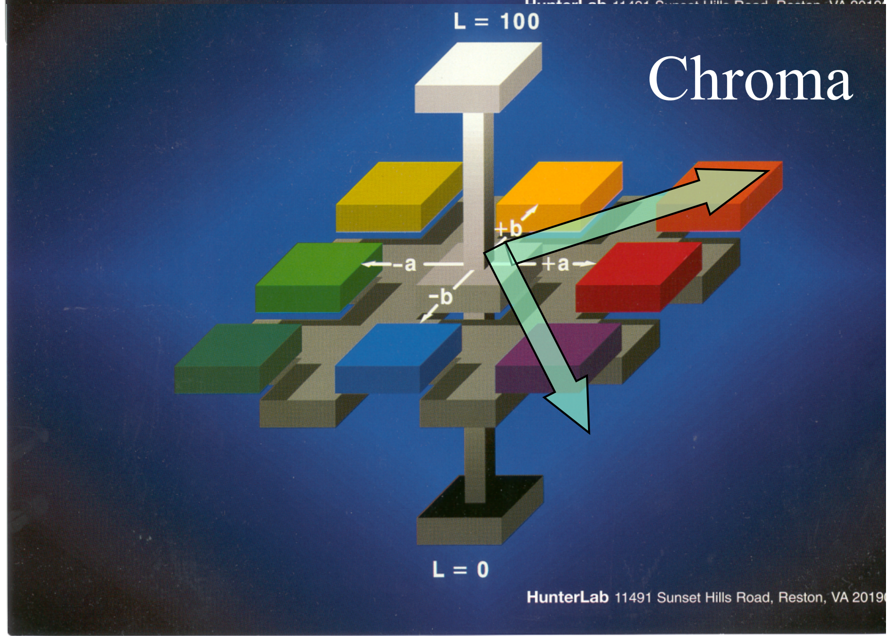
CIELAB derived quantities

- Distance between two colors designed to predict color difference visibility

$$\Delta E_{ab} = \left(\left(\Delta L^*\right)^2 + \left(\Delta a^*\right)^2 + \left(\Delta b^*\right)^2 \right)^{1/2}$$

- CIE 2000 replaces this formula with a computer program





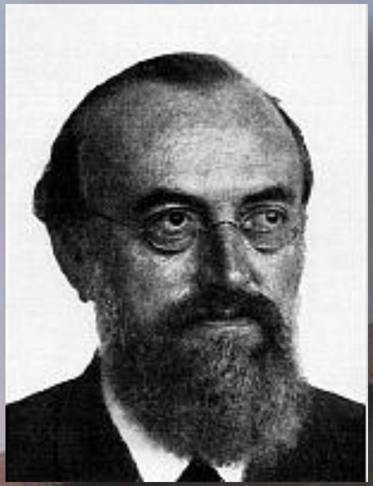
<u>Hue and chroma terms</u>

$$h_{ab} = \arctan(b^* / a^*)$$

$$C^*_{ab} = (a^{*2} + b^{*2})^{1/2}$$

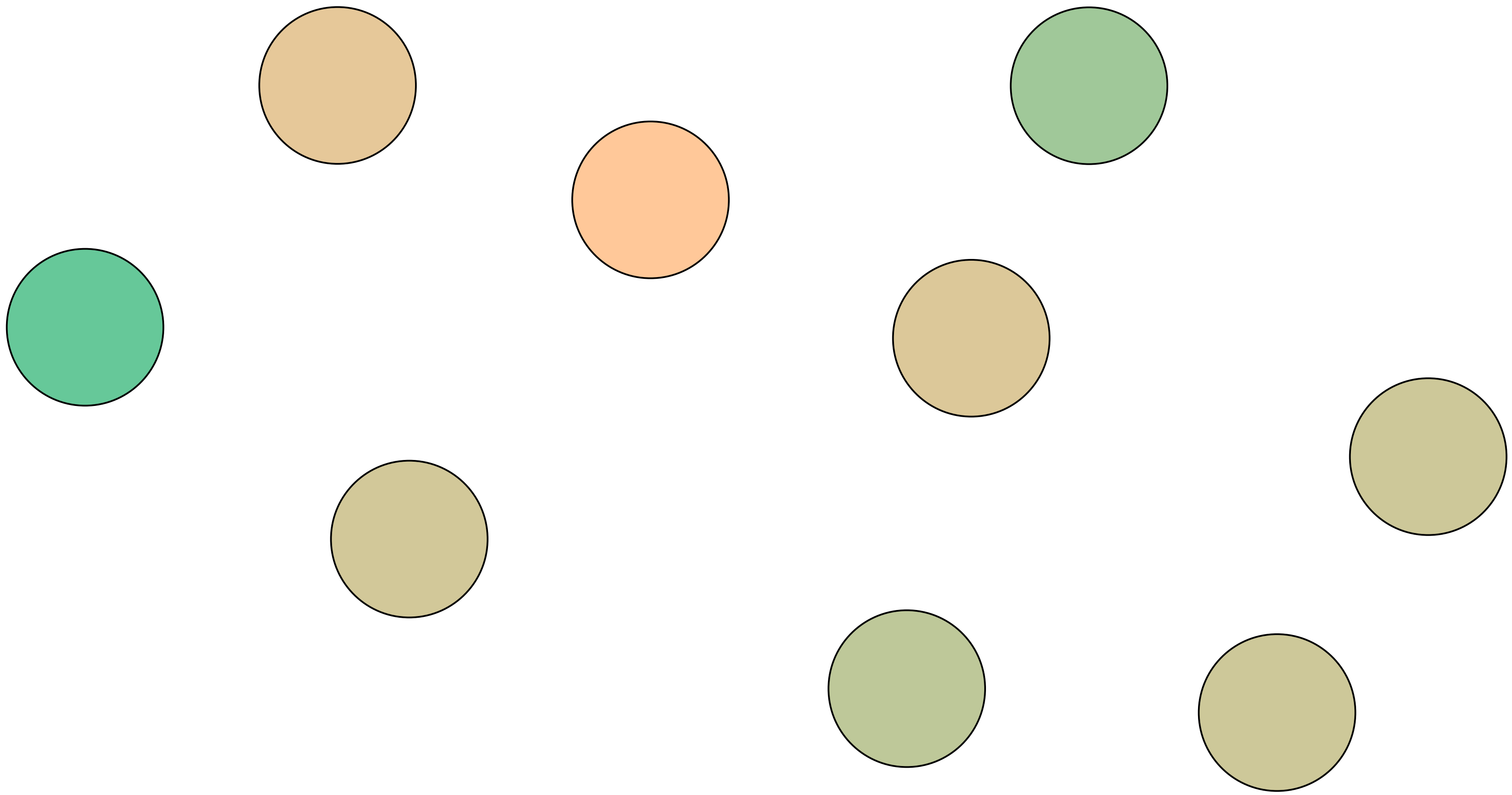
Opponent colors

``that modern tendency in sensory physiology, which has found its most acute expression in the Physiological Optics of Helmholtz, is not leading us to the truth, and whoever wishes to open up new avenues of research in this area, must first free himself from the theories which now prevail."



