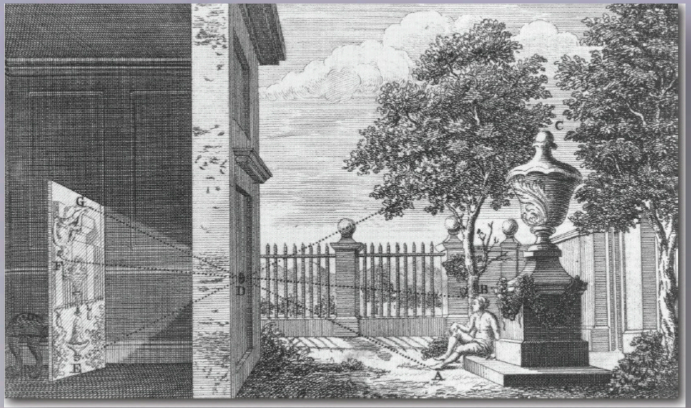
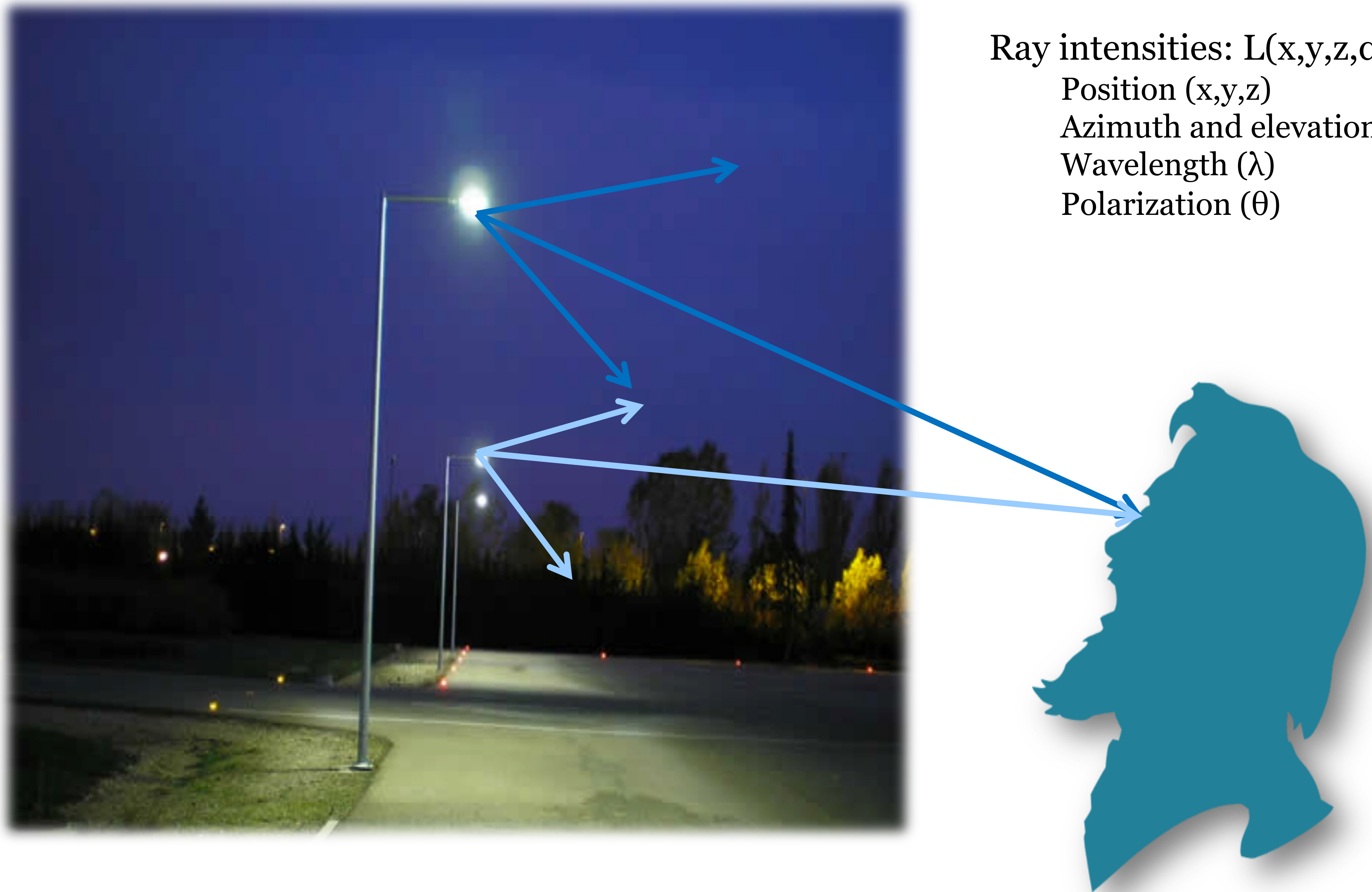
Introduction to image formation



- Scene radiance
- Pinhole cameras
- Diffraction
- Lenses, Snell's Law, lens maker's equation
- Geometric blur, depth of field

Scene spectral radiance in the world

Gershun (1936)



Ray intensities: L(x,y,z,α,β,λ,θ)

Position (x,y,z)

Azimuth and elevation (α, β)

Wavelength (λ)

Polarization (θ)

Light field at the eye (plenoptic function)

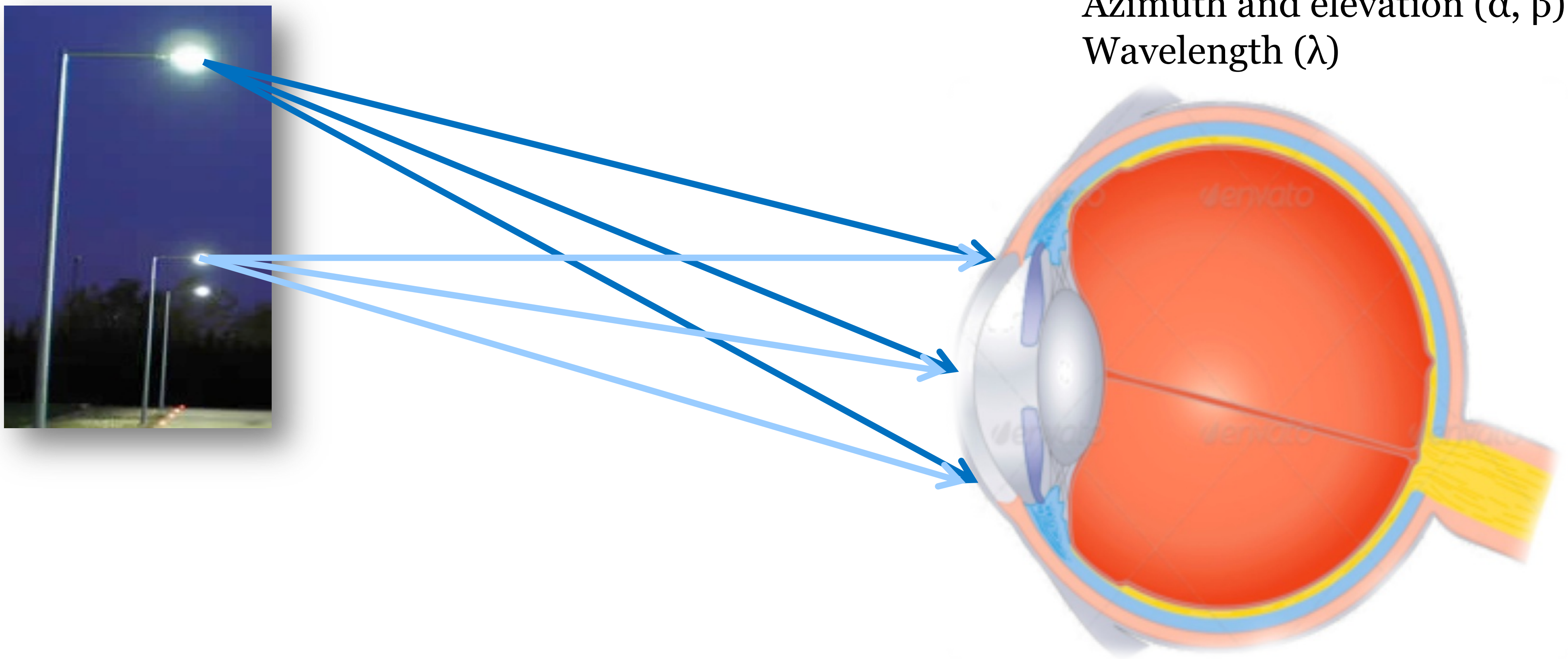
(Adelson and Bergen, 1991)

Ray intensities: $L(u,v,\alpha,\beta,\lambda)$

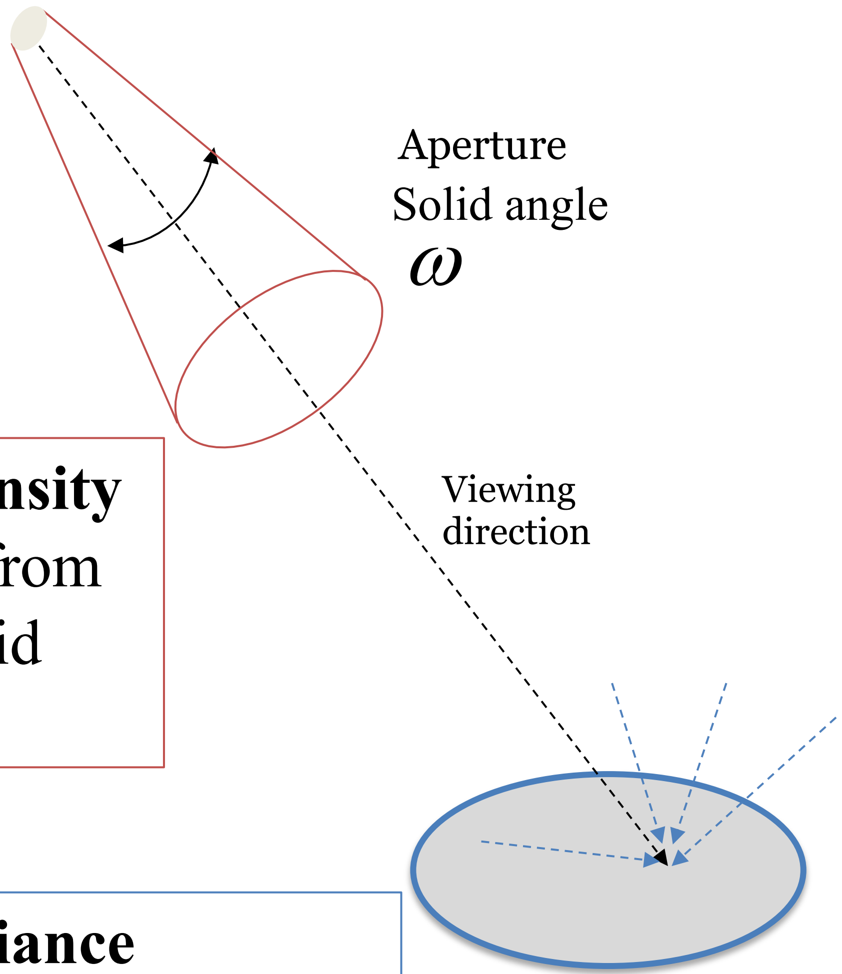
Position (u,v)

Azimuth and elevation (α, β)

Wavelength (λ)



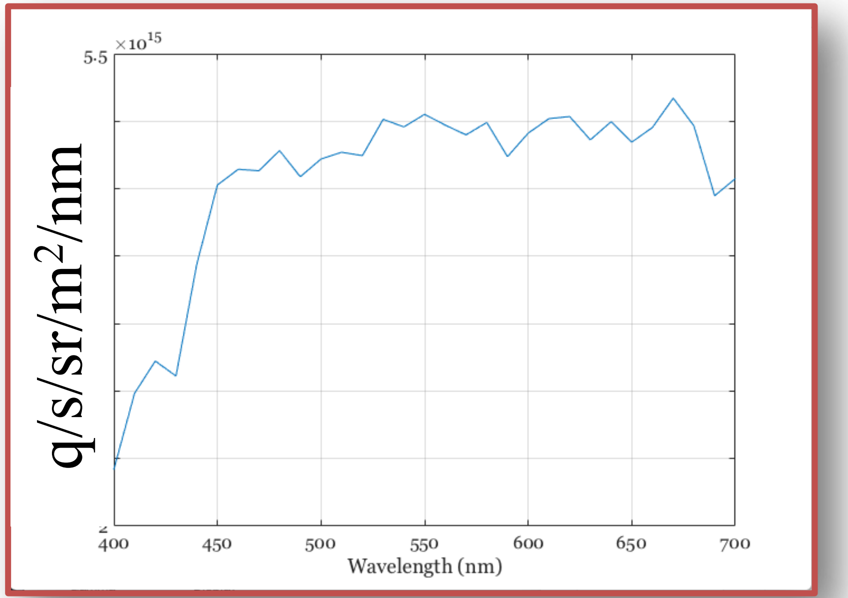
Scene **radiance** vs surface **irradiance**