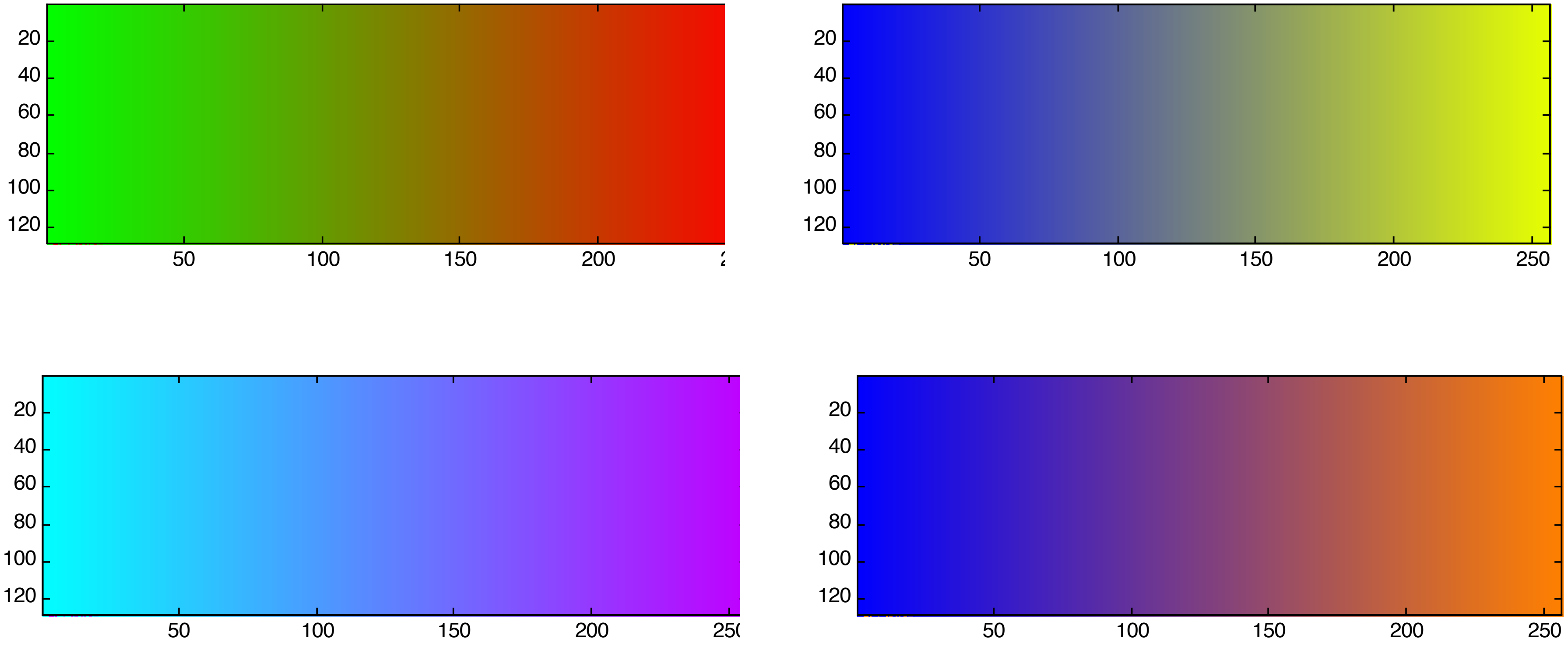
Karl Ewald Hering

Red/Green examples

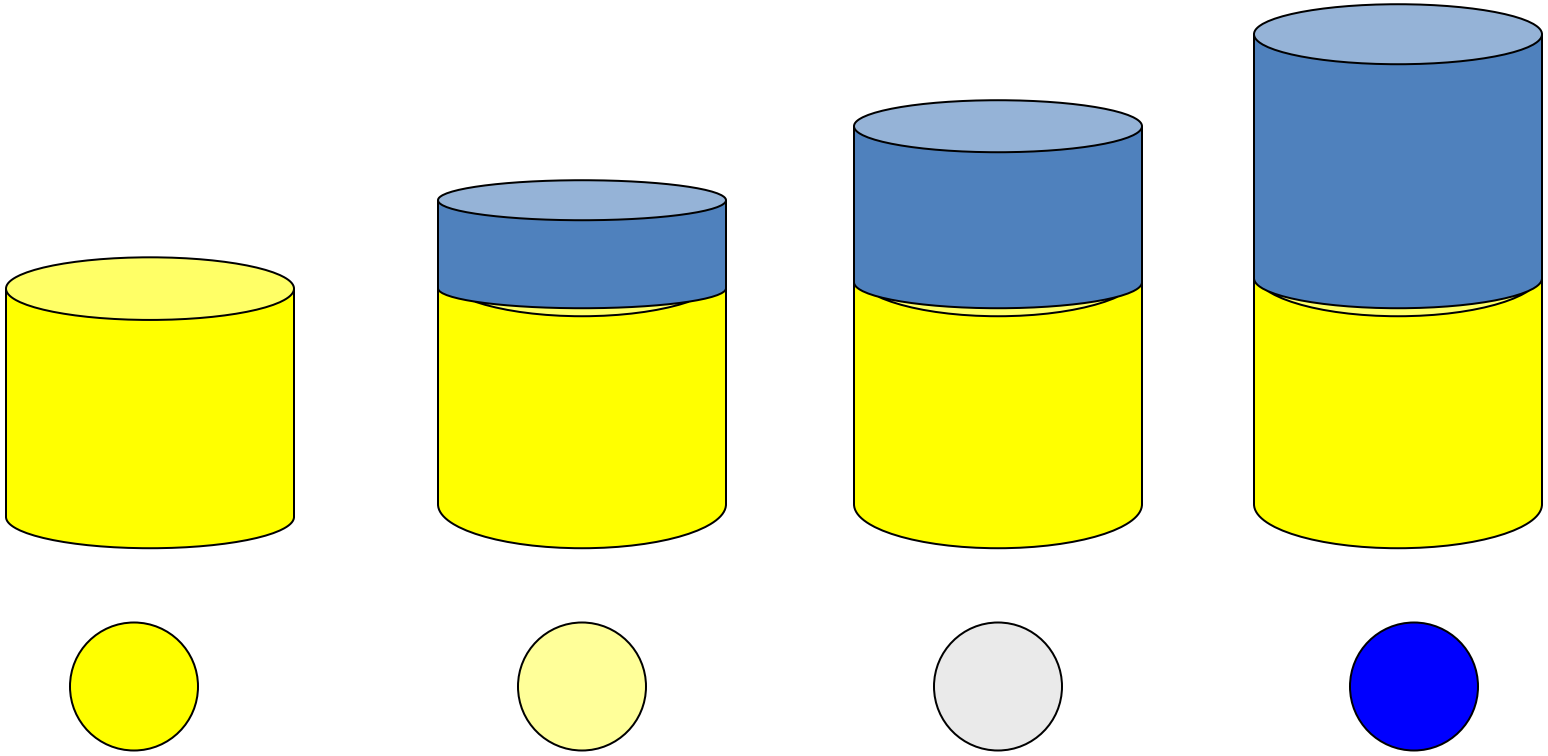


Opponent colors

Red, green, yellow, blue



Hue cancellation experiment



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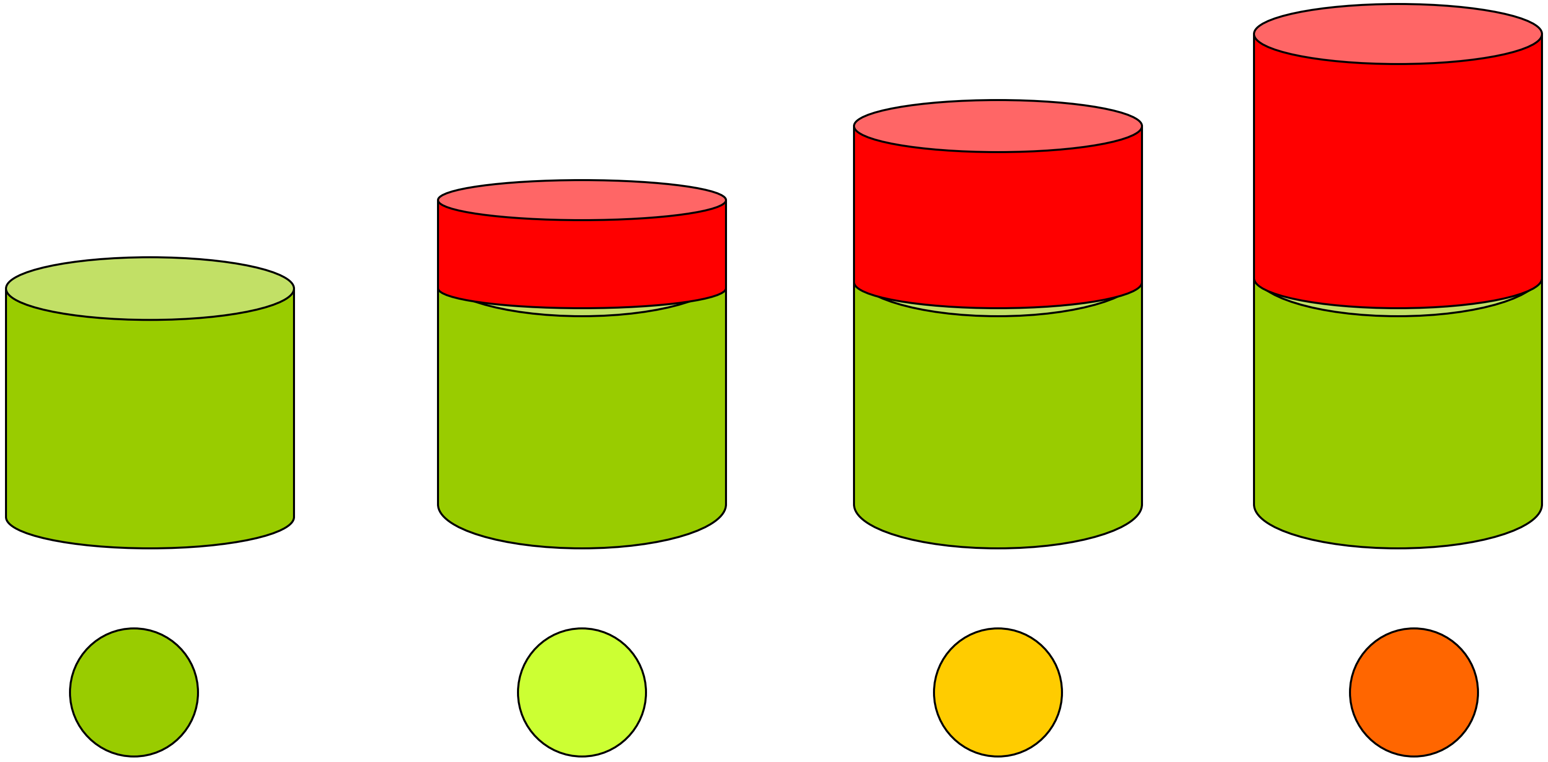
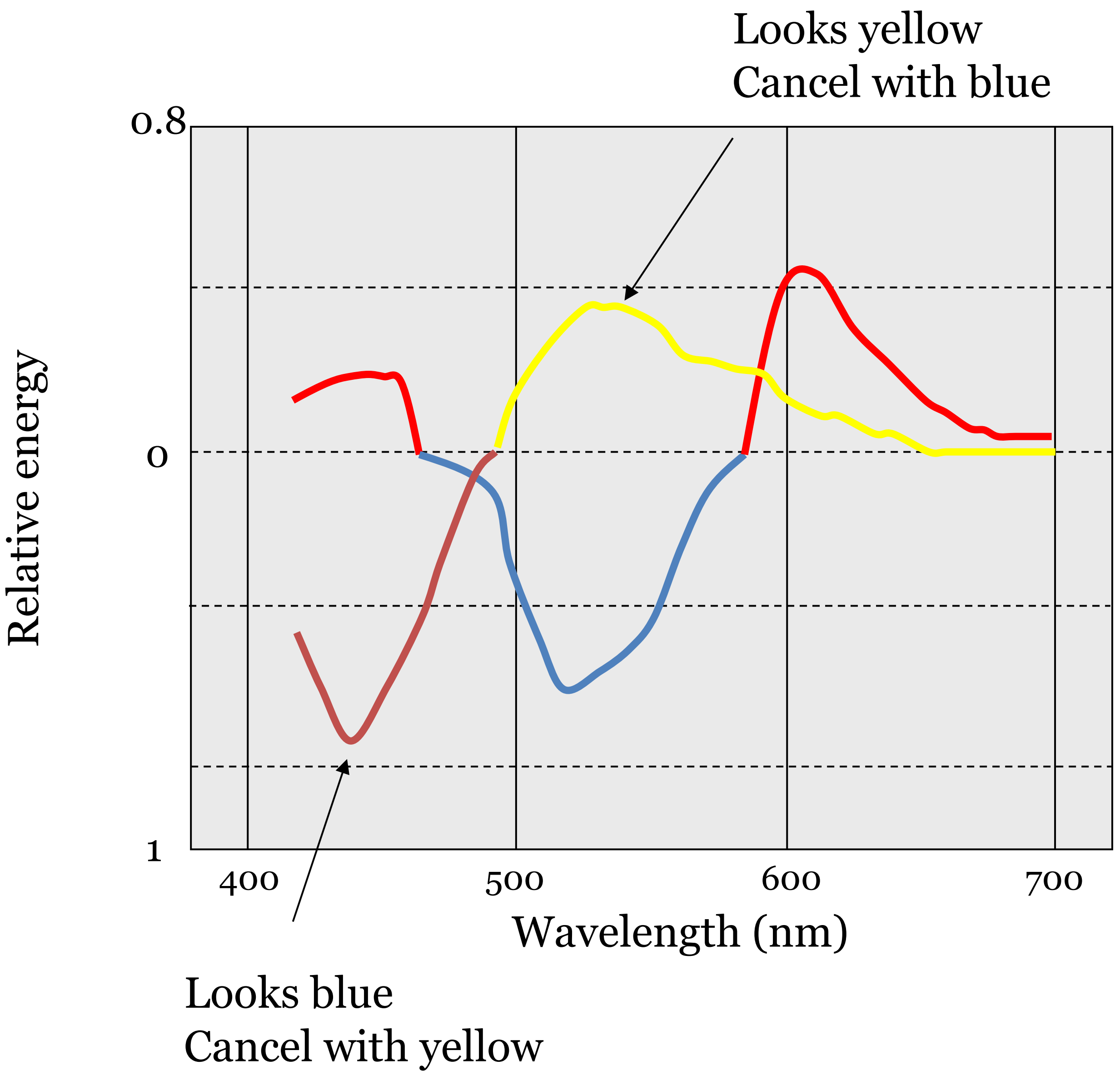
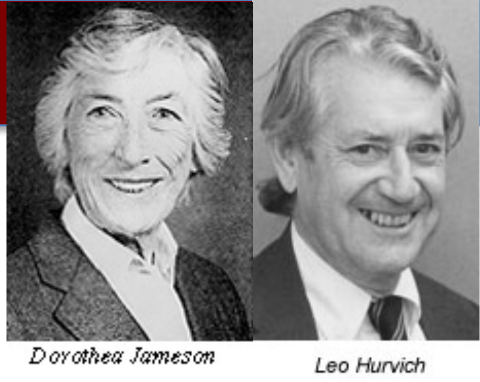


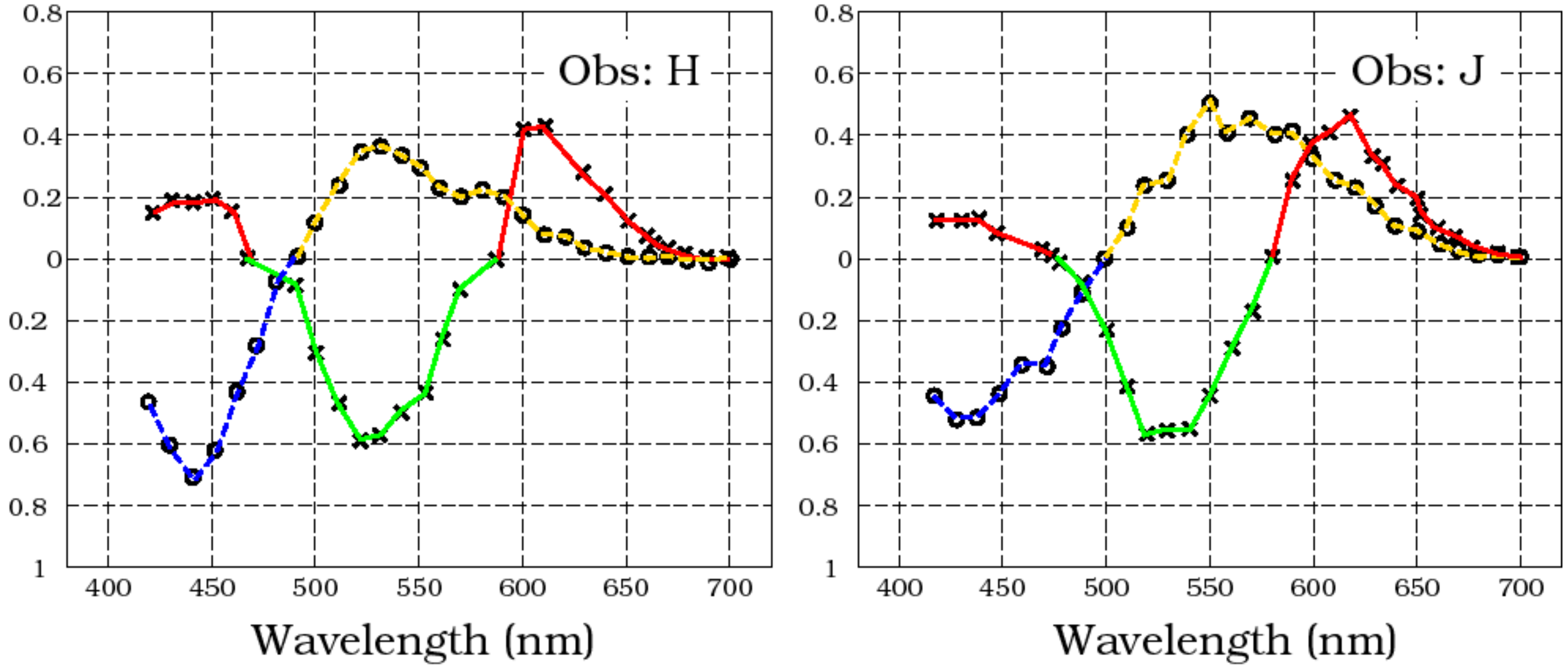
Hue cancellation curves



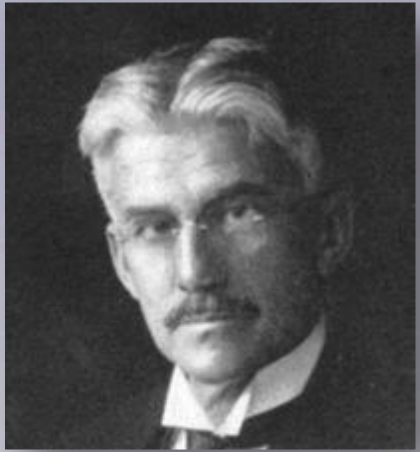


Hue cancellation curves from two observers

Opponent-colors organization is widespread in technology; next lecture

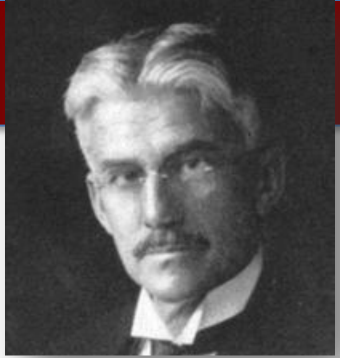


Color appearance space: Munsell



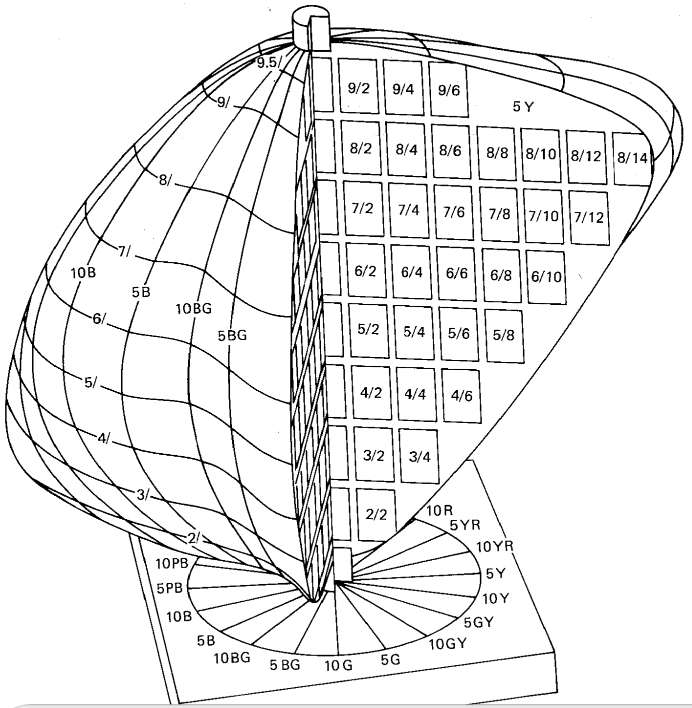
Albert Munsell

Munsell Color Notation



"rational way to describe color"

Not all colors are realized



- Munsell Book of Colors (A.H. Munsell)
- Hue, Value, Chroma (2.5YR 5/10)
- Cylindrical geometry and radial sampling (cf. OSA/UCS)
- Unique notation was calibrated to CIE standards in 'Munsell renotation system"

Outer circle are various **hues** (color circle)
Up and down are **values** (lightness)
Center to edge is **saturation**

Munsell Color Notation

"rational way to describe color"

http://en.wikipedia.org/wiki/Munsell

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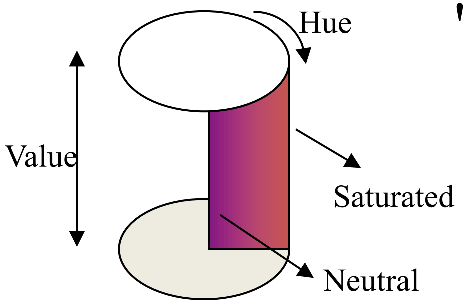
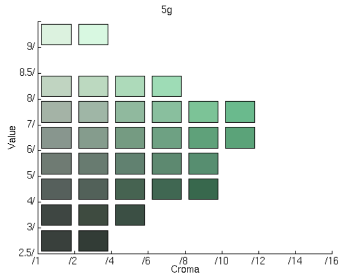
Outer circle are various **hues** (color circle)
Up and down are **values** (lightness)
Center to edge is **saturation**

Munsell Color Notation



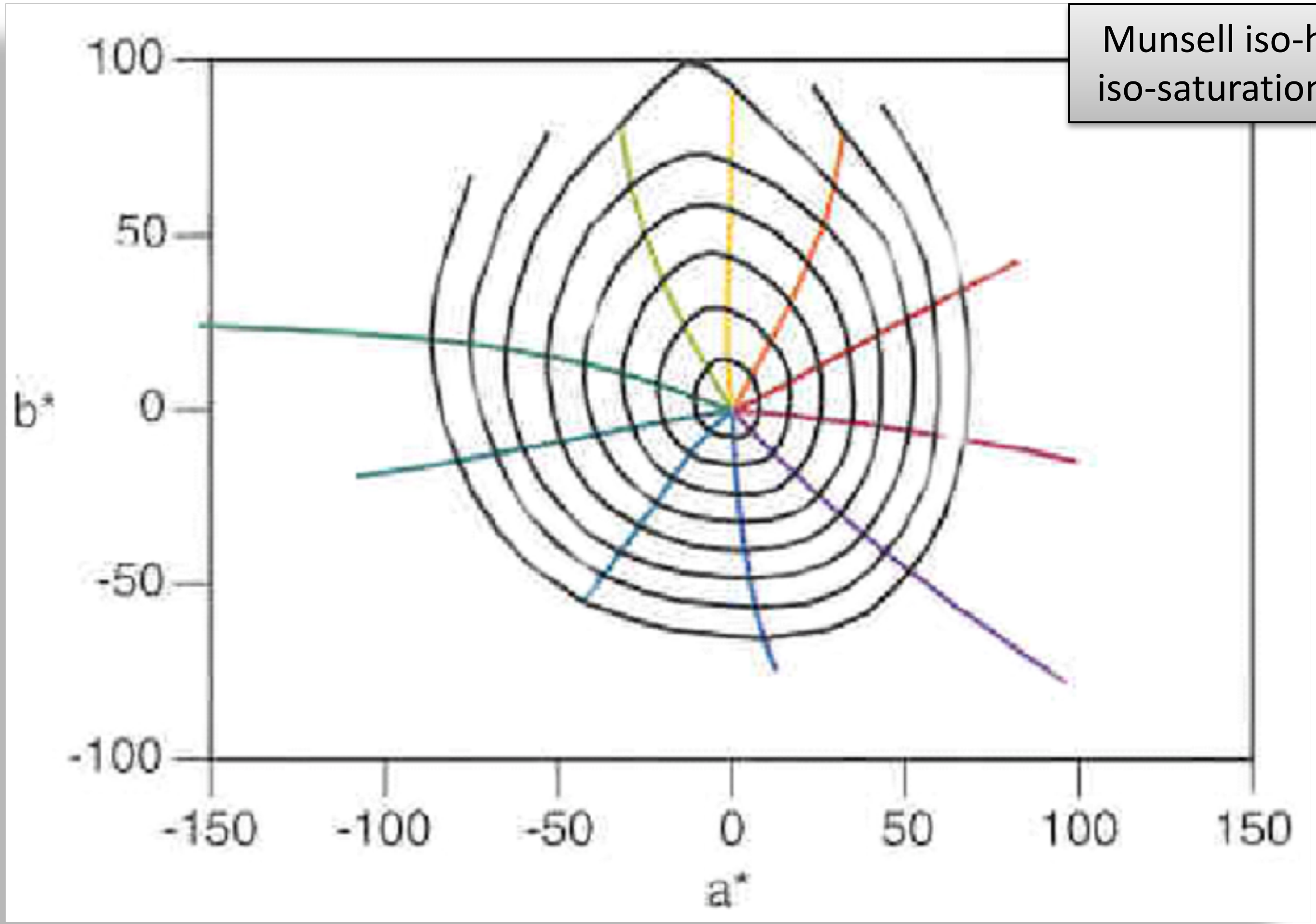
"rational way to describe color"



- Munsell Book of Colors (A.H. Munsell)
- Hue, Value, Chroma (2.5YR 5/10)
- Cylindrical geometry and radial sampling (cf. OSA/UCS)
- Unique notation was calibrated to CIE standards in ‘Munsell renotation system”

Outer circle are various **hues** (color circle)
Up and down are **values** (lightness)
Center to edge is **saturation**

CIELAB captures Munsell hue and saturation reasonably, but not perfectly

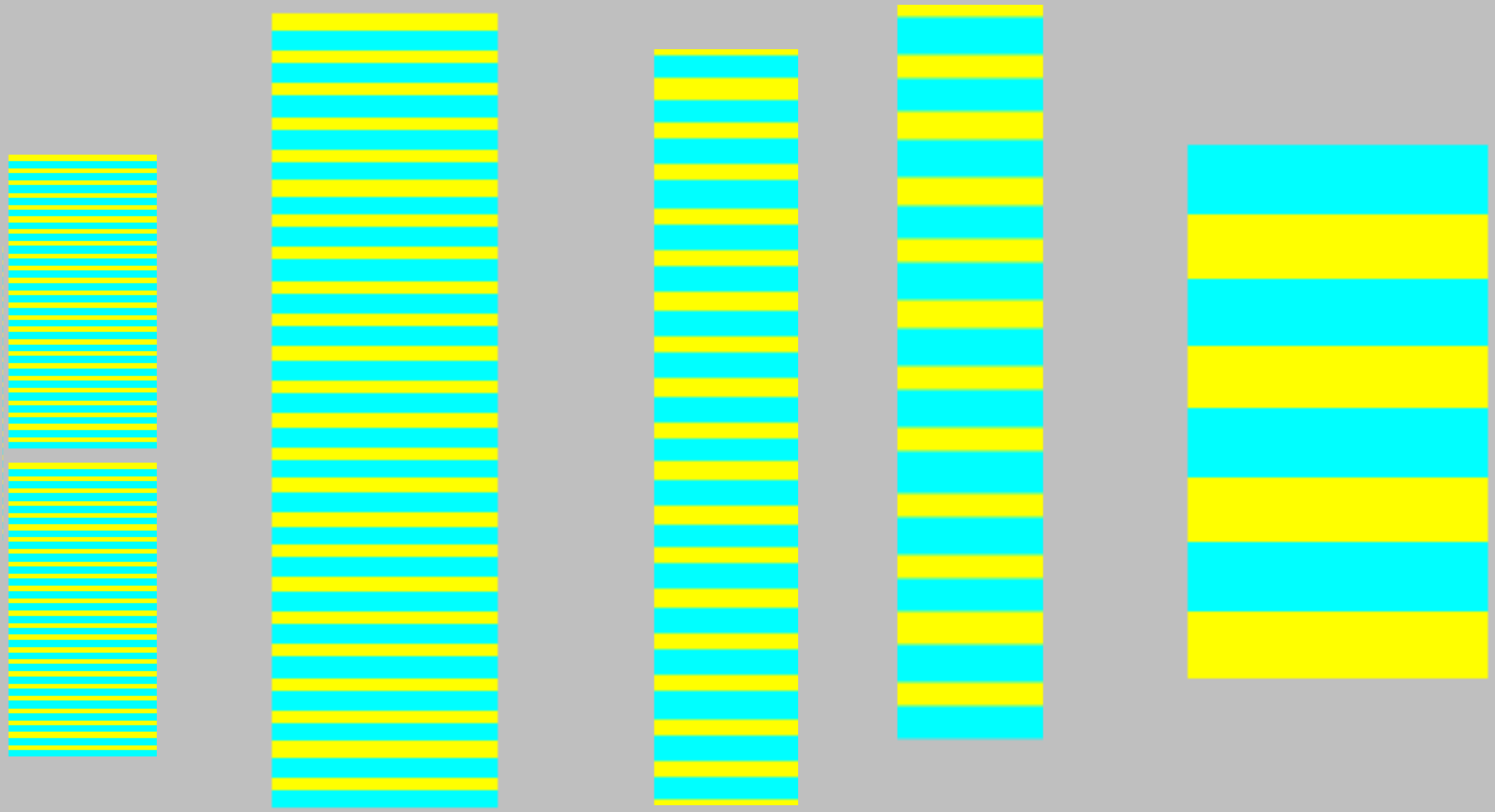


Munsell iso-hue and iso-saturation curves

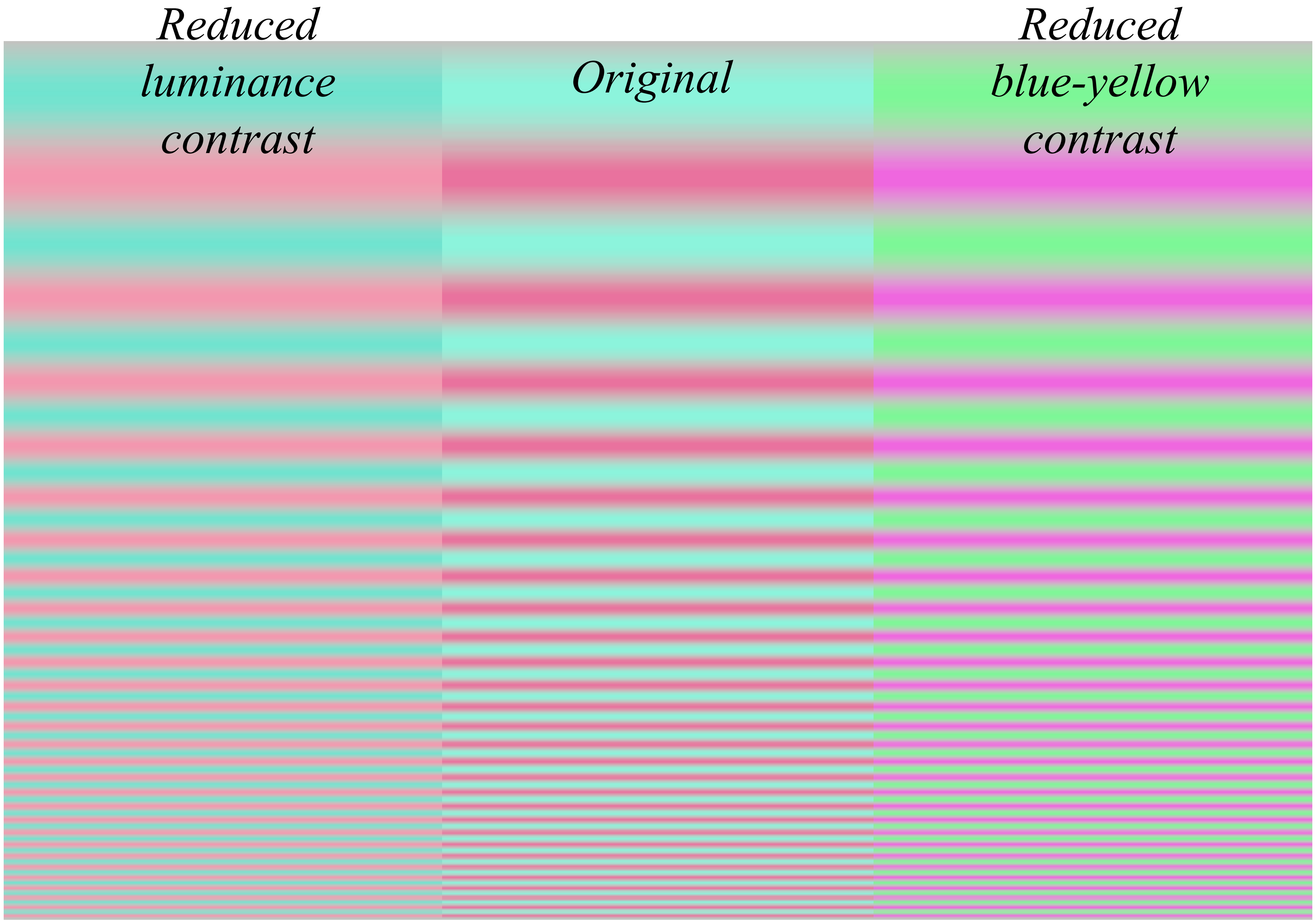
Color and pattern

- Color appearance depends on spatial pattern
- A metric for spatial pattern and color

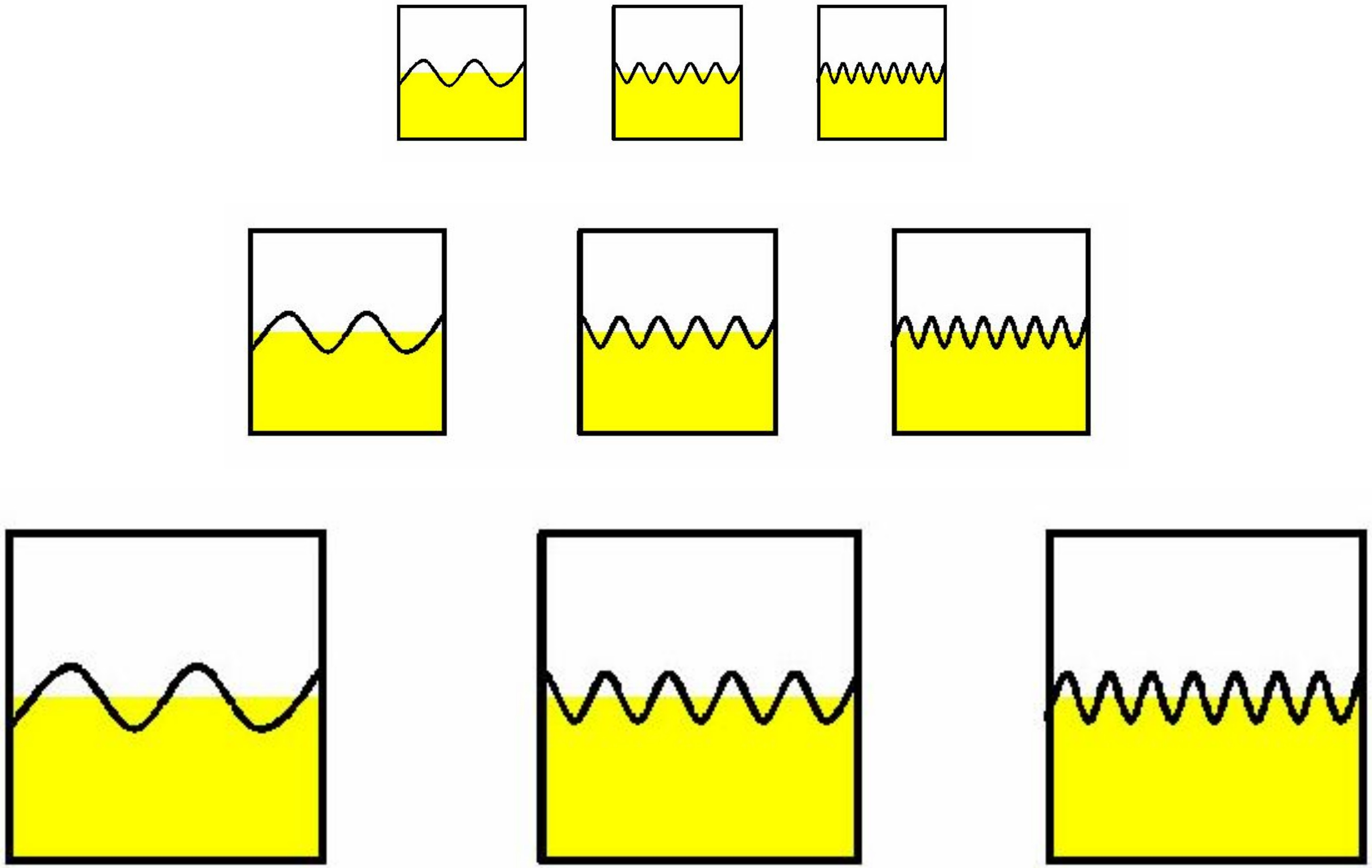
Space – The next frontier (Color is not invariant as we change pattern)