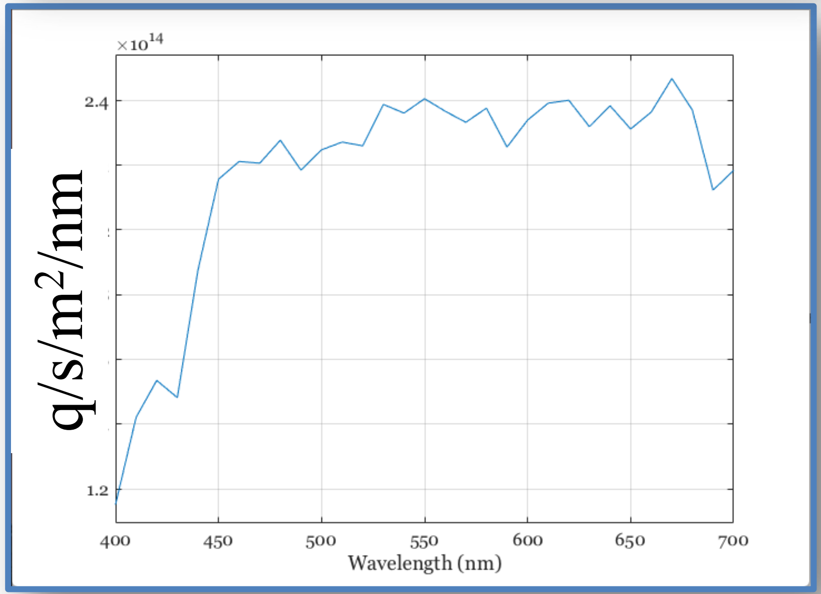


Radiance intensity
Light emitted from a point in a solid angle

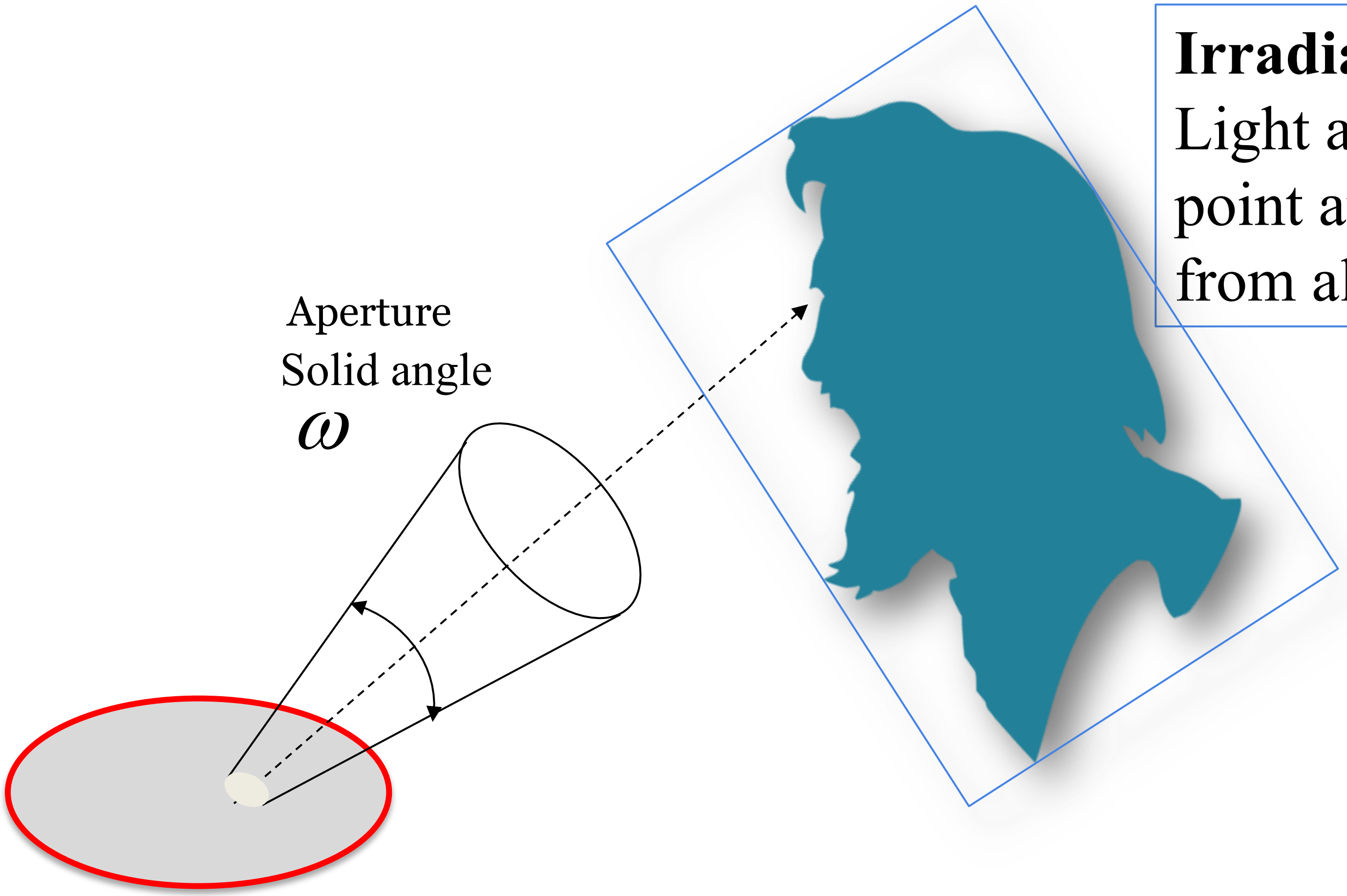
Irradiance
Light arriving at a point at a surface, from all angles

Notice the units

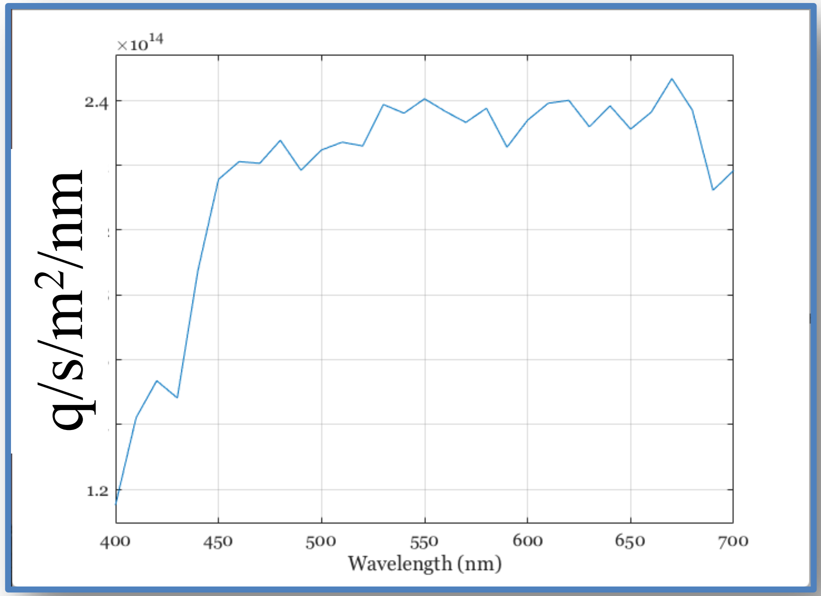


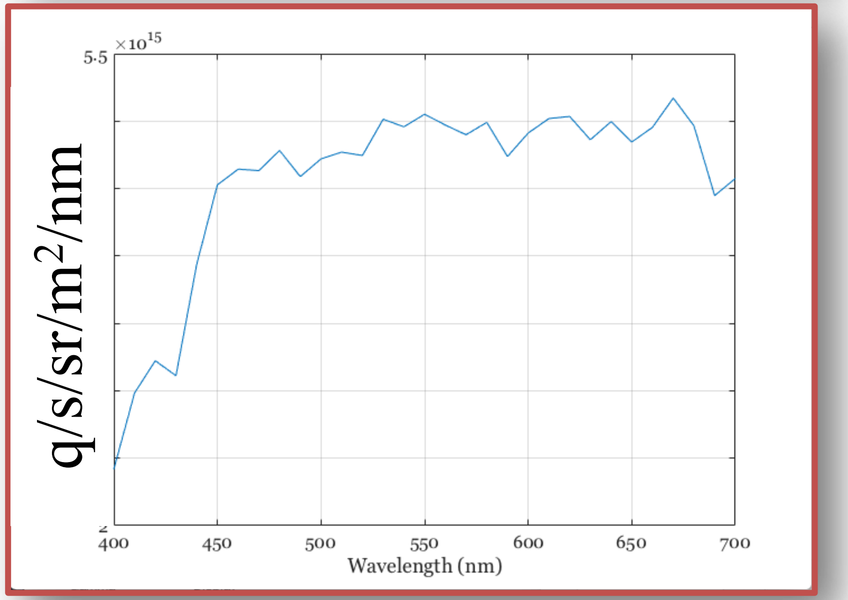


Scene radiance and surface irradiance



Irradiance
Light arriving at a point at a surface, from all angles

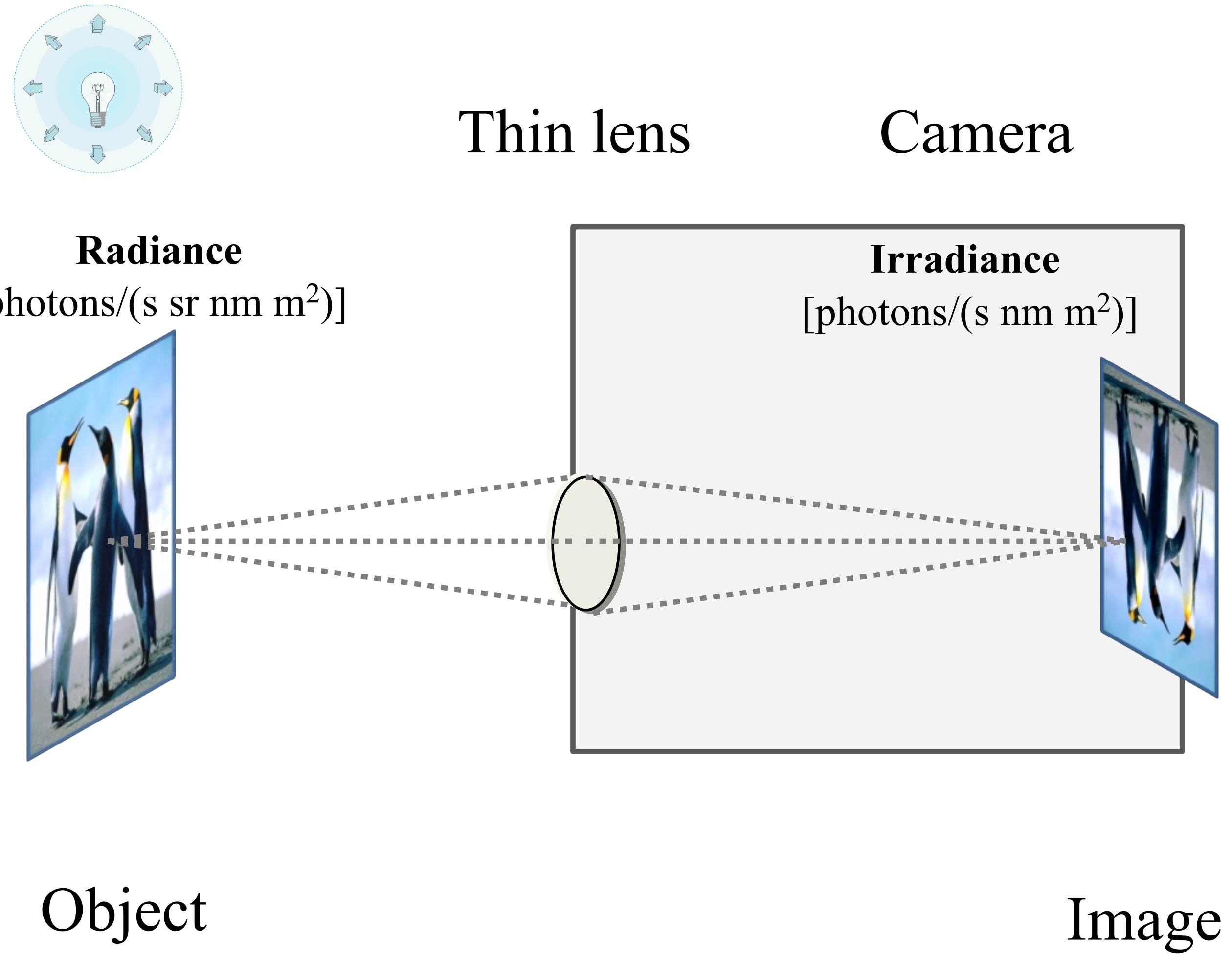




Radiance intensity
Light emitted from a point in a solid angle

Optics map scene radiance to image irradiance

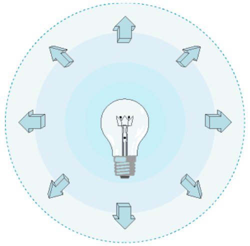
- A point in object space should map to a unique point in image space
- Capture as many photons from the object space as possible



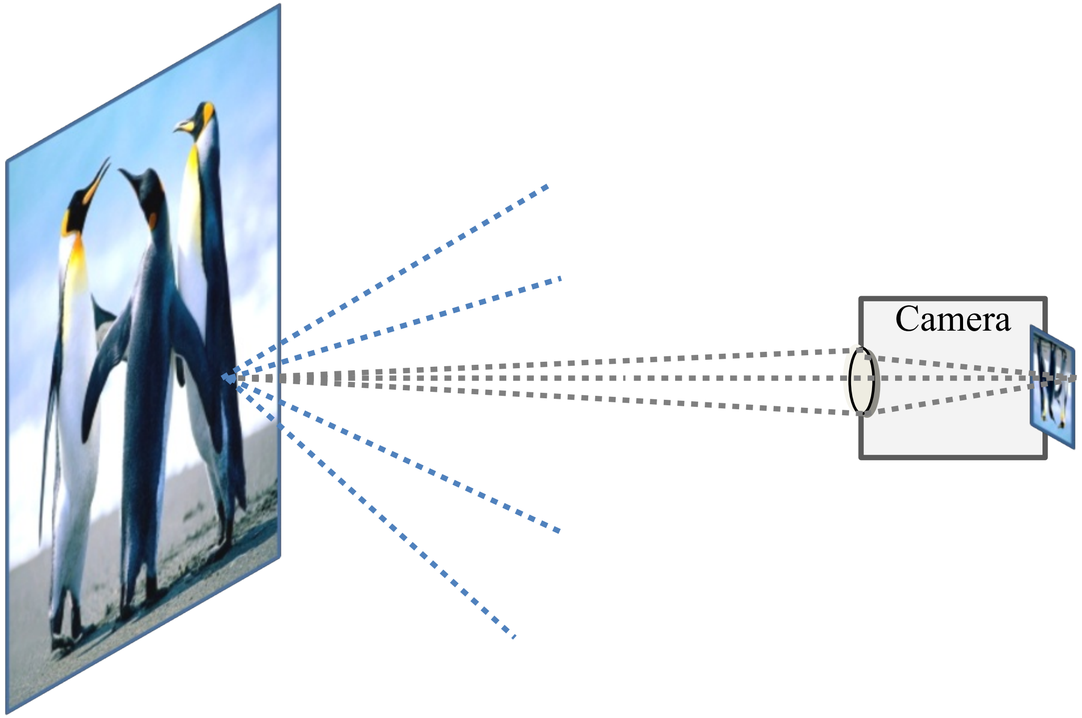
Scenes are not planes! We start this way for mathematical simplicity. We will provide computational tools for 3D

Optics map scene radiance to image irradiance

- Remember that compared to the camera/eye objects for image sensing are large
- Rays are everywhere; the camera captures a very small fraction of ambient radiation
- The exception is the microscope!



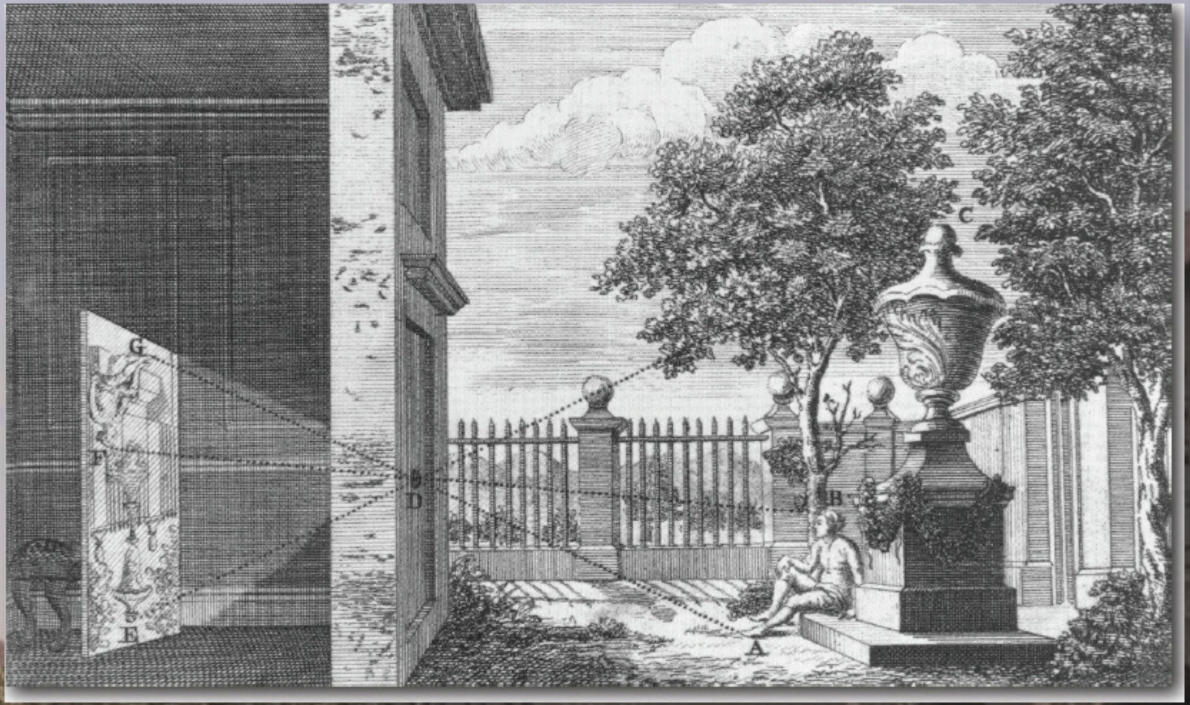
Radiance
[photons/(s sr nm m^2)]



Object

Pinhole cameras

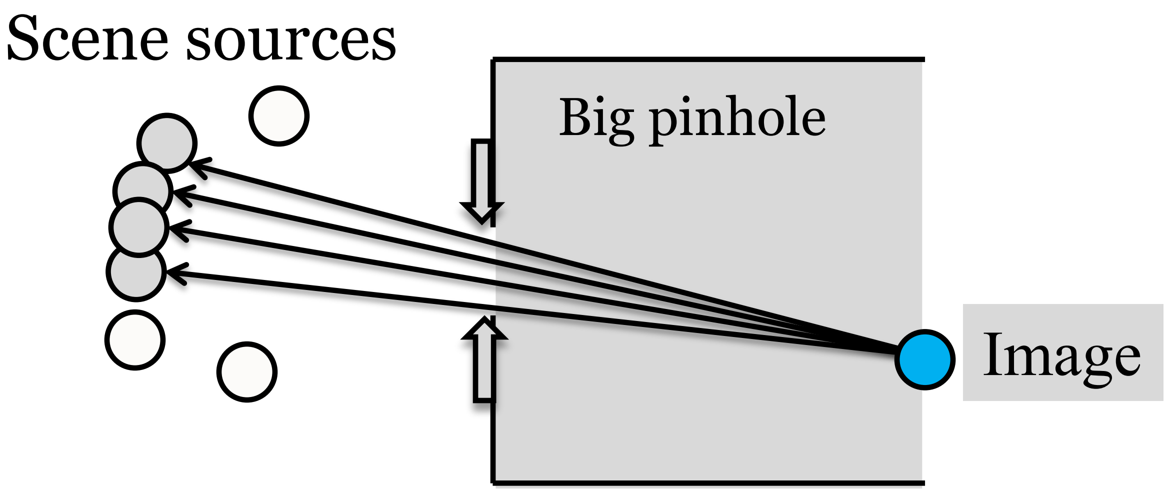
Eighteenth century engraving showing a camera obscura



Wikimedia Commons

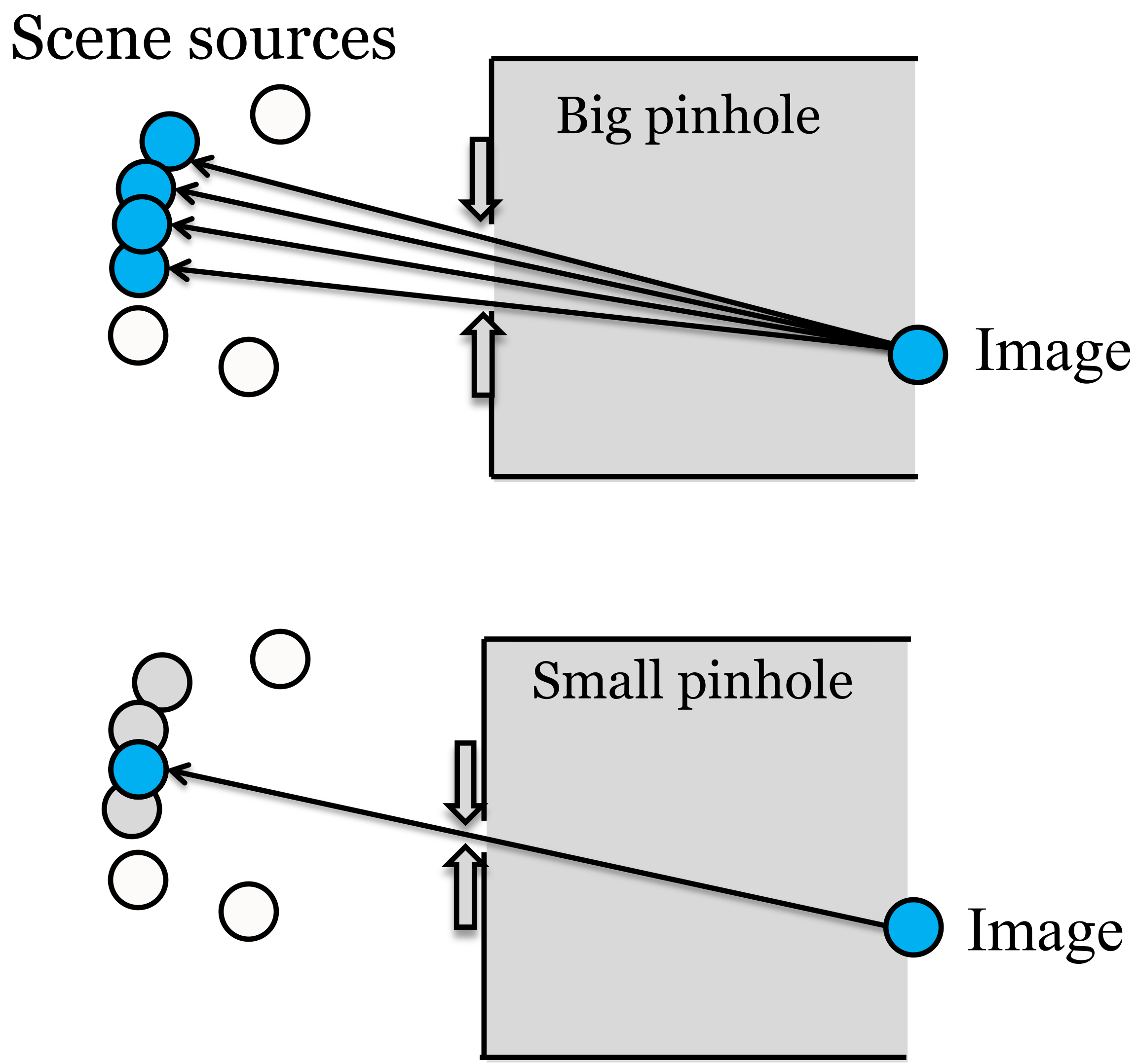
Pinhole optics: What arrives at a single image point?