Spatial and temporal variations in color sensitivity are significant

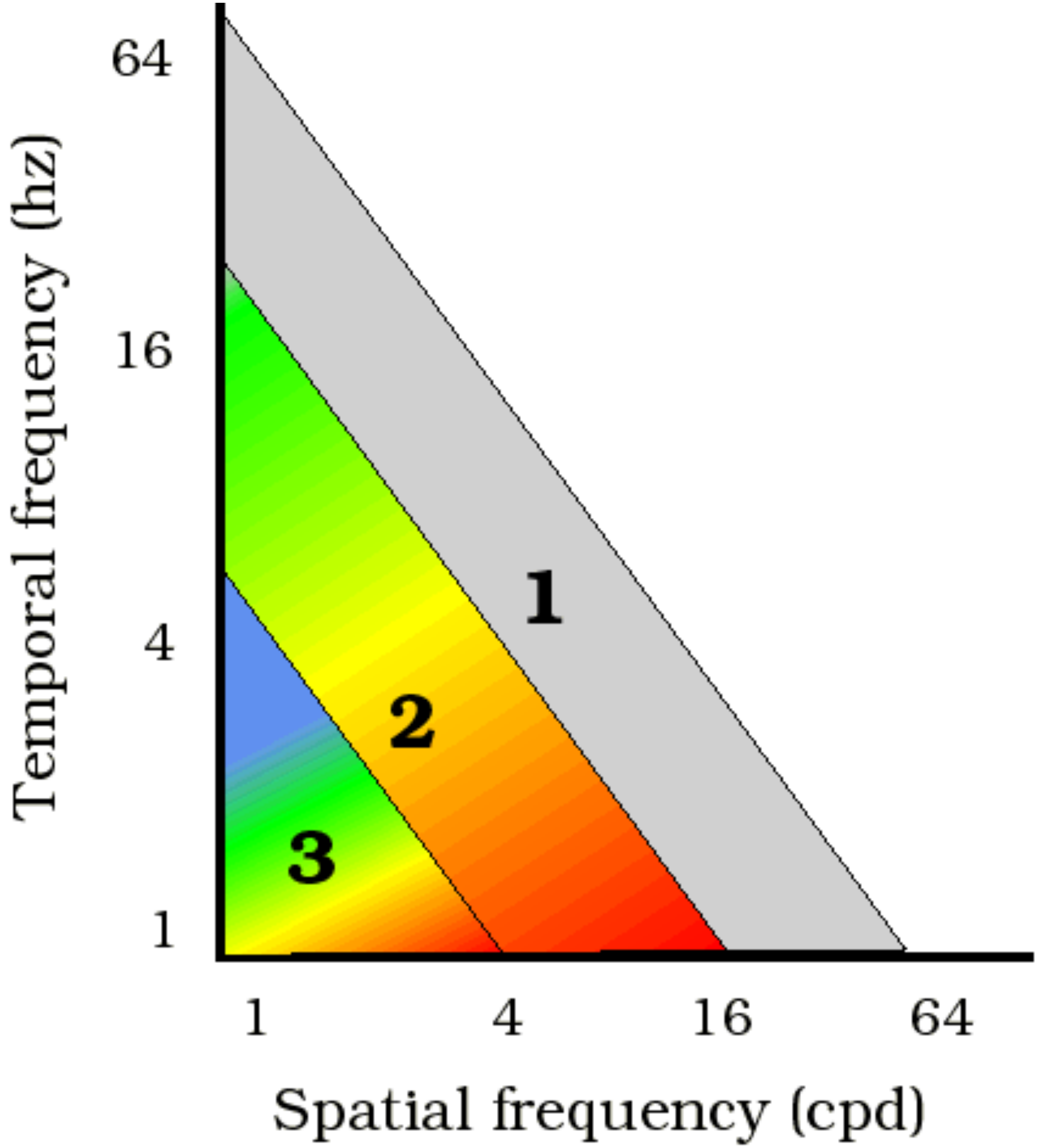


Pattern color interactions



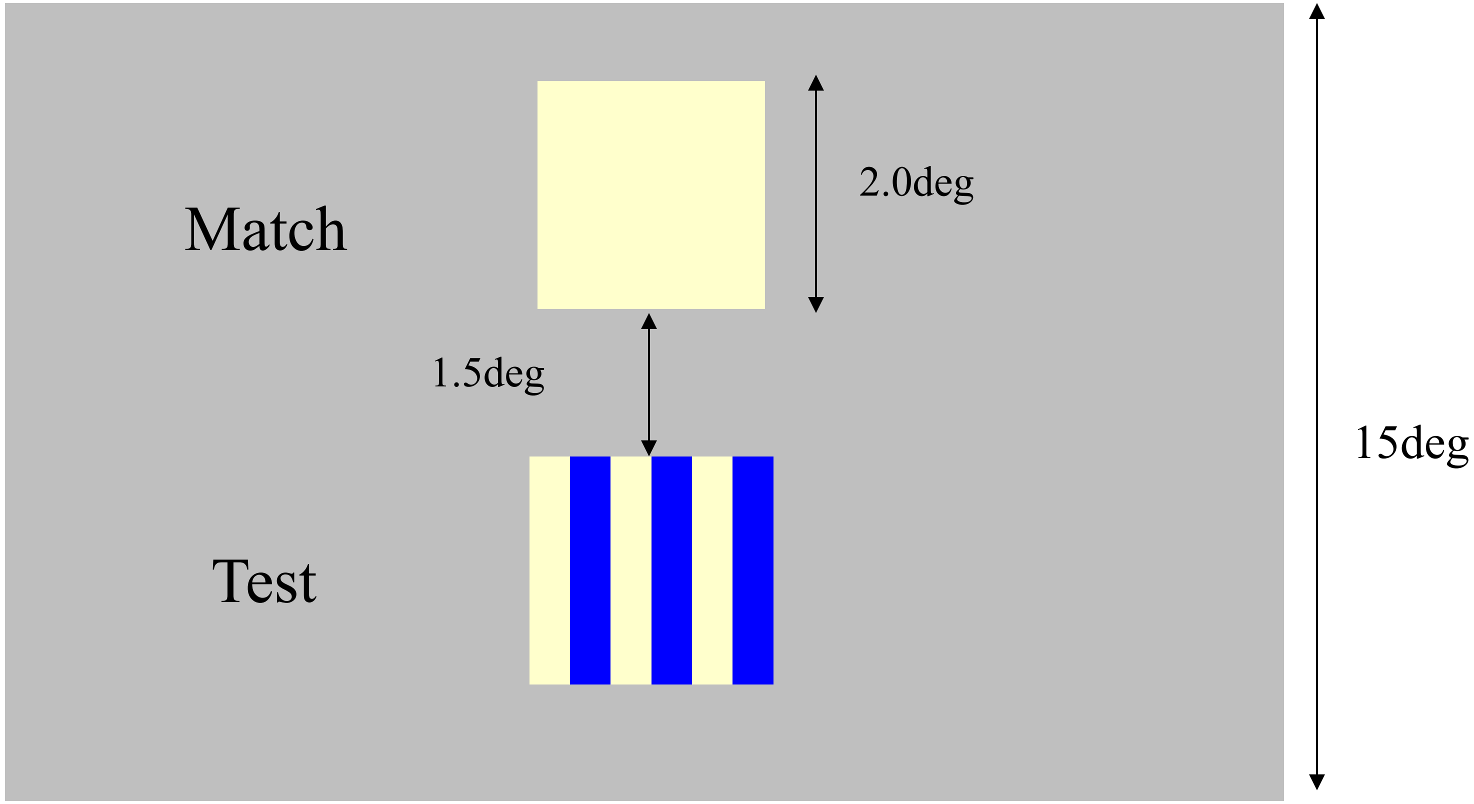
Space-Time-Color sensitivity

- Luminance is visible over the largest range
- Blue-yellow sensitivity is lost at relatively low spatial and temporal frequencies (e.g., chromatic aberration)
- Red-green is intermediate



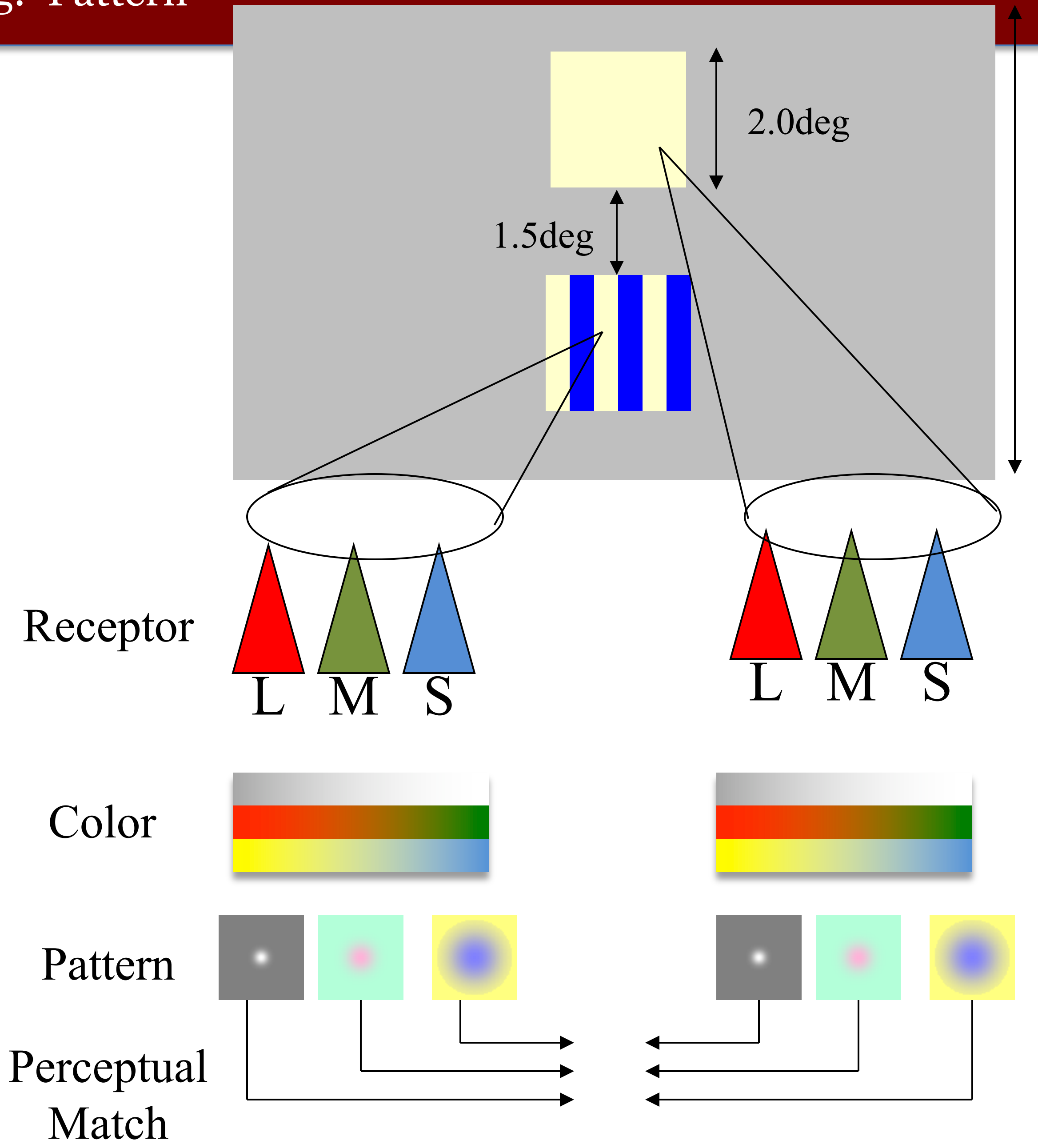
Spatial CIELAB (S-CIELAB)