- Pinhole optics achieve the point to point mapping by limiting which scene points contribute to an image point



Pinhole optics

- Pinhole optics achieve the point to point mapping by limiting which scene points contribute to an image point
- Small pinholes make sharper images, but fail to capture many photons



Pinhole cameras illustrated

Eighteenth century engraving showing a camera obscura

Accidental pinhole cameras (Torralba and Freeman)

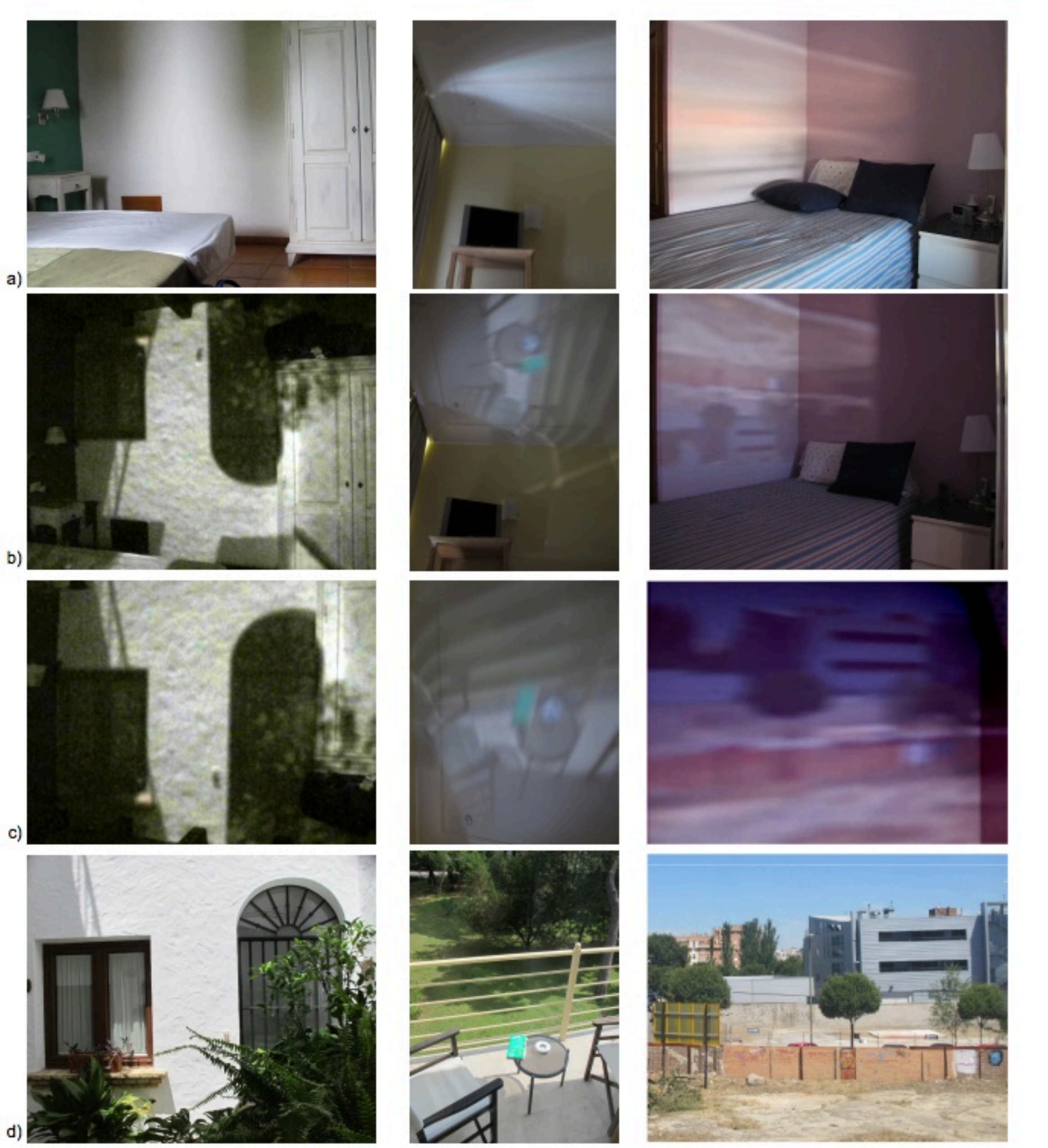
Accidental pinhole cameras (Torralba and Freeman)

- Three different rooms

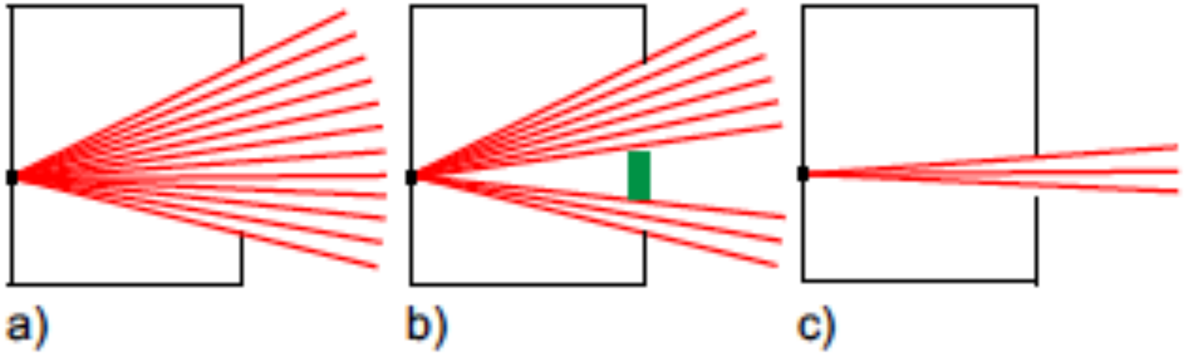
- Narrowing the window reveals the 'camera obscura' image

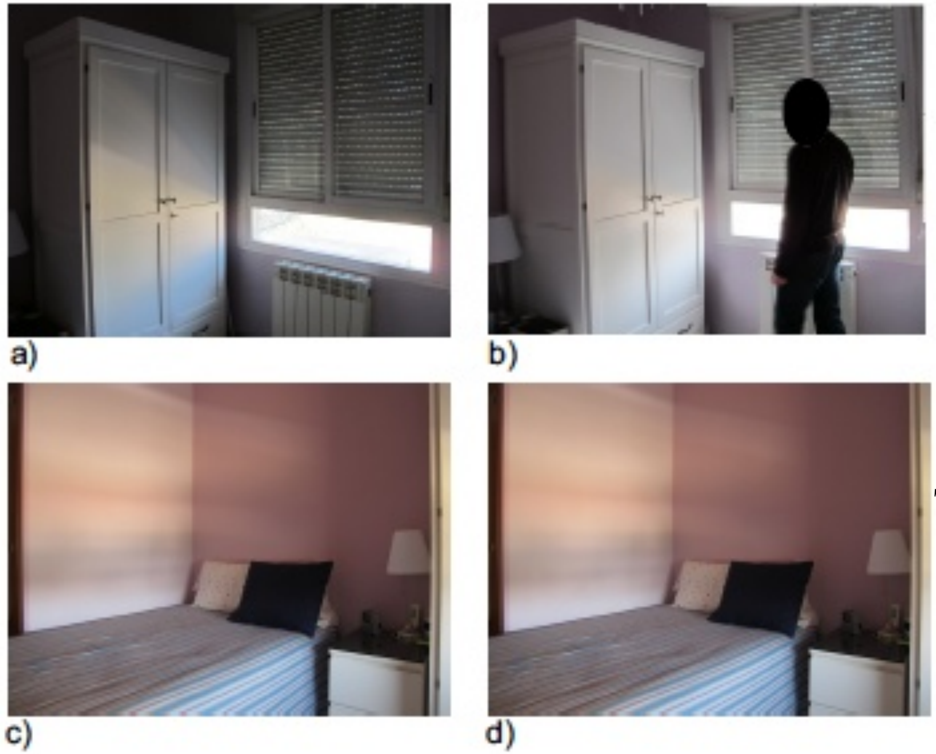
- Flipping up down

- The scene outside – showing the match



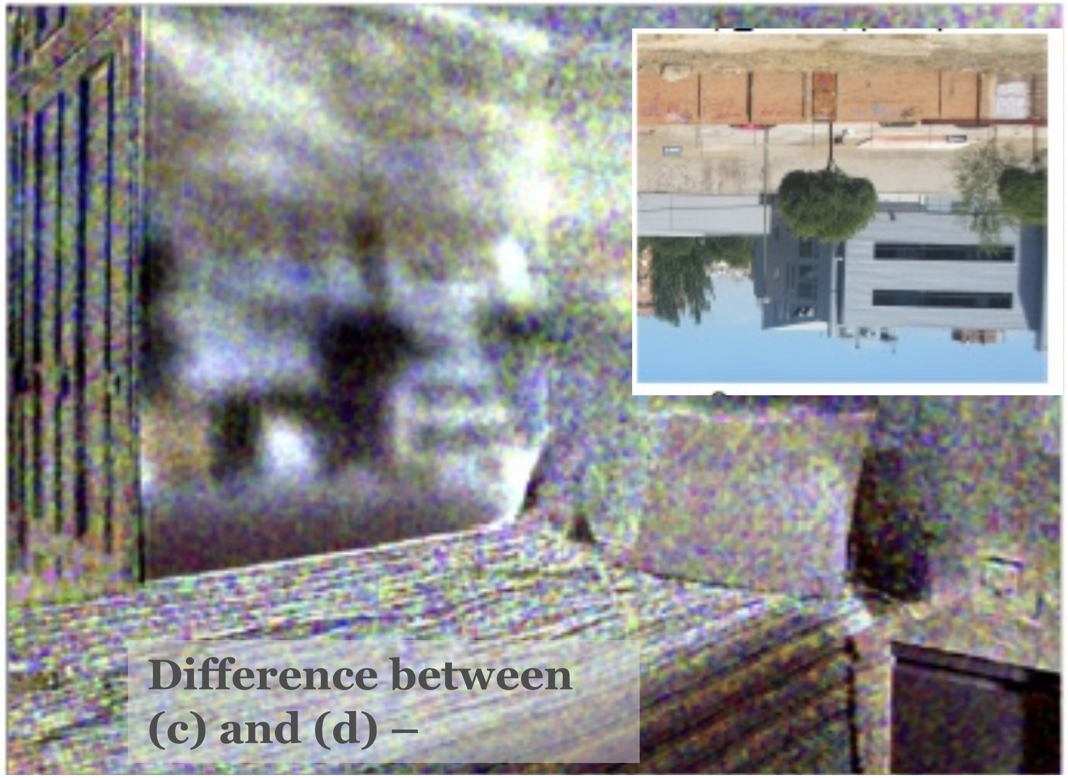
Making a pinhole camera - The spy scenario (Torralba and Freeman)





Window and partially occluded window by the spy

Two associated images



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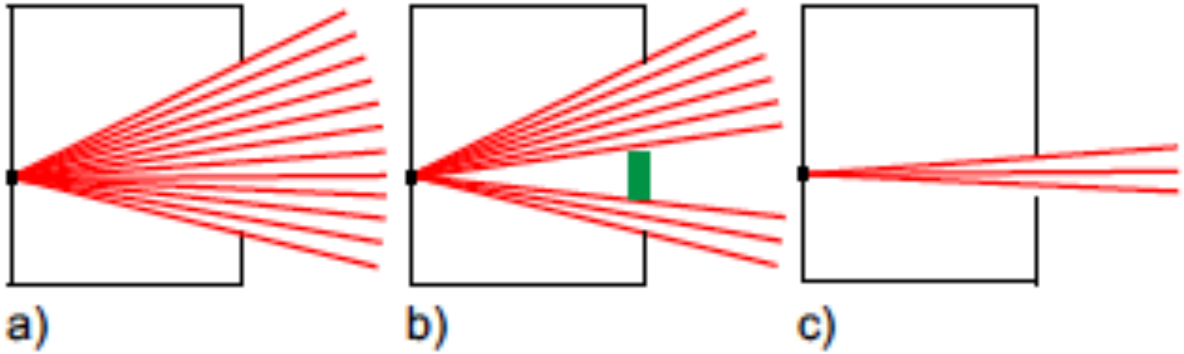

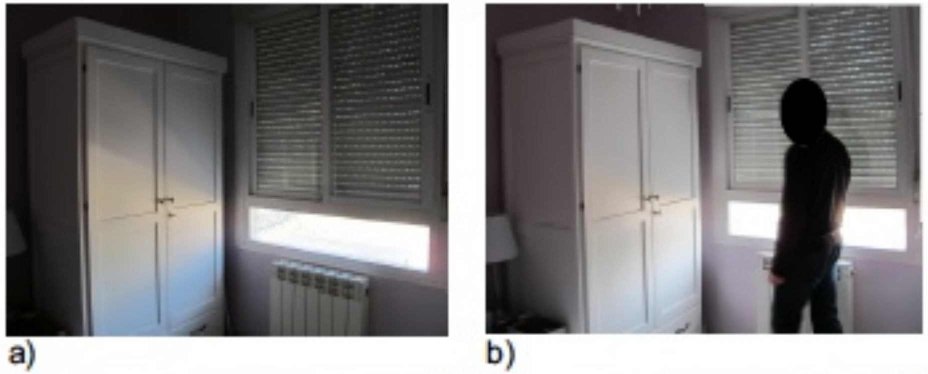

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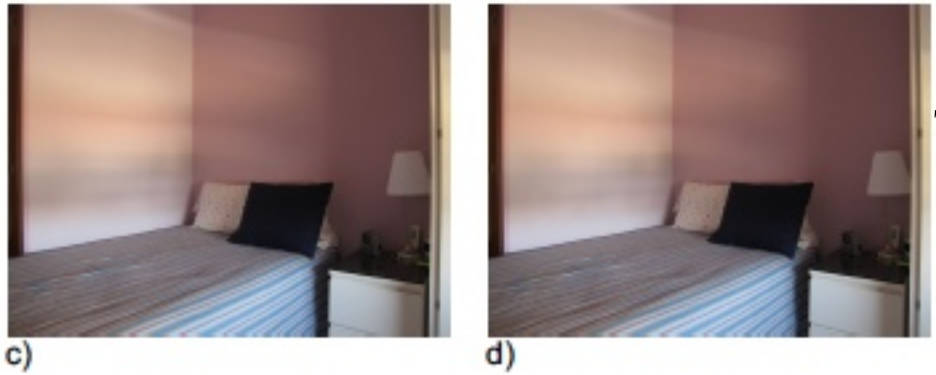
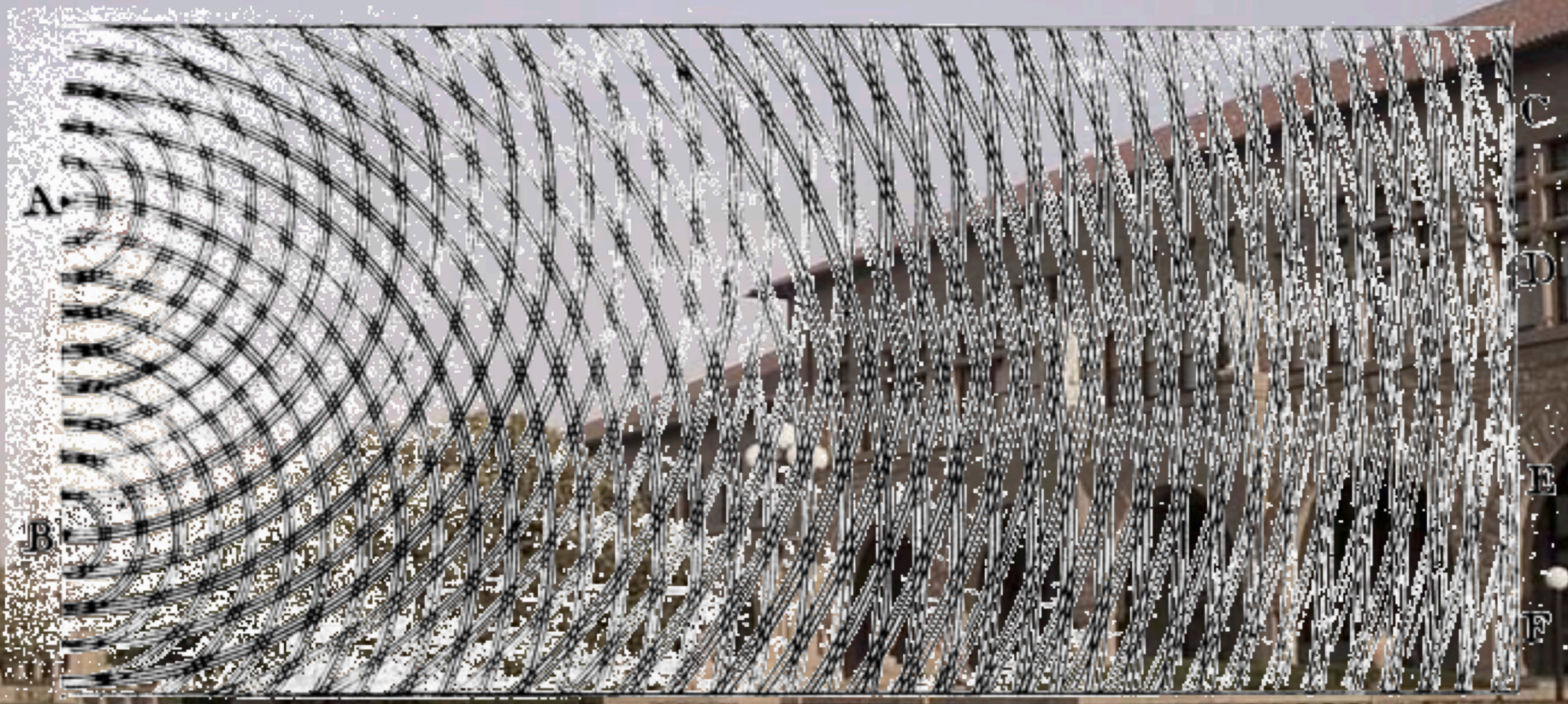


Two associated images

Diffraction

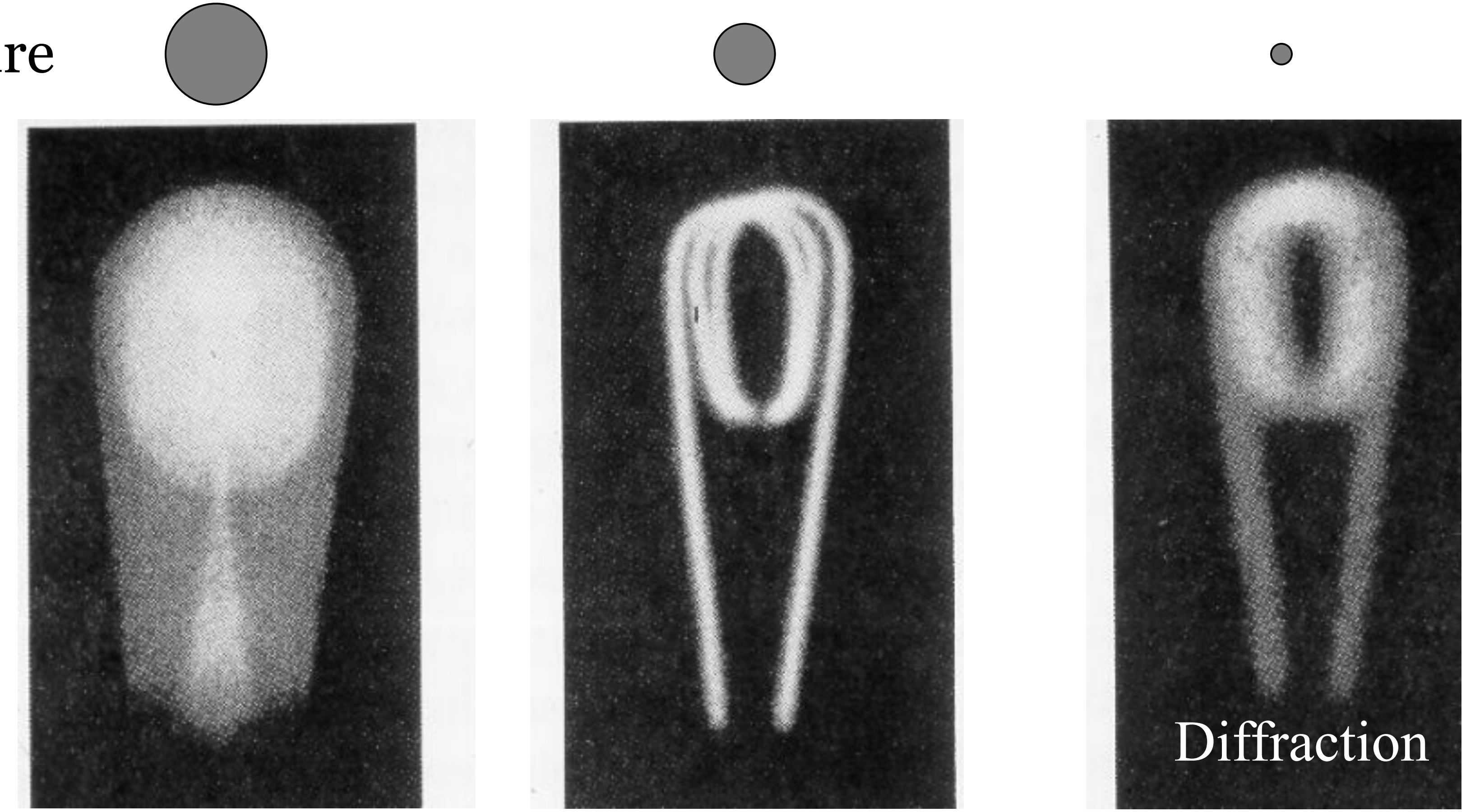
Suggestion: Before Tuesday's class, please have a look at the two video overviews of ISET. Also, please download ISETCam and the github homework repository, as explained on the Canvas web-site.

These two videos are intended to help you get started with the software. We will help more on Tuesday in class, as well: [Explaining ISET simulation.] - overview; [ISET Overview (short version).]



Diffraction: Another consequence of shrinking the pinhole

Aperture



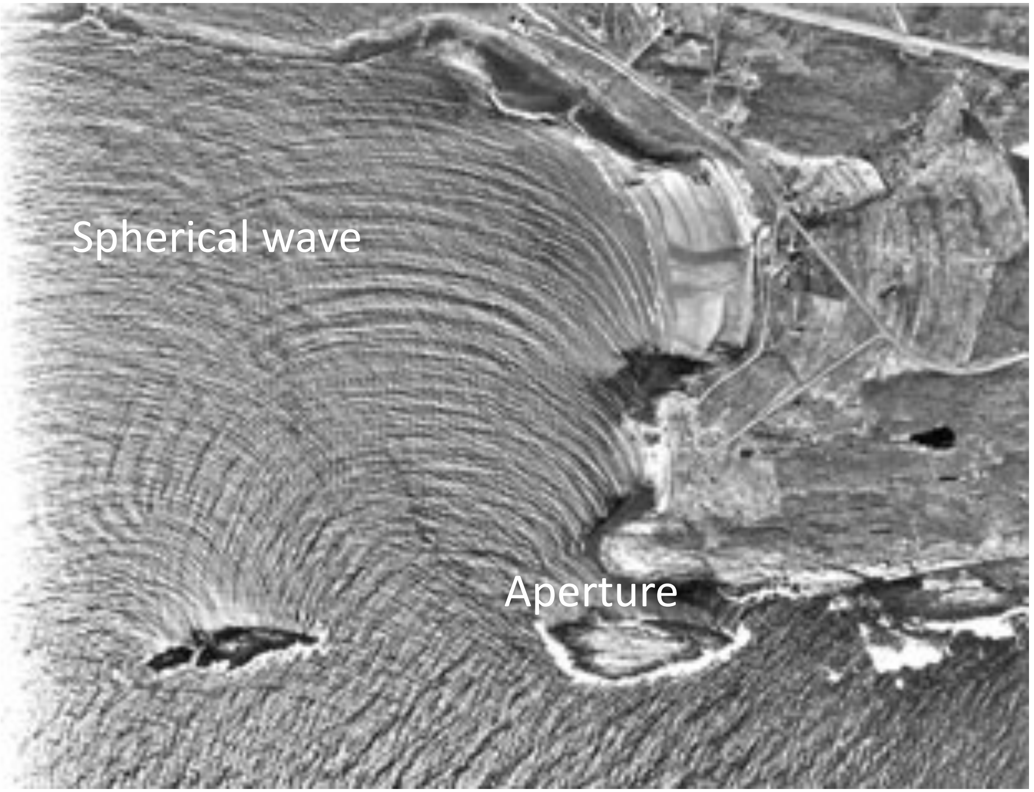
(From Jenkins and White)