Asymmetric color matching experiment

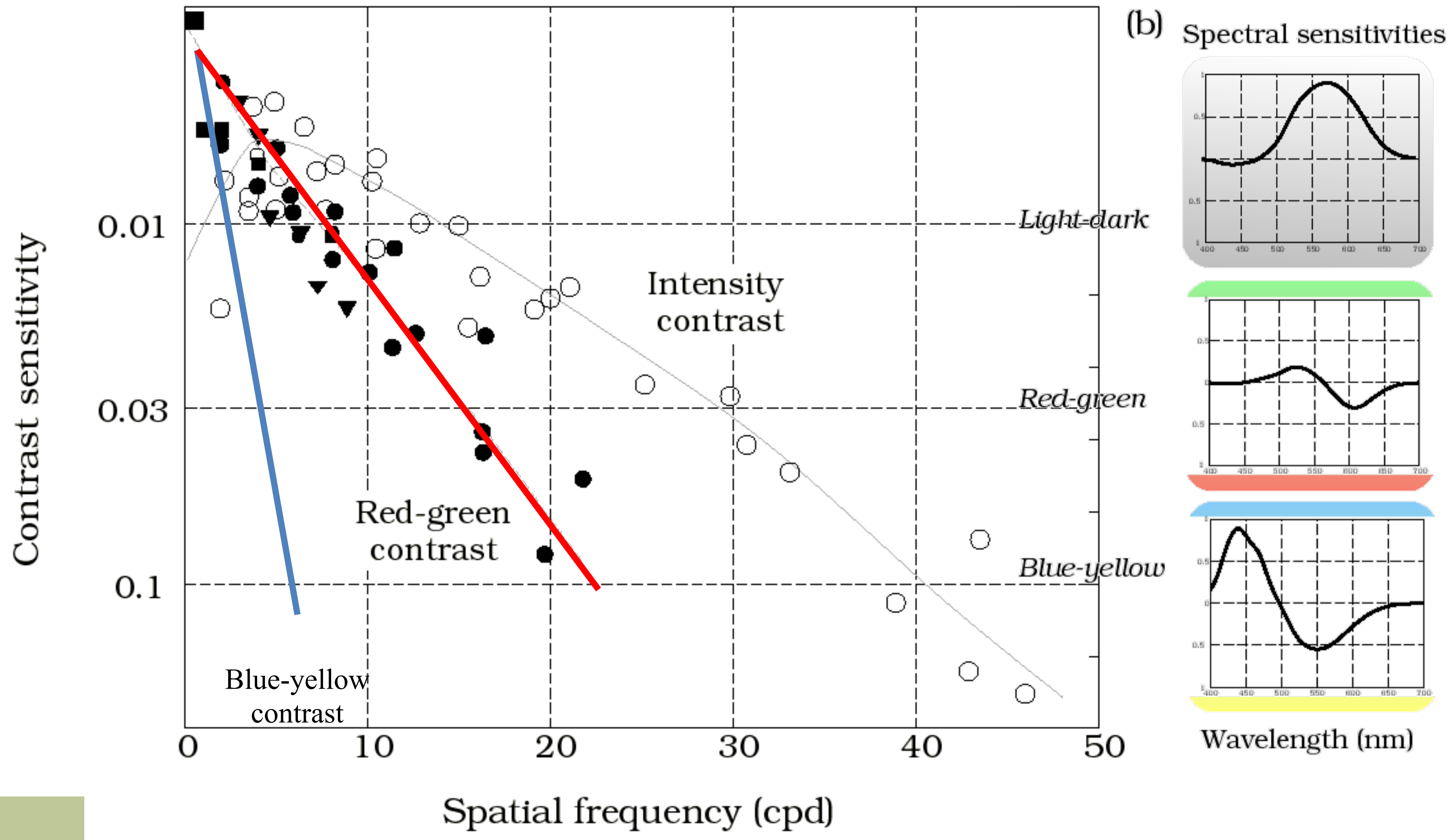


Poirson and Wandell,
J. Opt. Soc. Am A, 1993, **10**,
no. 12, pp. 2458-2470

Asymmetric Color Matching: Pattern



Pattern color sensitivity: 3-channel model fit

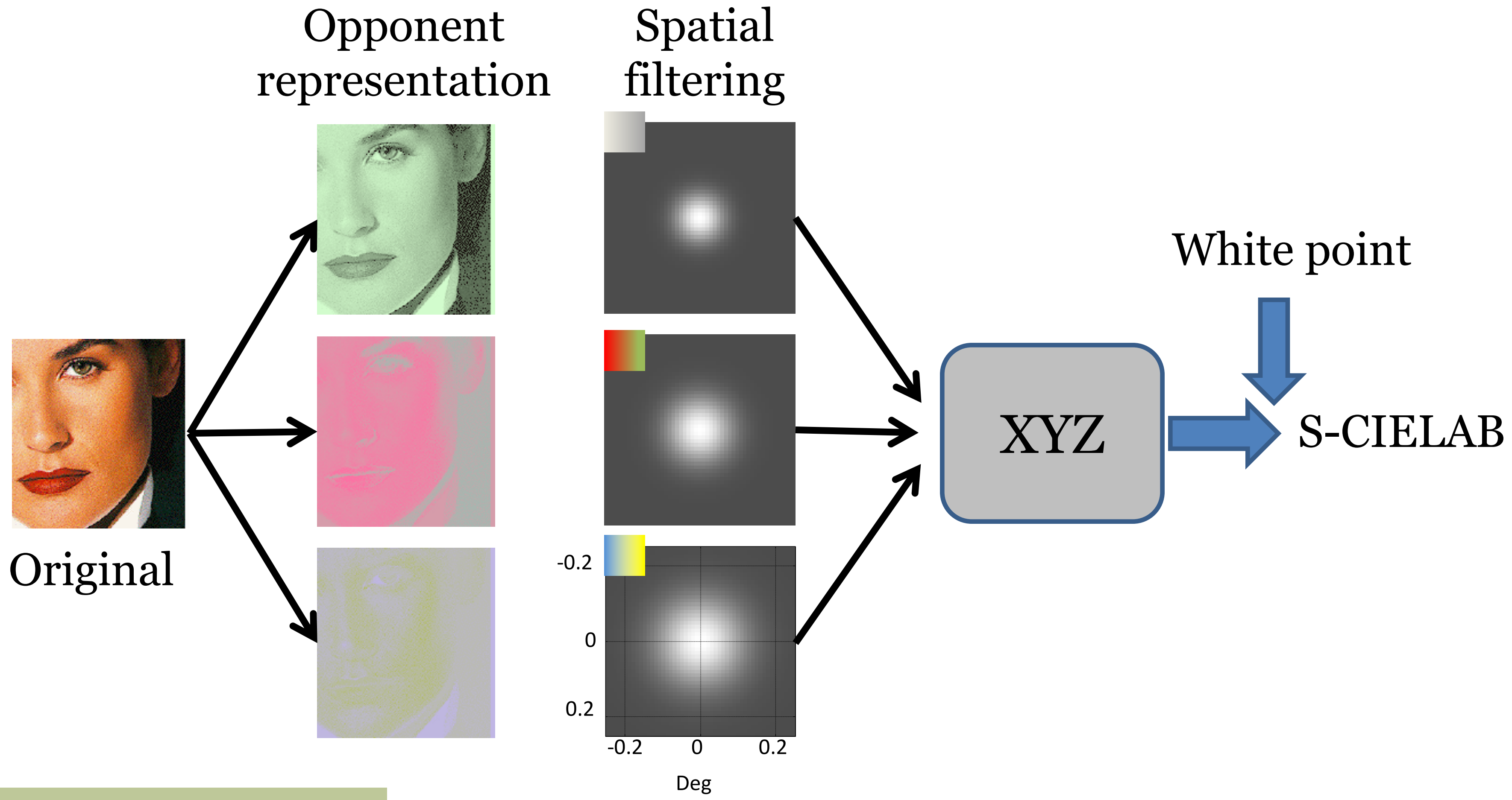


Poirson and Wandell,
J. Opt. Soc. Am A, 1993, **10**,
no. 12, pp. 2458-2470

Spatial CIELAB Representation

Merging CIELAB and human spatial sensitivity

s_scielabExample.m

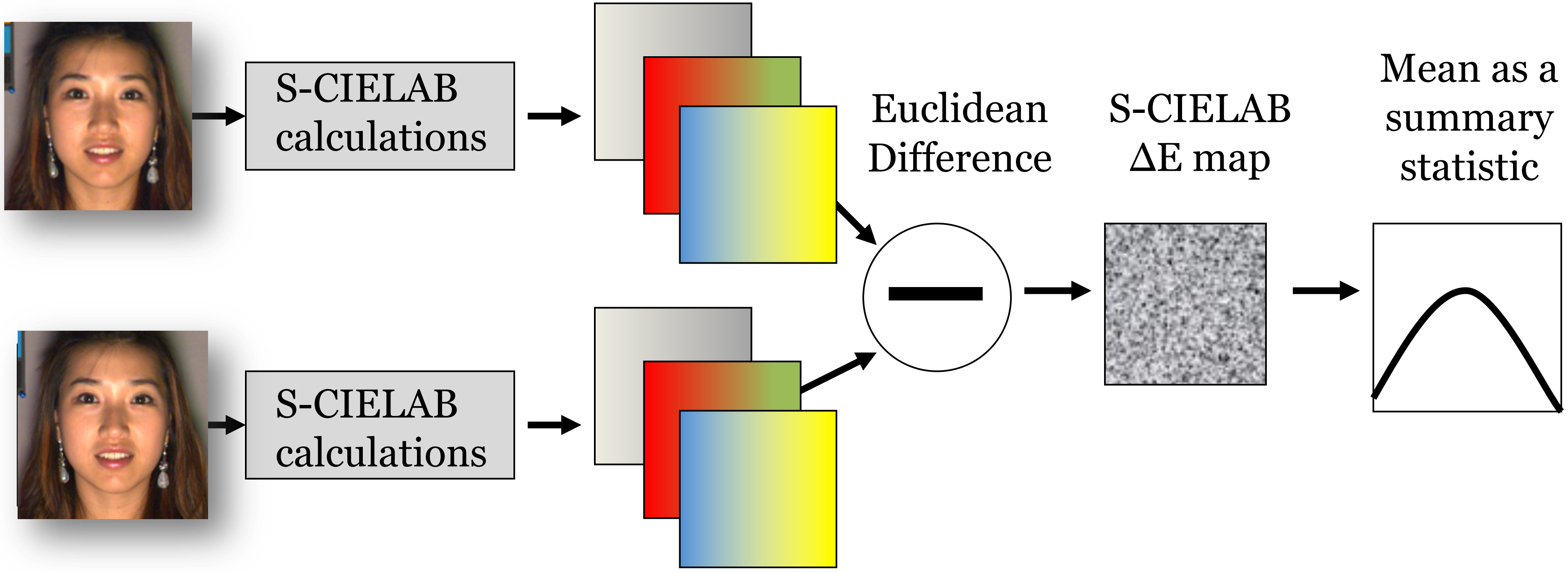


A spatial extension of CIELAB for digital color reproduction
Zhang and Wandell (1996)
Proceedings of the SID Symposiums, pp. 731–734

S-CIELAB ΔE maps

Difference between reference and test image

S-CIELAB difference distributions



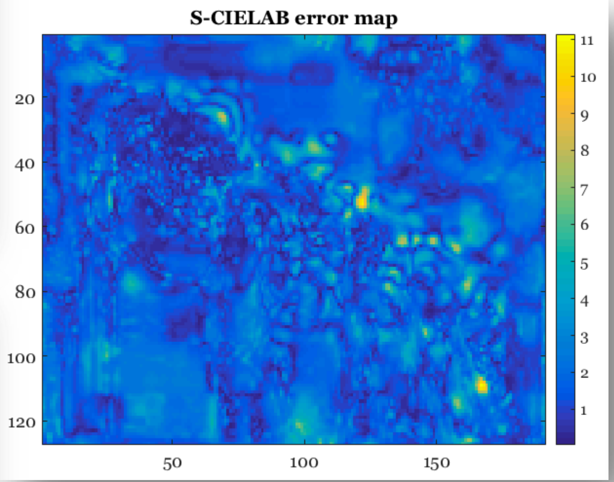
S-CIELAB ΔE maps

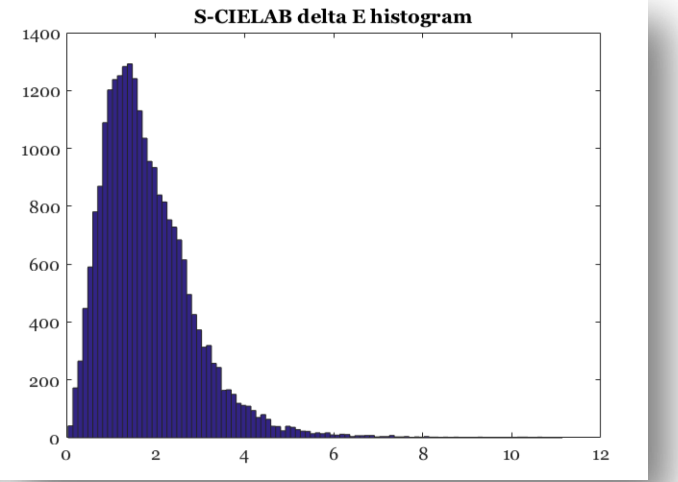
s_scielabExample.m

Difference between reference and test image
and

Original

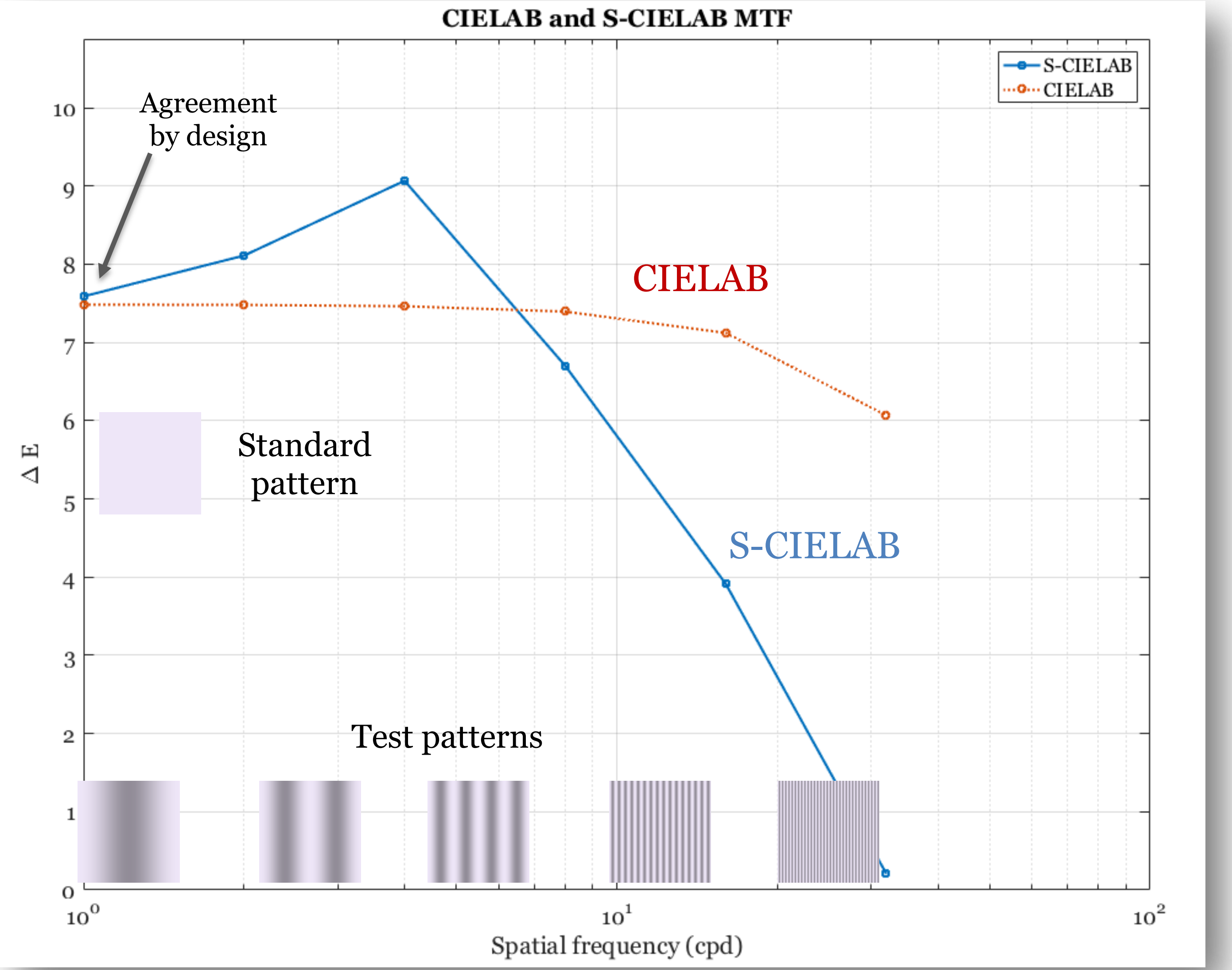
Compressed





Color error frequency dependence

- Find the mean ΔE difference between a uniform patch and harmonic patterns
- CIELAB has very little spatial dependency – the ΔE average is about the same for all the spatial frequency patterns
- S-CIELAB vs. uniform ΔE varies a lot with the spatial pattern



Low-dimensional linear models

Low dimensional linear models

- In most applications and natural images, the spectral functions we encounter are a subset of all possible functions – they are smooth

- There is so much regularity, that we can accurately summarize these functions by the weighted (w_i) sum of a small number (N) of spectral basis functions (B_i)

The linear model

Spectral function

Weights

Basis functions

$$s_j(\lambda) \approx \sum_{i=1}^{N} w_{ij} B_i(\lambda)$$

The basis functions are fixed – so each spectral function is summarized by only the N-weights

Why are linear models useful?

- Compact representation of the data
- Work smoothly with conventional linear matrix computations
- Help make design decisions such as the number of sensors needed to measure a set of signals: One for each basis function

Two-dimensional linear model in matrix tableau

$$
\begin{array}{ccc}
\text{Spectral function} & \text{Basis functions} & \text{Weights} \\
\begin{pmatrix} | \\ s_j(\lambda) \\ | \end{pmatrix} & \approx \begin{pmatrix} | & | \\ B_1(\lambda) & B_2(\lambda) \\ | & | \end{pmatrix} & \begin{pmatrix} w_{1,j} \\ w_{2,j} \end{pmatrix}
\end{array}
$$

Data dimensionality reduction: matrix tableau

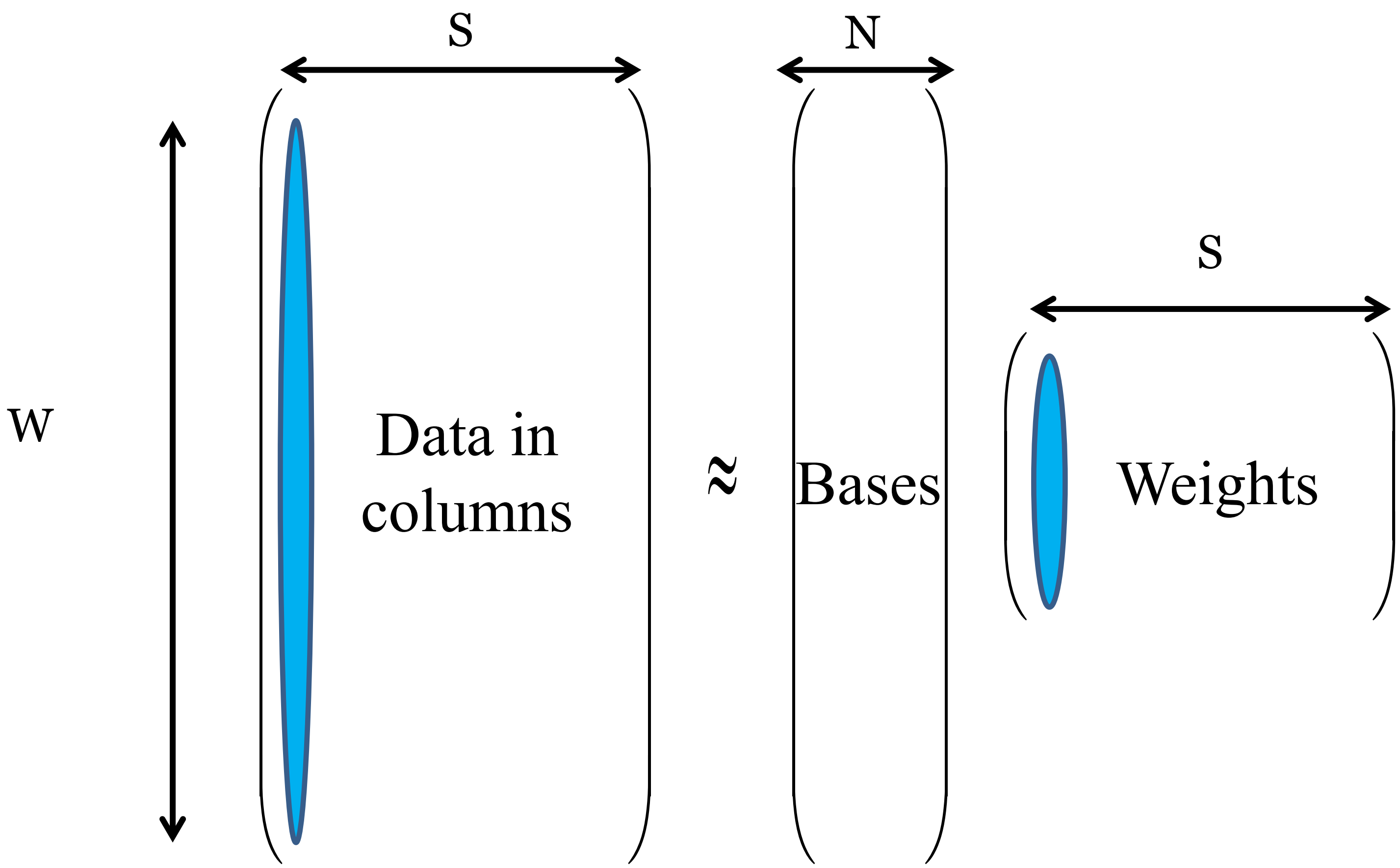
The original data contain (W x S) values

The linear model representation contains

(N x S) + (W x N) = (S + W) x N

Suppose we have W = 76wavelength (samples, 400:4:700) and S = 1000 measurements (76,000) data values.

If we can approximate with a three dimensional (N = 3) linear model, then we need 3228 values, a reduction by a factor of 20.