Diffraction: Waves

[Diffraction movie](https://www.youtube.com/watch?v=BH0NfVUTWG4) (YouTube)

https://www.youtube.com/watch?v=BH0NfVUTWG4

- Parallel waves incident on an aperture
- After the aperture, the waves form a spherical wavefront
- This can be explained if we consider light as a wave phenomenon



Parallel waves at the aperture

Diffraction: Rays

Diffraction movie (YouTube)

https://www.youtube.com/watch?v=BH0NfVUTWG4

- Parallel waves incident on an aperture
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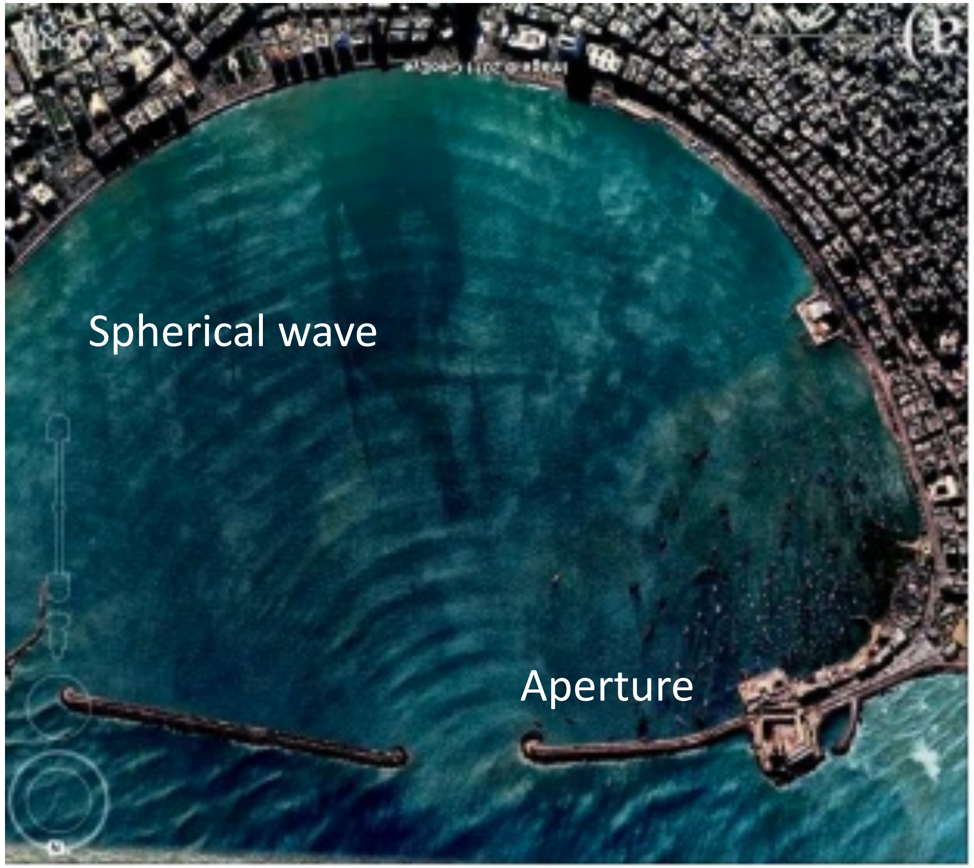
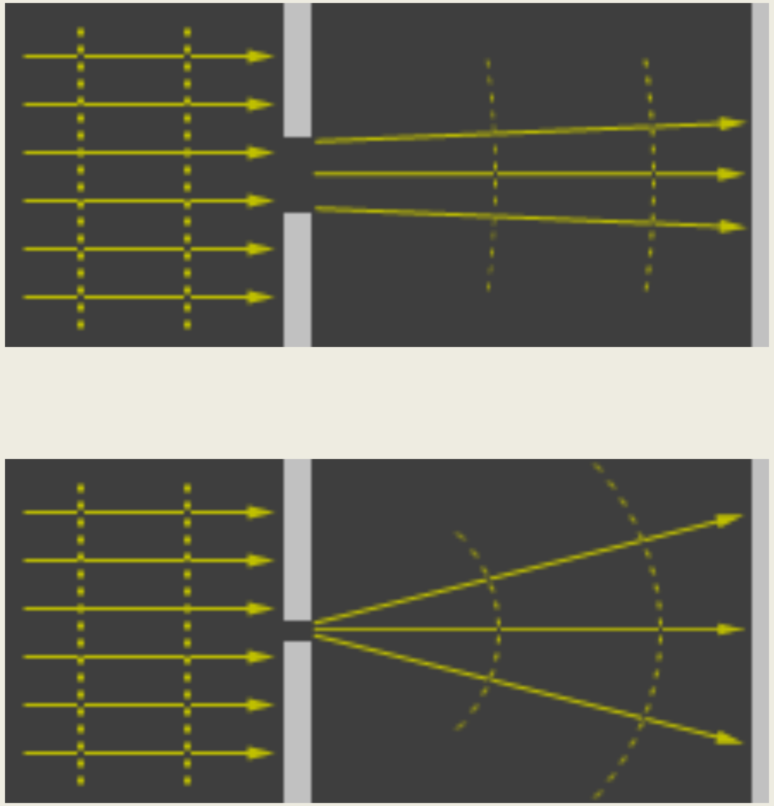


Parallel waves at the aperture

Diffraction: Rays

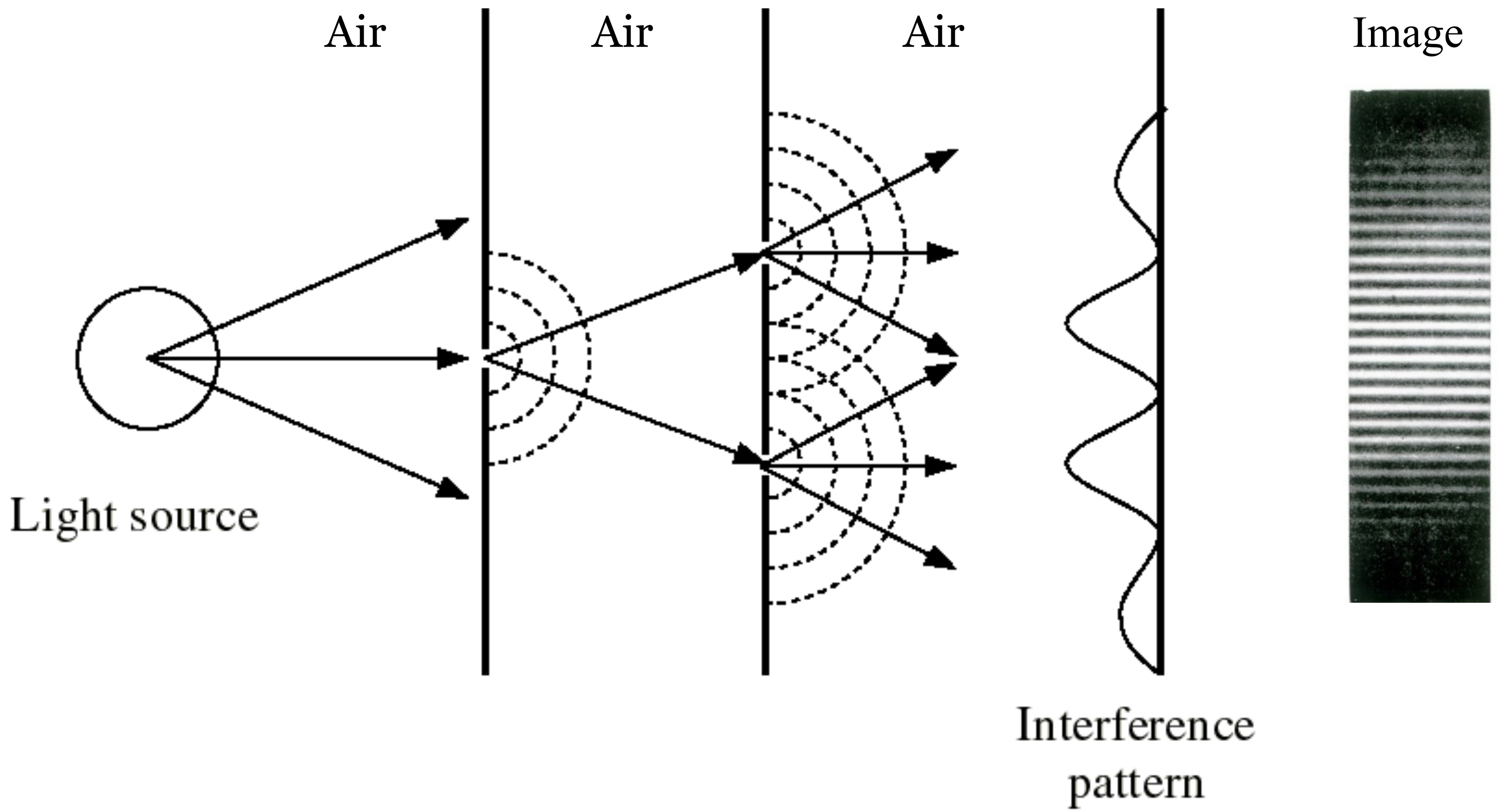
https://www.youtube.com/watch?v=AXaZc-VQzWk
https://www.youtube.com/watch?v=cKY9KzMezdY
()

- Parallel rays incident on an aperture, say rays from a point source at infinity, begin to diverge.
- The smaller the aperture, the larger the divergence.
- This can be explained if we consider light as a wave phenomenon



Light as a wave phenomenon

Thomas Young's double-slit experiments

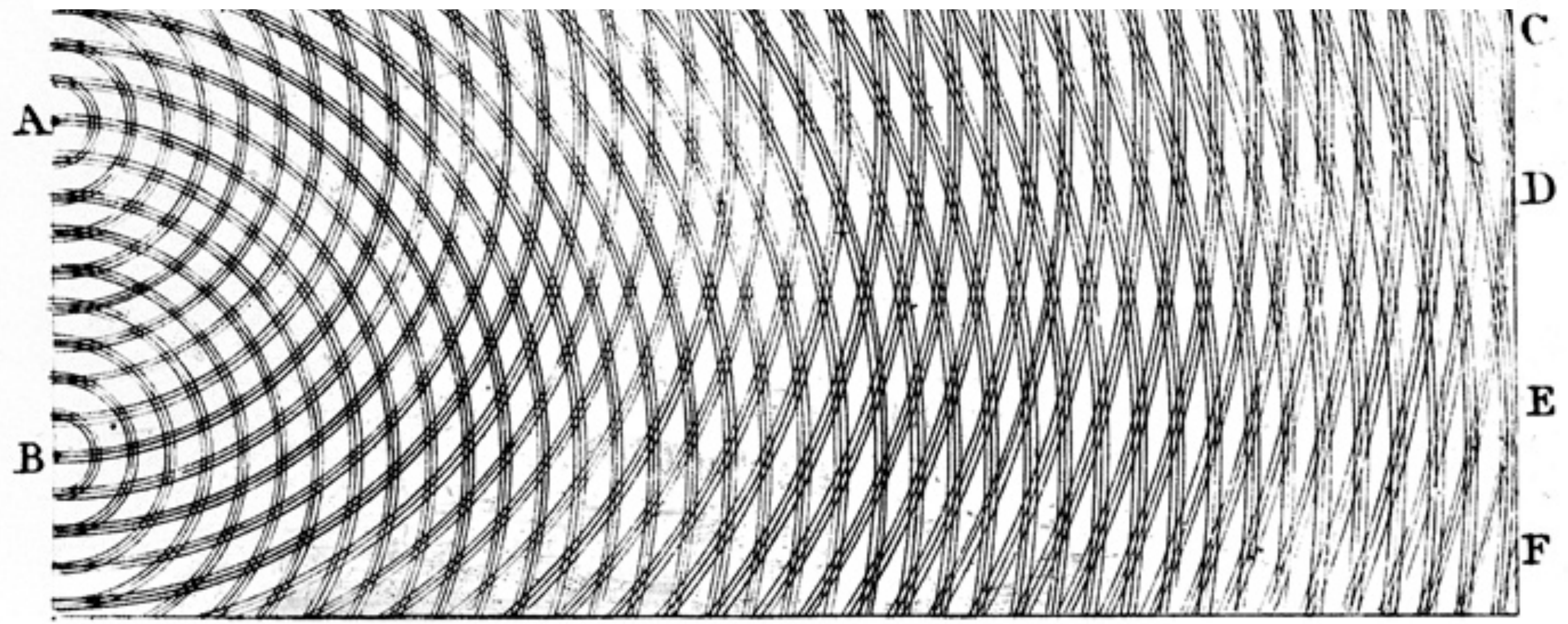


Light as a wave phenomenon

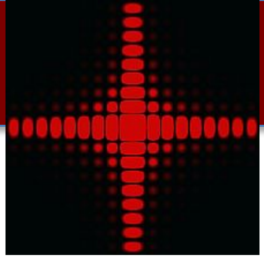
Thomas Young's double-slit experiments

Thomas Young's sketch presented to the Royal Society in 1803

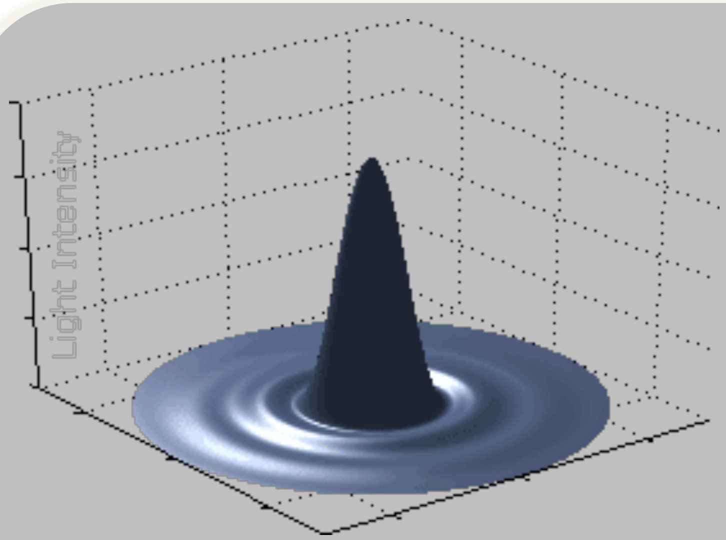
http://en.wikipedia.org/wiki/Diffraction



Diffraction in space domain



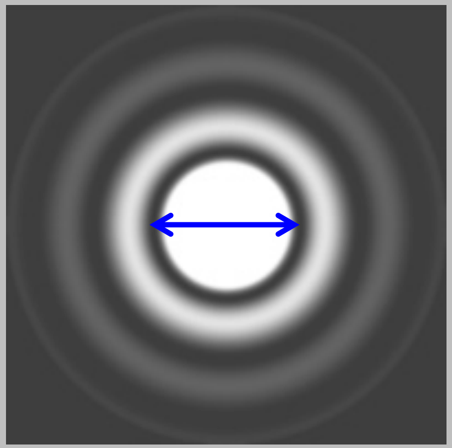
Square aperture



$$PSF(x) = \frac{[2 \cdot J_1(u)]^2}{u^2}$$

$$\text{with} \quad u = \frac{\pi \cdot x}{\lambda \cdot f/\#}$$

$$\text{and} \quad x_{cuttoff} = 1.22 \cdot \lambda \cdot f/\#$$



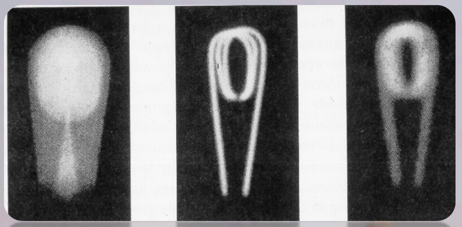
Airy disk diameter: 2.44 λ f/#

For an optical system with circular aperture
the finite image spot forms an Airy Disk (Point Spread Function)

Shift-invariant linear systems calculations in the space domain (convolution)

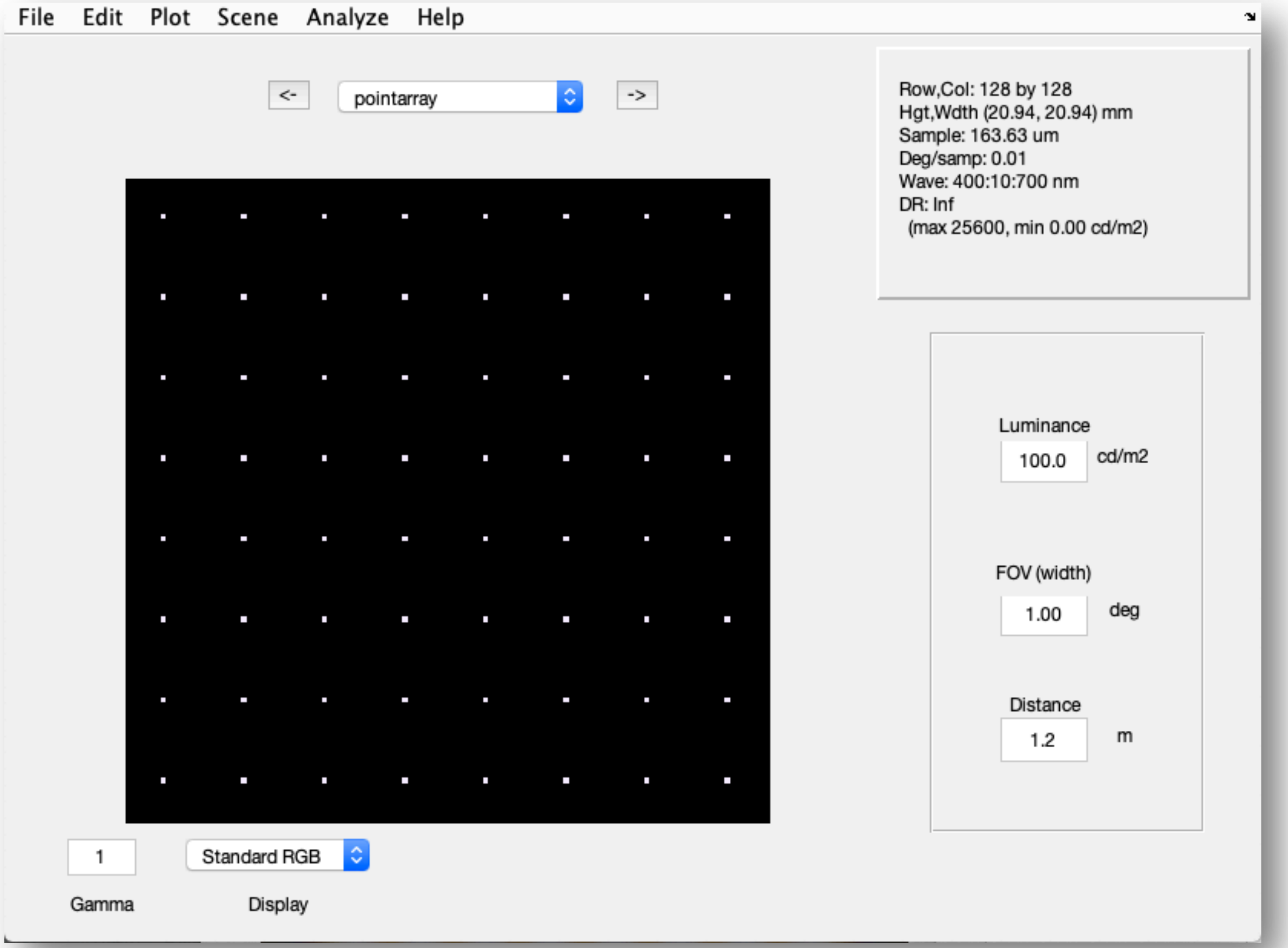
- General shift-invariant point spread functions (isoplanatic)

$$irrad(x,\lambda) = S \times \int radiance(z,\lambda)\; psf(x-z,\lambda)\, dz$$



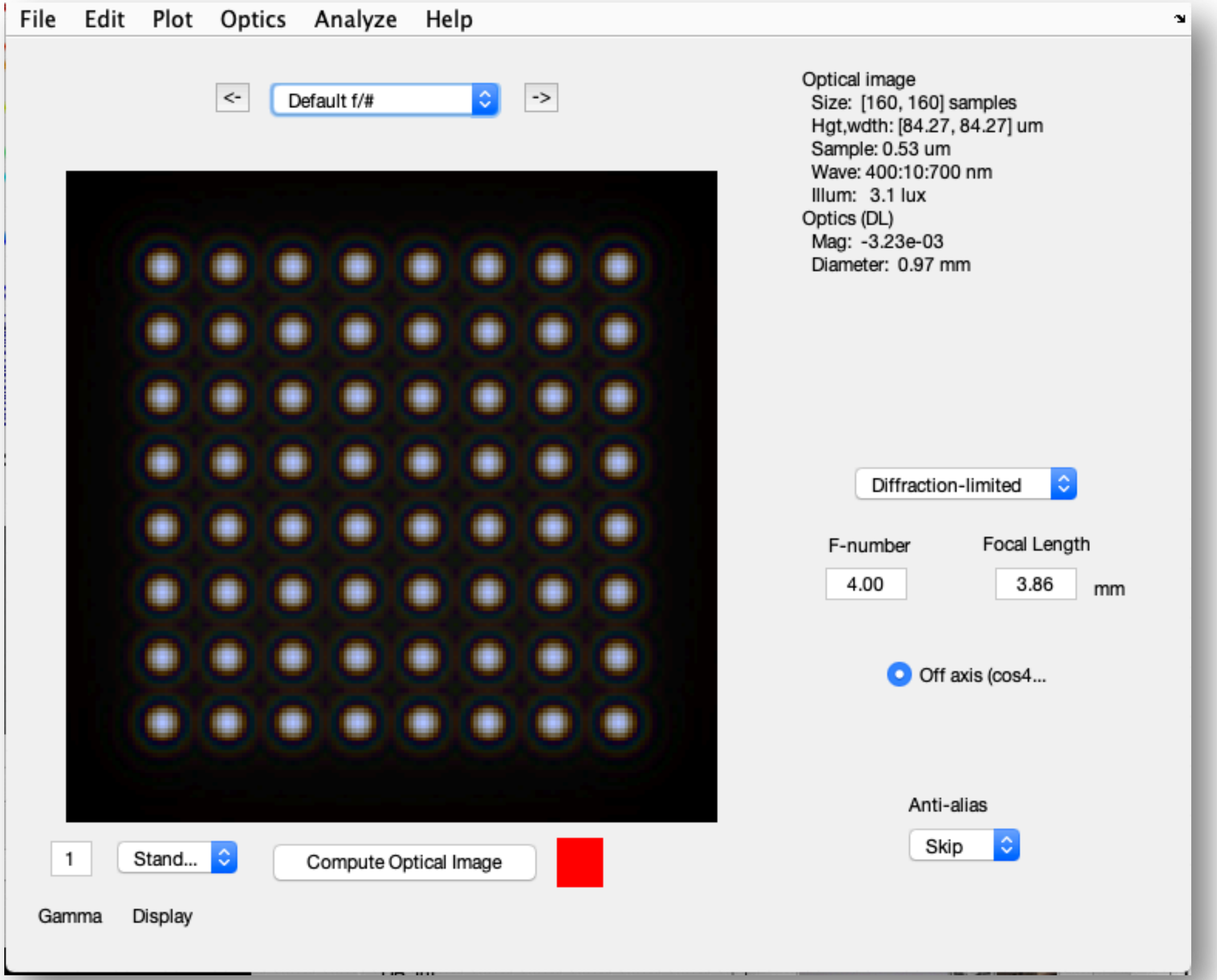
ISET: Diffraction illustration

```
%% Create a point array scene
scene = sceneCreate('point array');

% Degrees
scene = sceneSet(scene,'h fov',1);

sceneWindow(scene);
```