Solving for the basis functions

For an N-dimensional linear model, we find the basis functions and weights that minimizes the error across samples (j)

Data

Linear model approximation

$$\sum_j \left(\sum_\lambda [s_j(\lambda) - \sum_{i=1}^{N} B_i(\lambda) w_i]^2 \right)$$

The singular value decomposition (SVD)

- The SVD is a solution for finding the basis functions that achieve the best least-squares approximation
- The data are placed in the columns of a matrix, D. Typically, the mean is removed first, but not always.
- The SVD is a matrix factorization into three terms; two orthonormal matrices (U,V) and one diagonal matrix (S)
- The basis functions and weights correspond to the groupings at the right

SVD definition

$$D = USV^t$$

$$UU^t = U^tU = Identity$$

$$VV^t = V^tV = Identity$$

$$S \text{ is diagonal with } s_i \geq s_{i+1}$$

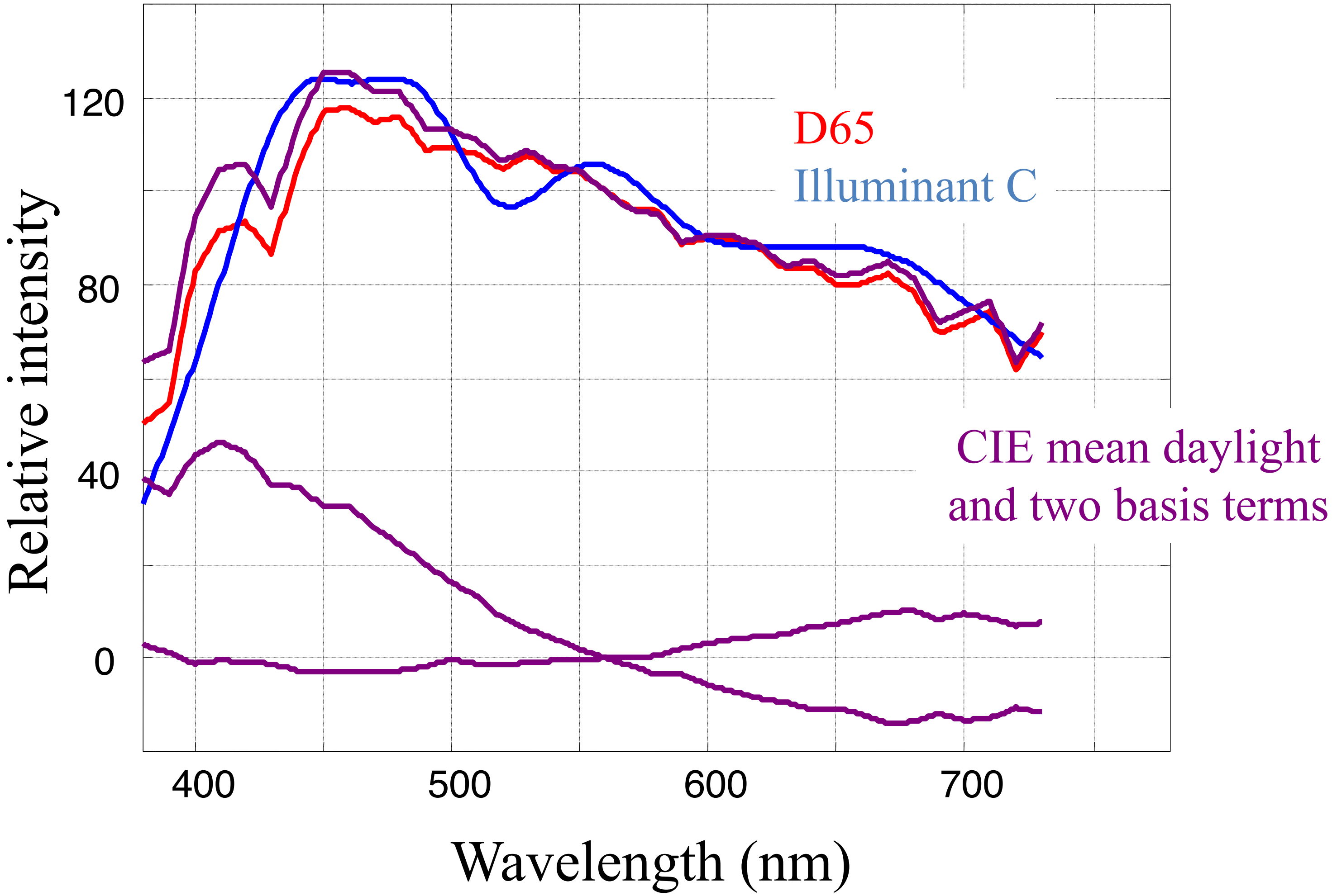
We reduce the dimensionality by zeroing out all but the first N of the diagonal weights

The first N columns of U are the basis functions

The product (SV^t) is the weights

Example: CIE daylight basis

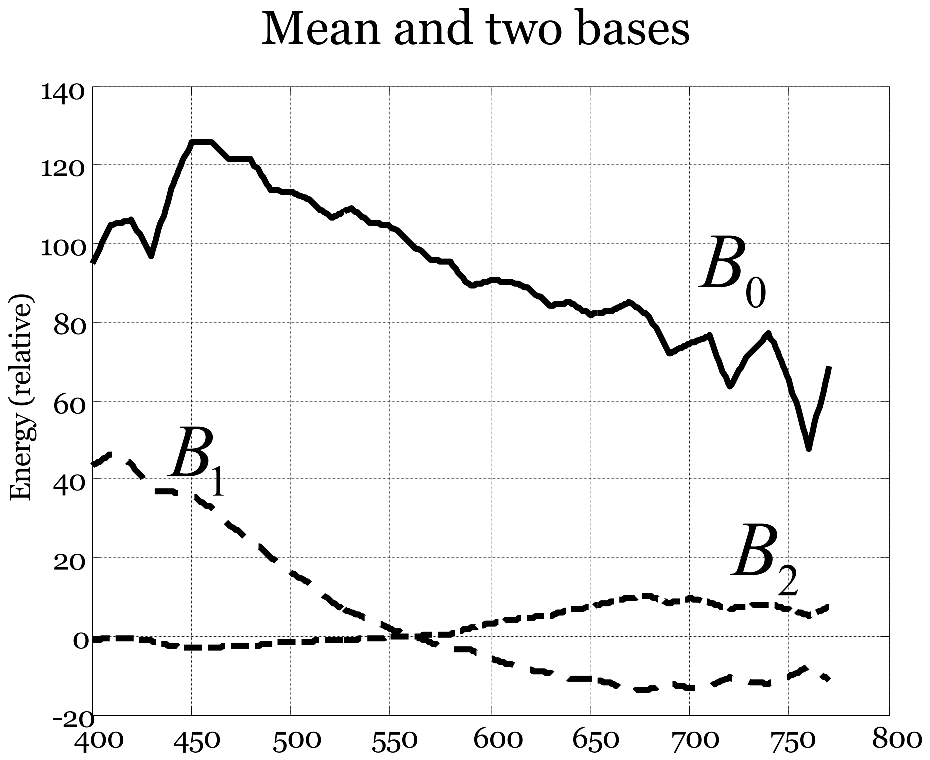
$$E(\lambda) \approx B_0 + \sum_{i=1,2} w_i B_i(\lambda)$$

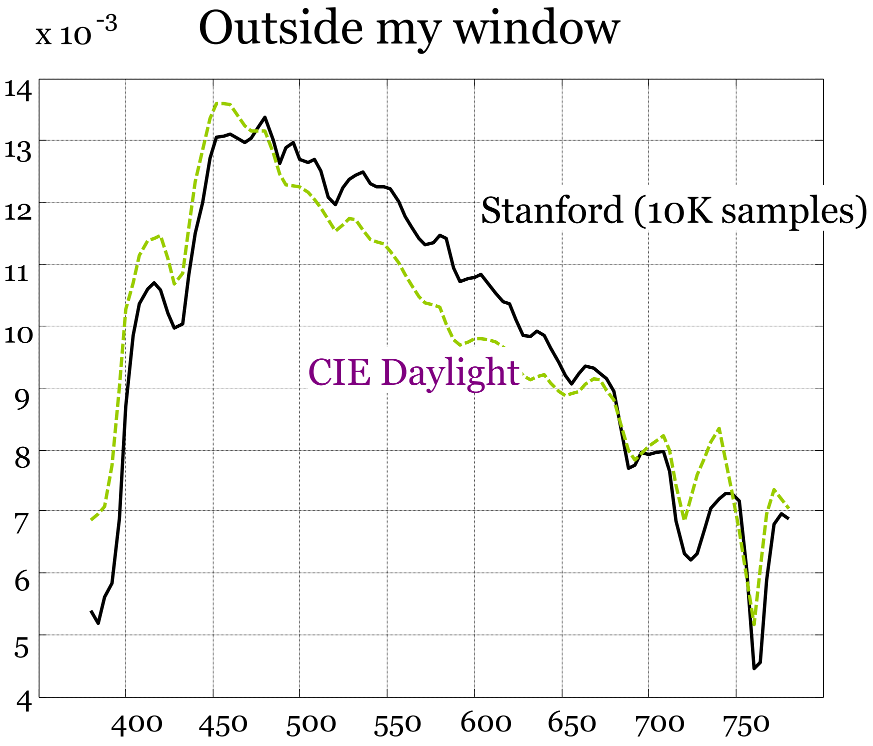


CIE daylight

ISET: s_sceneDaylight.m

$$E(\lambda) \approx B_0 + \sum_{i=1,2} w_i B_i(\lambda)$$



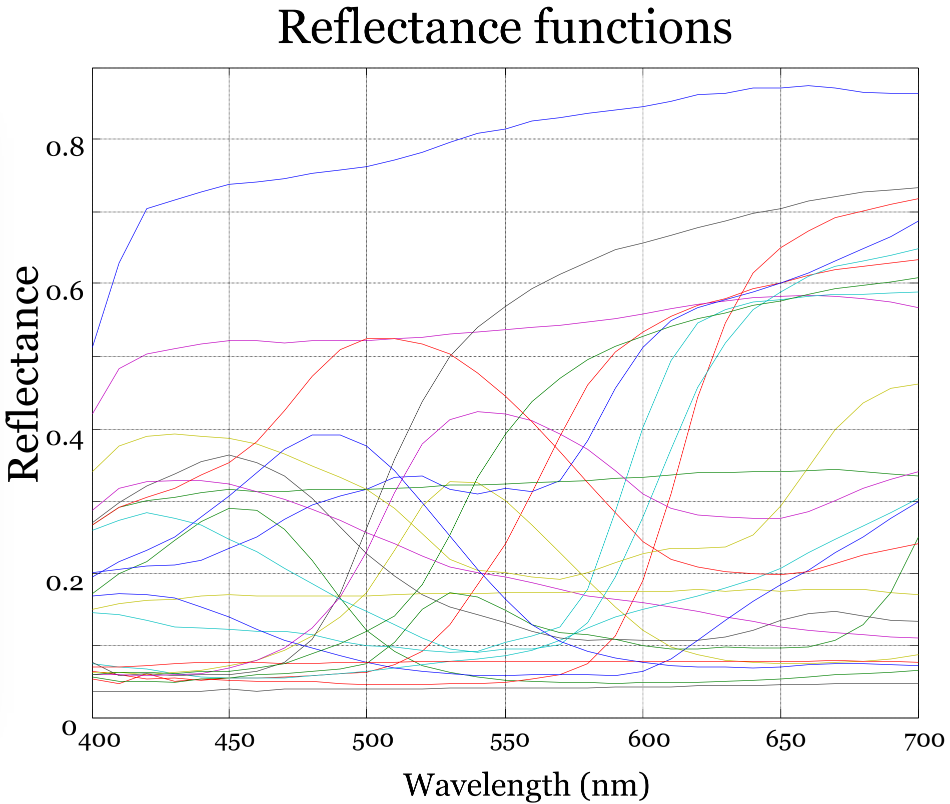


Wavelength(nm)

Surface reflectance linear models

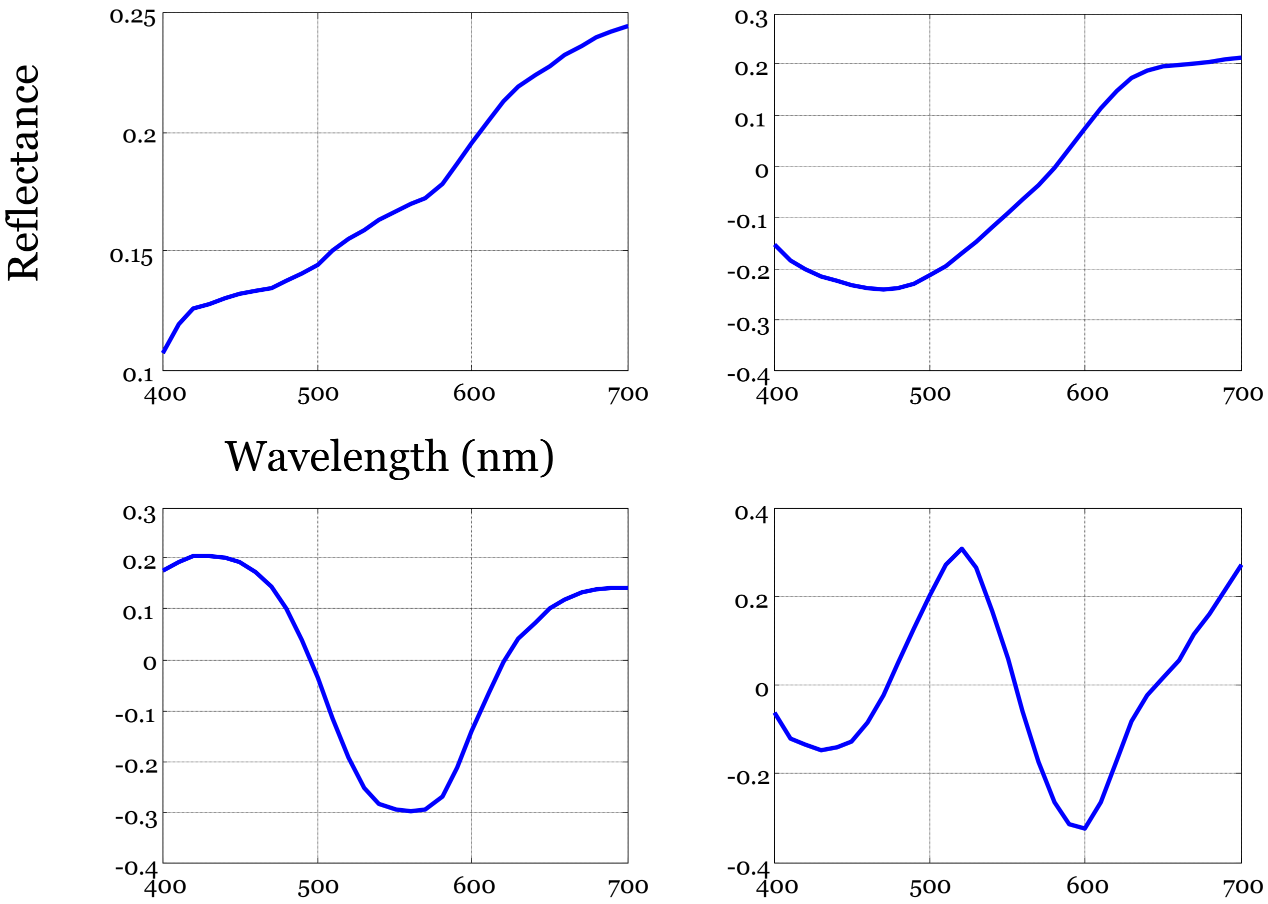
Macbeth (Gretag) ColorChecker

- Reflectances chosen to match difficult to reproduce natural objects
- Neutral series
- Simple and widely used
- Reflectance functions published along with (x,y) values



Macbeth reflectance basis functions

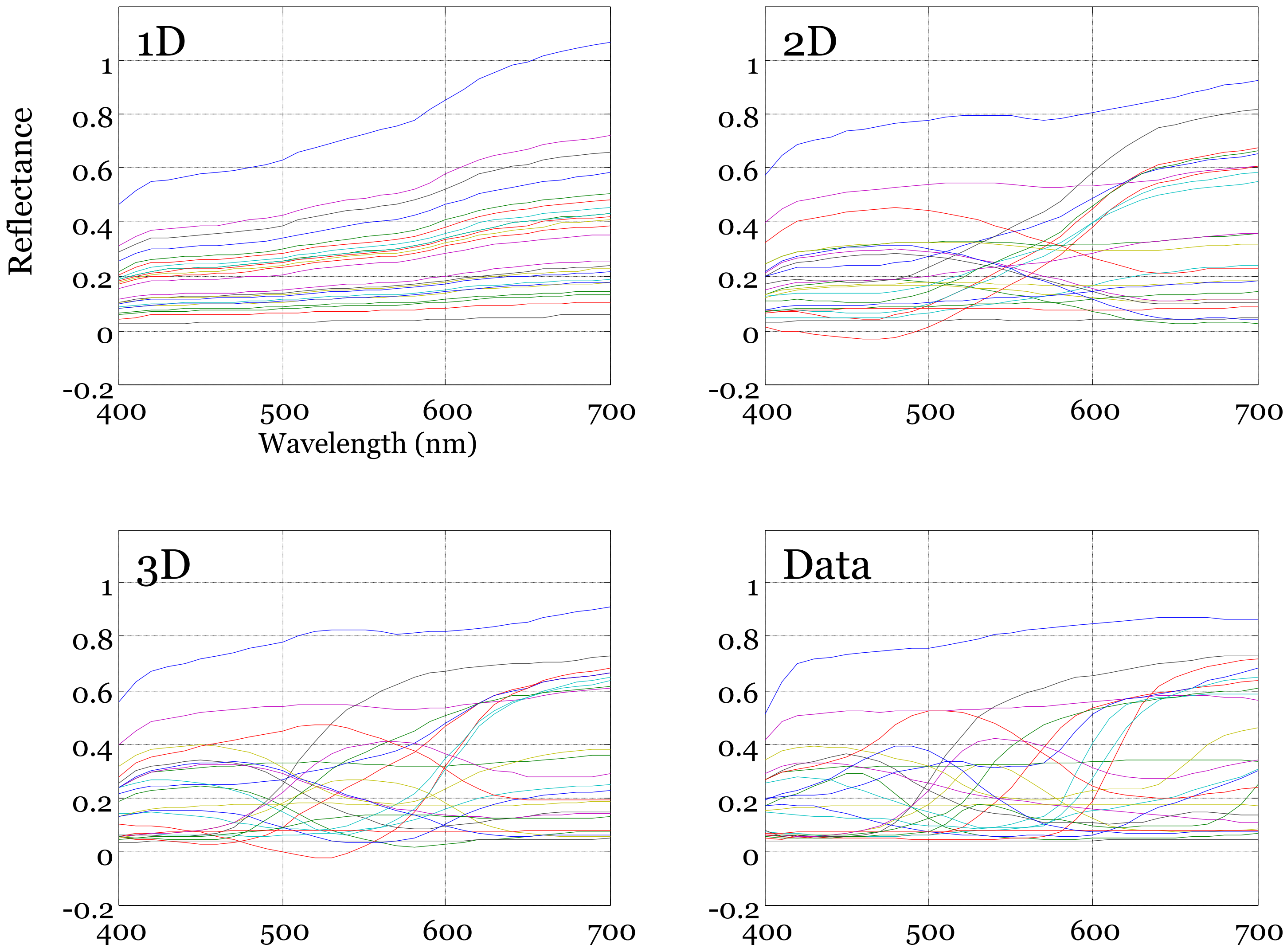
ISET: s_SurfaceModels



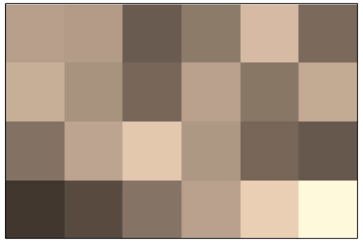

```
ref = macbethReadReflectance(wave);
[U S V]= svd(ref);
plot(wave,U(:,1:4))
```

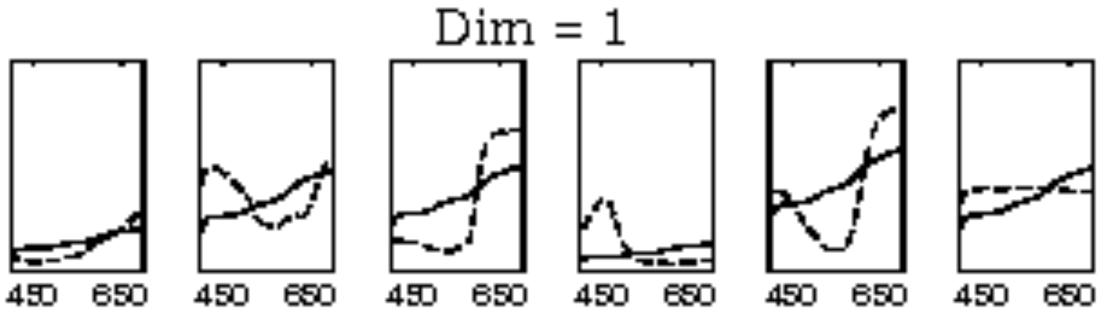
MCC approximations

ISET: s_SurfaceModels

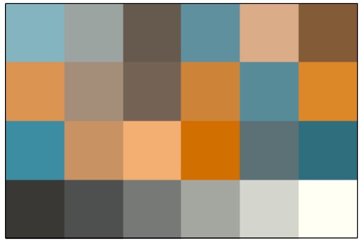


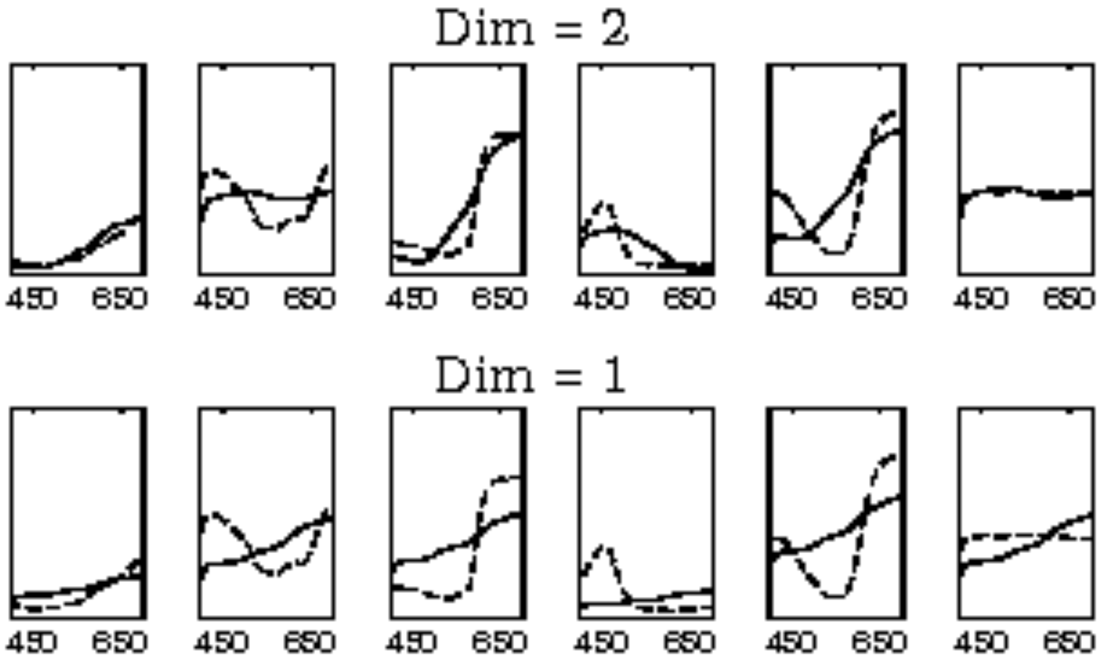
Visualizing the number of bases



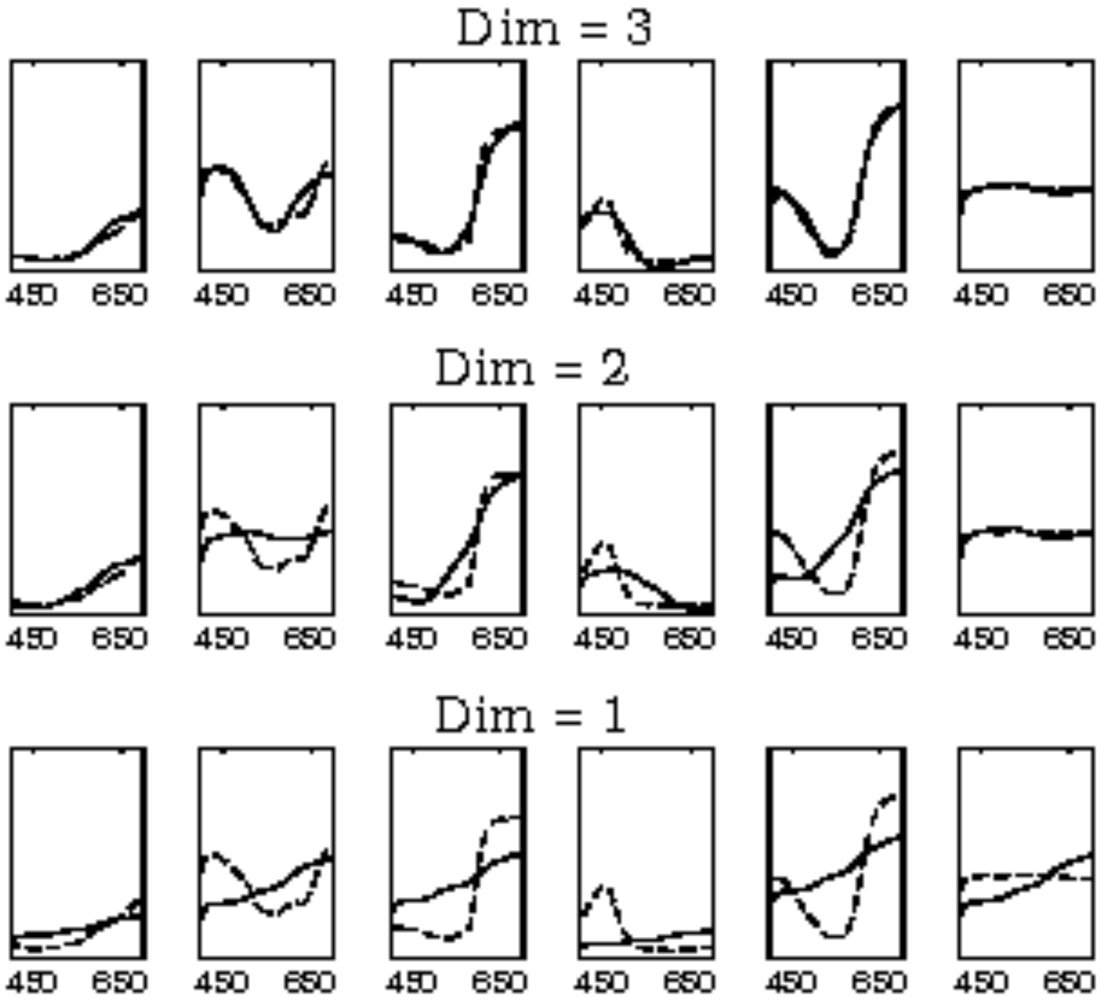


Increasing the number of bases



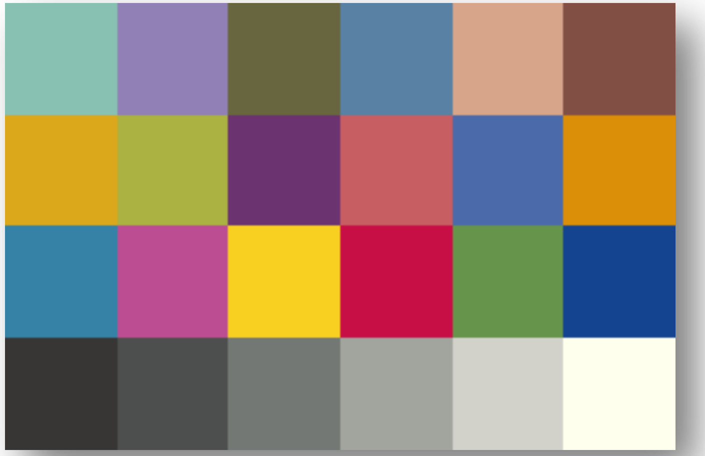


Increasing the number of bases

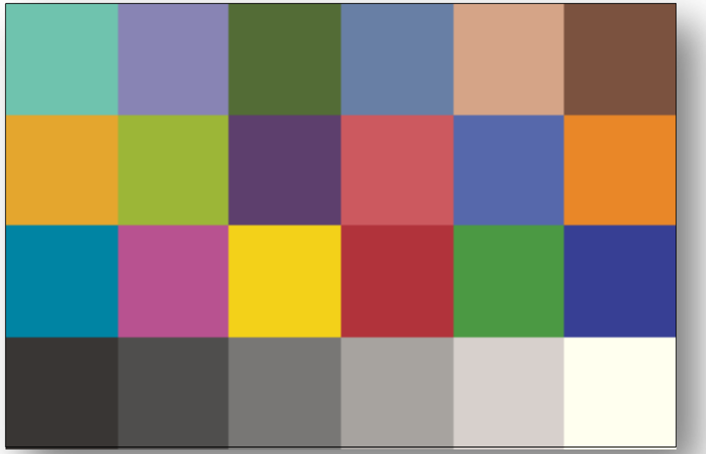


Macbeth approximations: Appearance

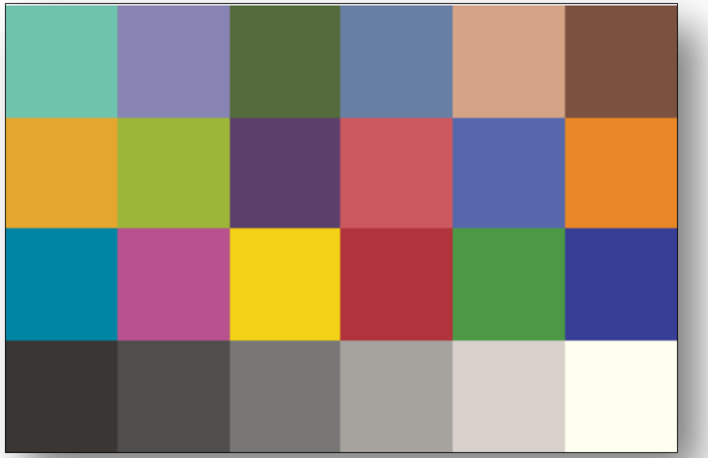
N = 3



N = 7



N = 8



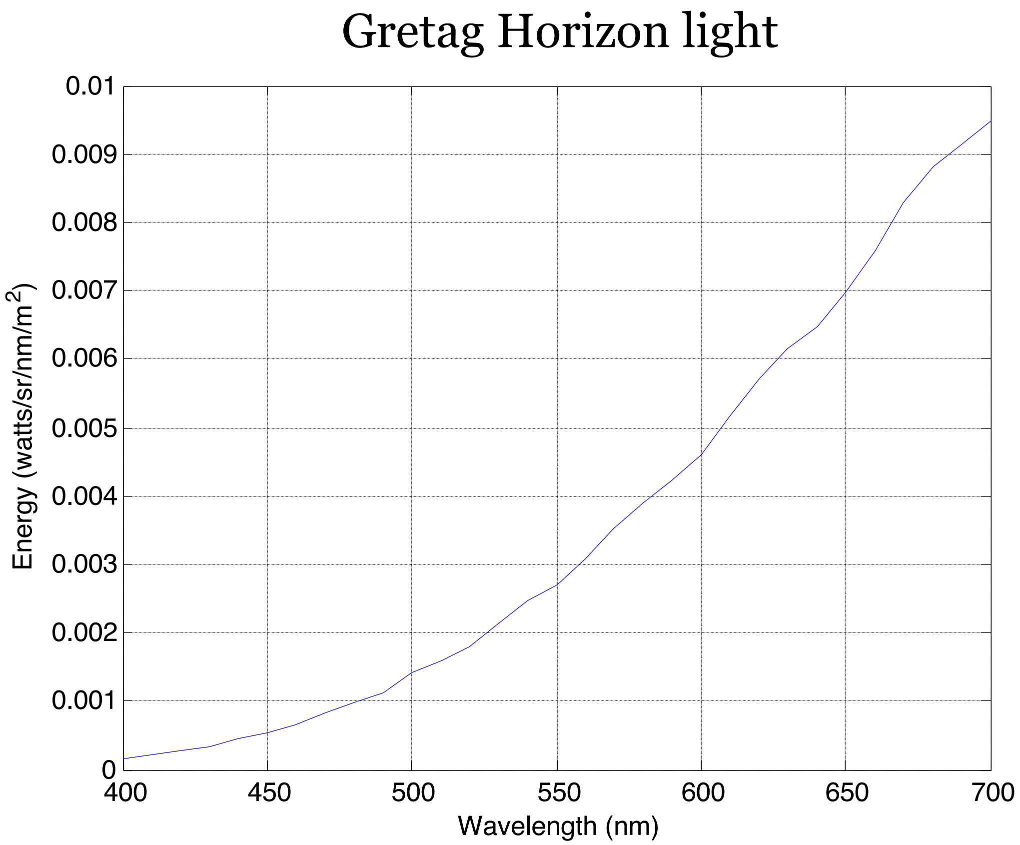
Next Generation Scenes: Computer Graphics

Future: depth and viewing angle

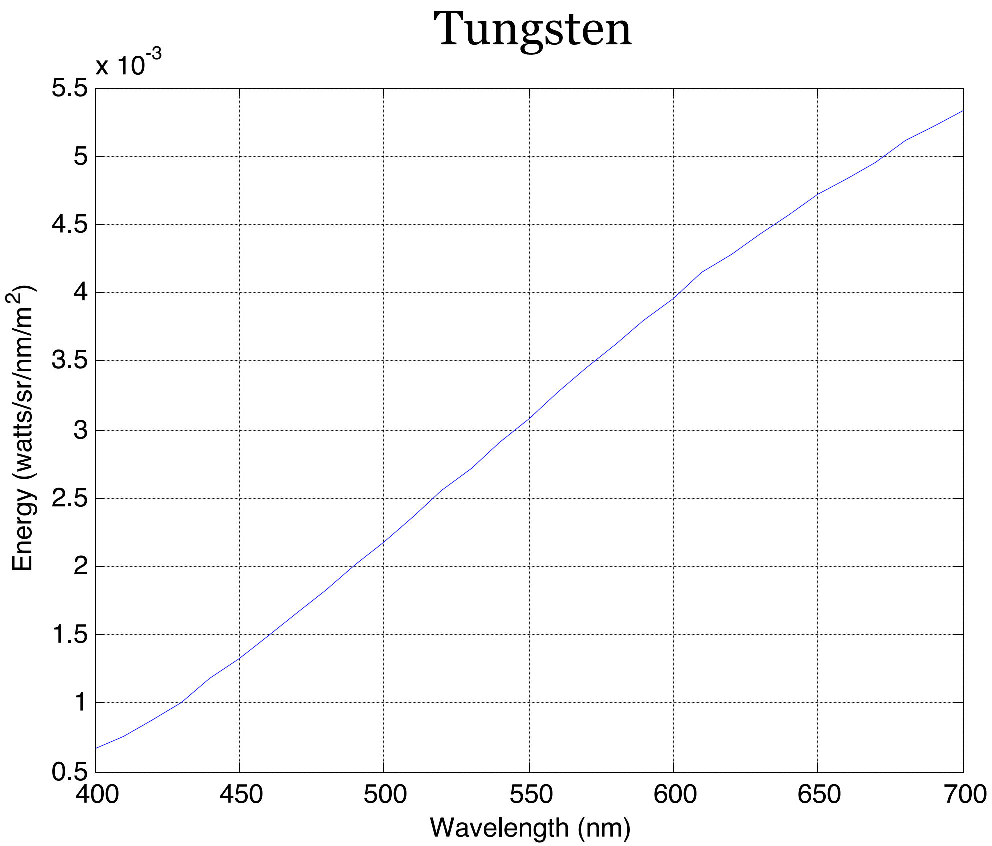
- 3D representations of the scene – account for defocus and depth of field
- Lift the Lambertian assumption to use full bidirectional reflectance distribution functions (BRDF); allows highlights

The **bidirectional reflectance distribution function** (**BRDF**; $f_r(\omega_i,\omega_o)$) is a four-dimensional function that defines how light is reflected at an opaque surface. The function takes an incoming light direction, ω_i, and outgoing direction, ω_o, both defined with respect to the surface normal n, and returns the ratio of reflected radiance exiting along ω_o to the irradiance incident on the surface from direction ω_i. Note that each direction ω is itself parameterized by azimuth angle $\varphi$and zenith angle θ, therefore the BRDF as a whole is 4-dimensional. The BRDF has units sr^{-1}, with steradians (sr) being a unit of solid angle.

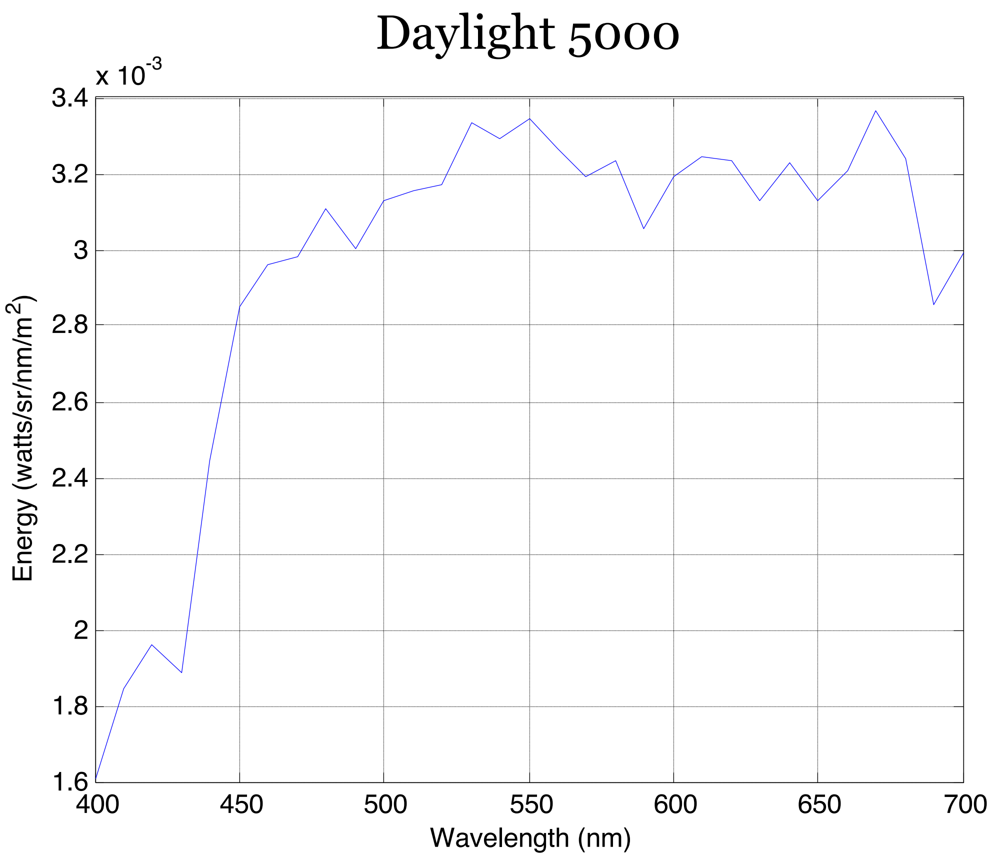
Illuminant changes



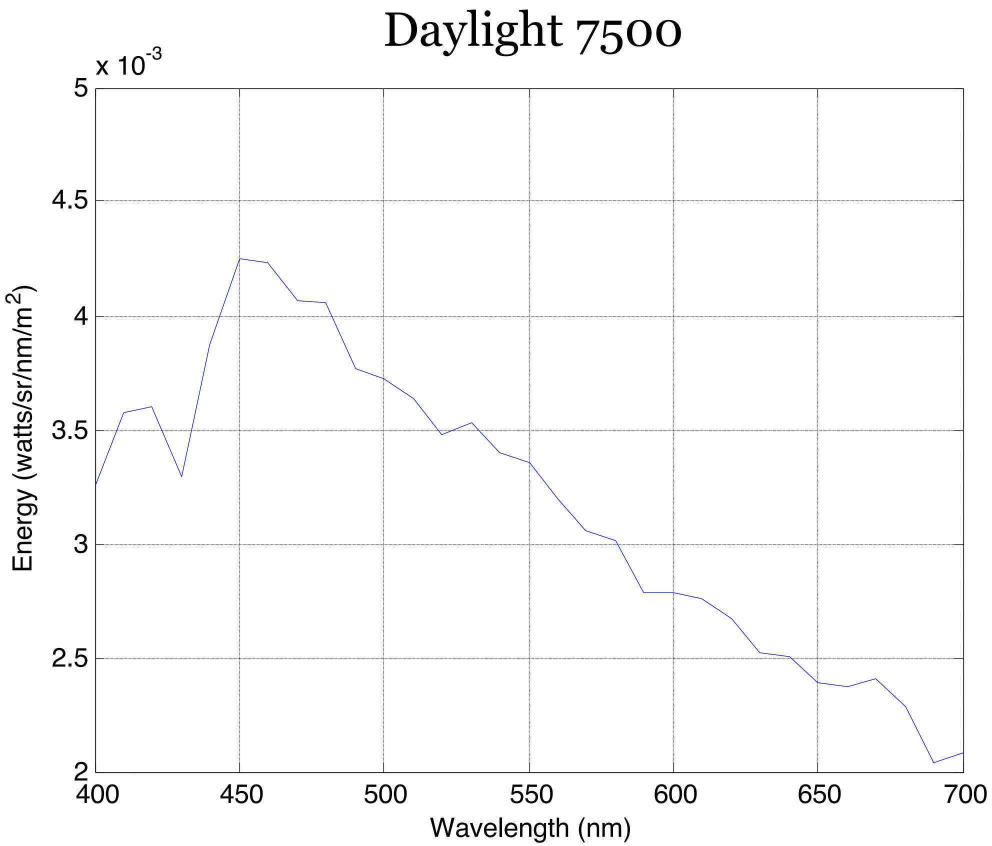
Illuminant changes



Illuminant changes



Illuminant changes



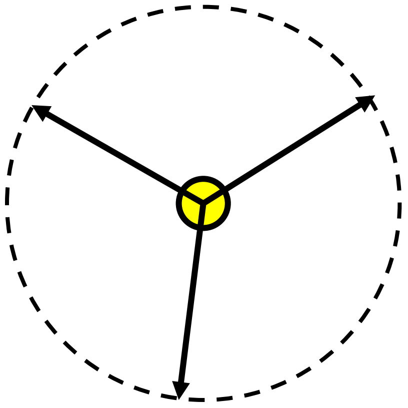
End

Radiance

Functions of wavelength

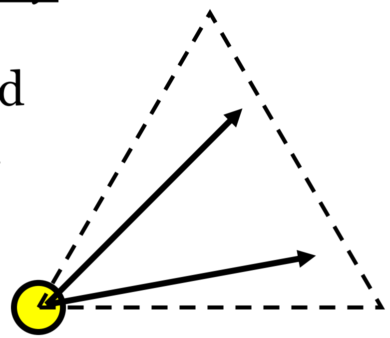
Radiant flux

Light emitted from a point in all directions



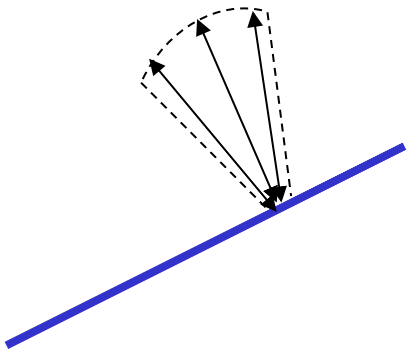
Radiant intensity

Light emitted from a point in a solid angle



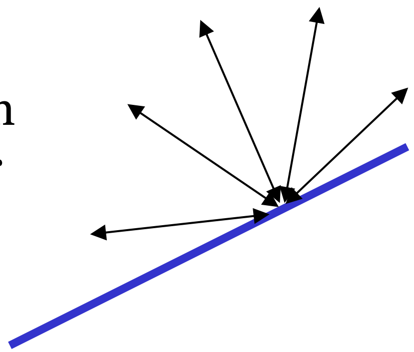
Radiance

Light scattered from an extended source in a given direction



Irradiance

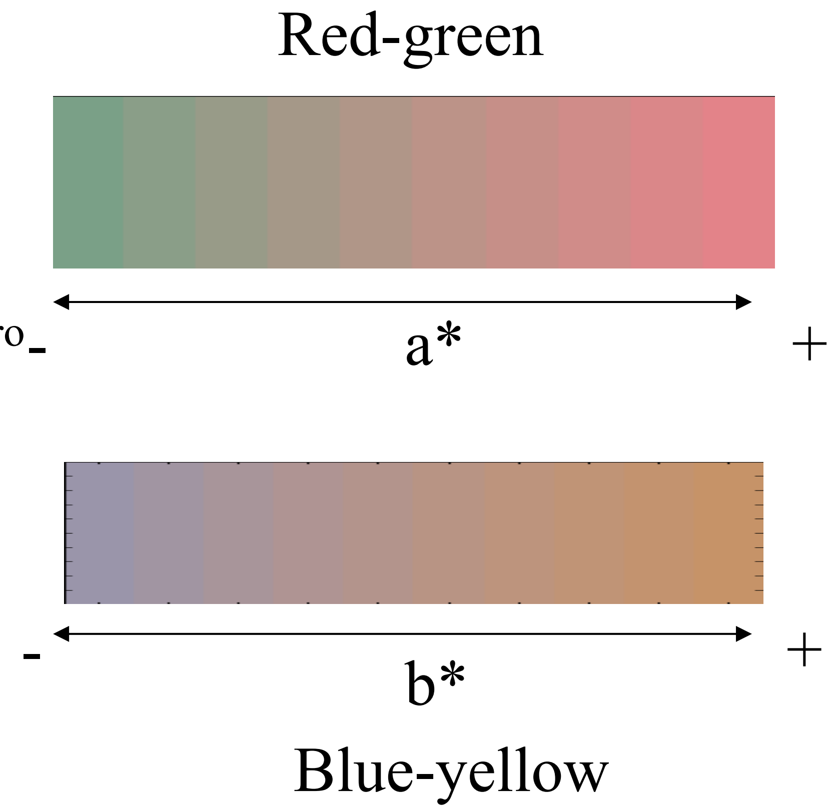
Total light falling on a surface (from all directions) per unit area



Tutorial: Imaging Lab Values

```
nChips = size(grayLAB,2);
aRamp = scale(1:nChips,-30,30);
varyAstar(1,:) = 70*ones(1,nChips);  % L is set to 70
varyAstar(2,:) = aRamp;               % a star varies
varyAstar(3,:) = zeros(1,nChips);     % b star is set ot zero

% Assuming a white point and convert from Lab to XYZ
varyXYZ = lab2xyz(varyAstar,whiteXYZ)
```



Tutorial: Imaging Lab Values

```
% You should understand these from basic color matching
rgb2xyz = XYZ'*phosphors;
xyz2rgb = inv(rgb2xyz);

whiteRGB = (xyz2rgb*whiteXYZ)'
whiteMax = max(whiteRGB)
varyRGB = (xyz2rgb*varyXYZ);
varyRGB = varyRGB/whiteMax;

% Use these routines to correct for gamma
invGamTable = mkInvGammaTable(monitorGam,1000);
fbRGB = round(dac2rgb(varyRGB',invGamTable))

% Display fbRGB values (see right)
```