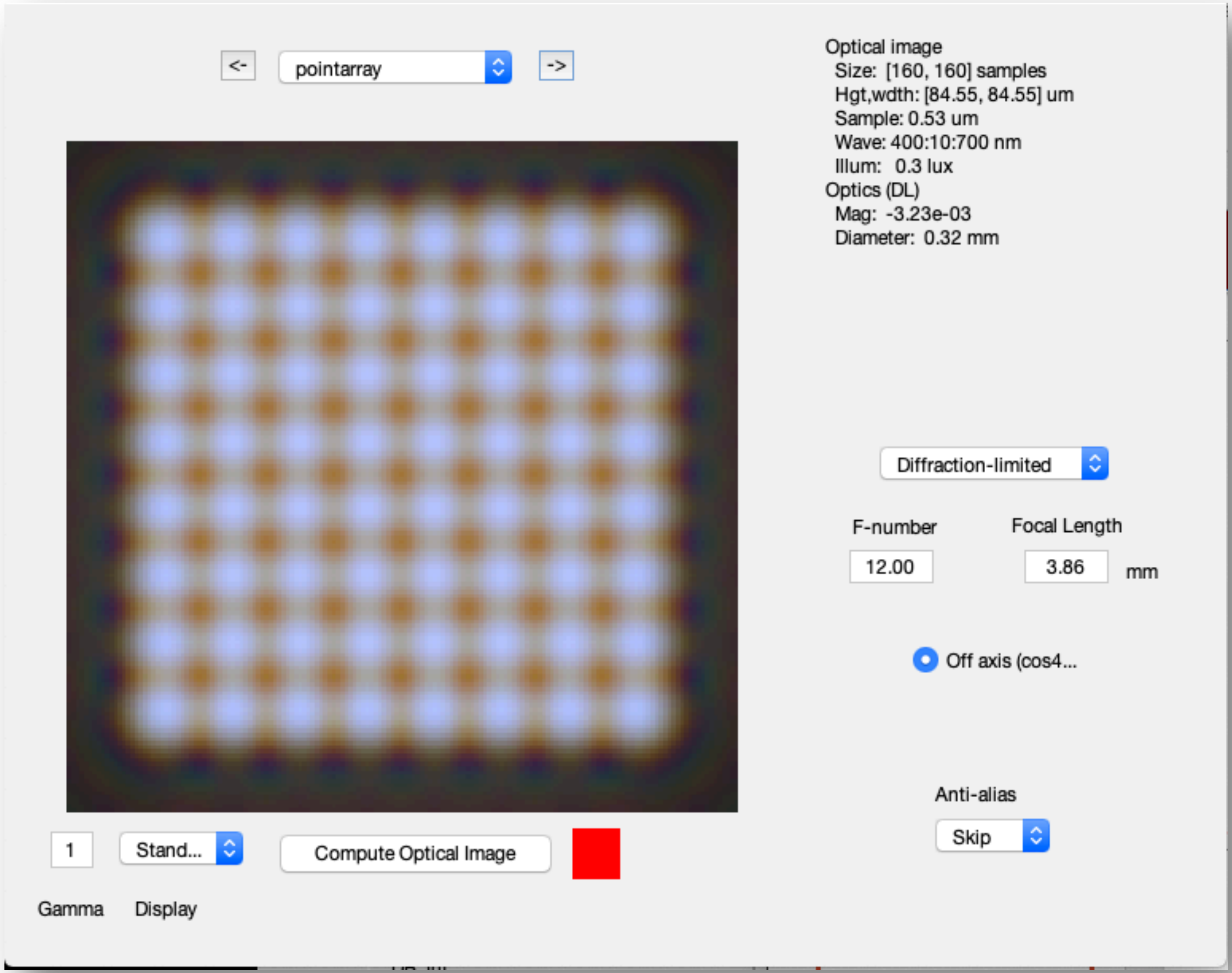


ISET: Diffraction illustration

```
%% Compute the sensor irradiance
oi = oiCreate;
oi = oiCompute(oi,scene);
oi = oiSet(oi,'name','Default f/#');

oiWindow(oi);
```

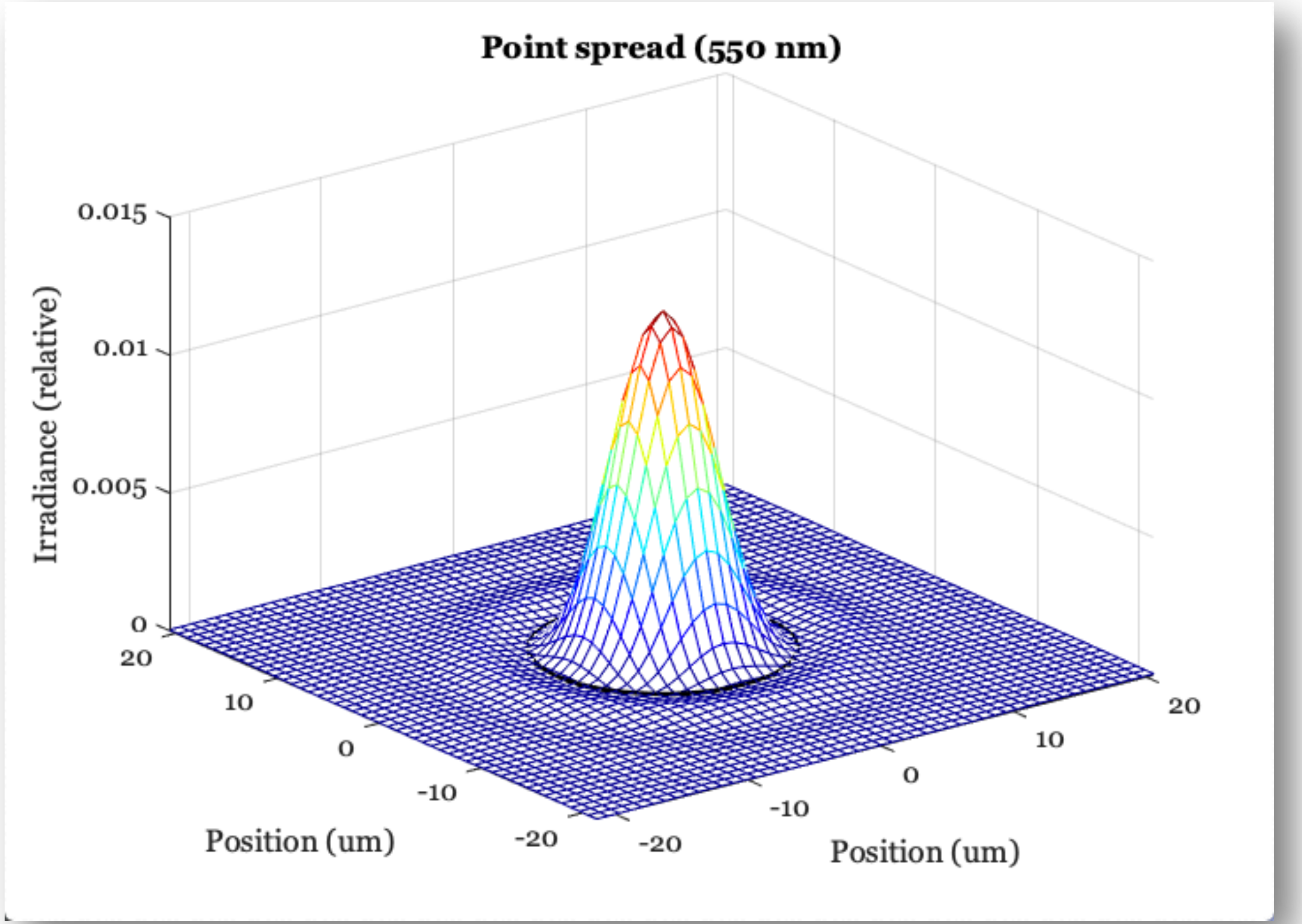


```
% Increase the f/#
% Blurs the image more
% It has larger depth of field

oi = oiSet(oi,'optics fnumber',12);
oi = oiSet(oi,'name','Large f/#');
oi = oiCompute(oi,scene);

oiWindow(oi);
```



ISET: Diffraction illustration

%% Plot the point spread
oiPlot(oi,'psf 550');

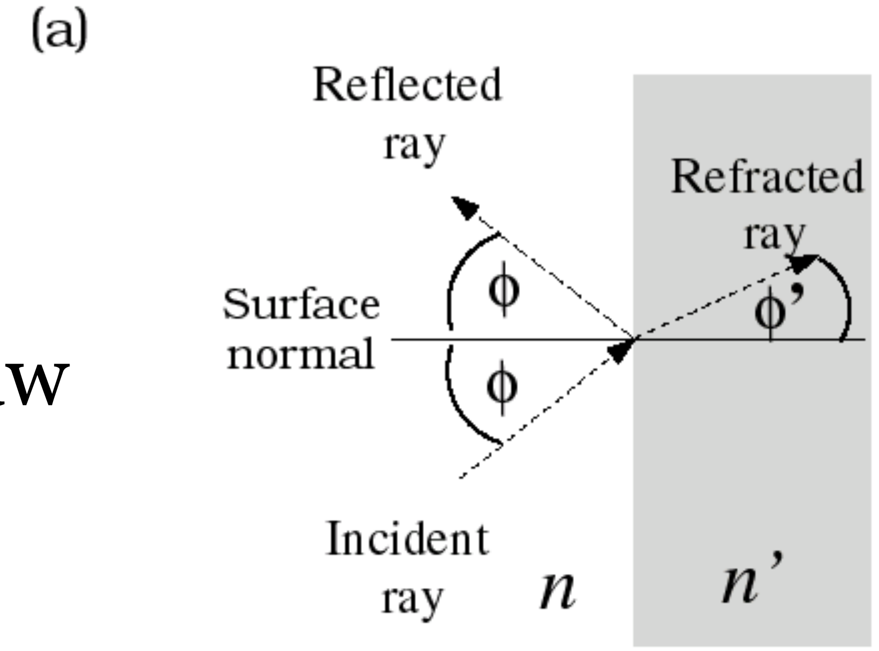


Lenses - formulae and parameters

- Radiance to irradiance
- Focal length and power
- F-number and Numerical aperture

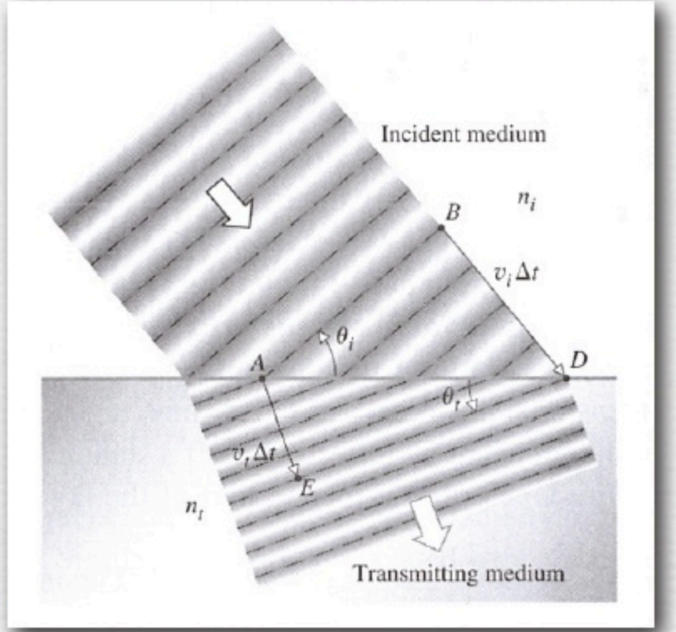
Willebrord Snellius

Lens Design: Snell's Law

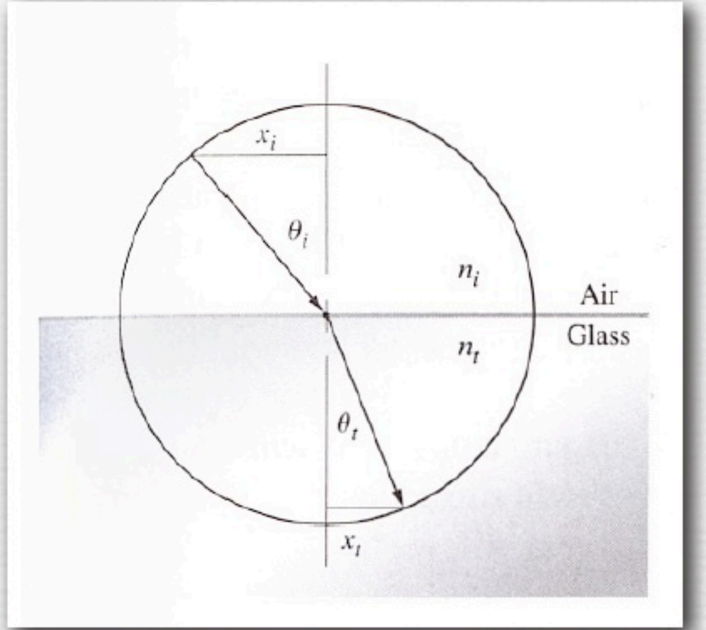


$$\frac{\sin(\phi)}{\sin(\phi')} = \frac{n'}{n}$$

Geometry of Snell's Law



(Hecht)

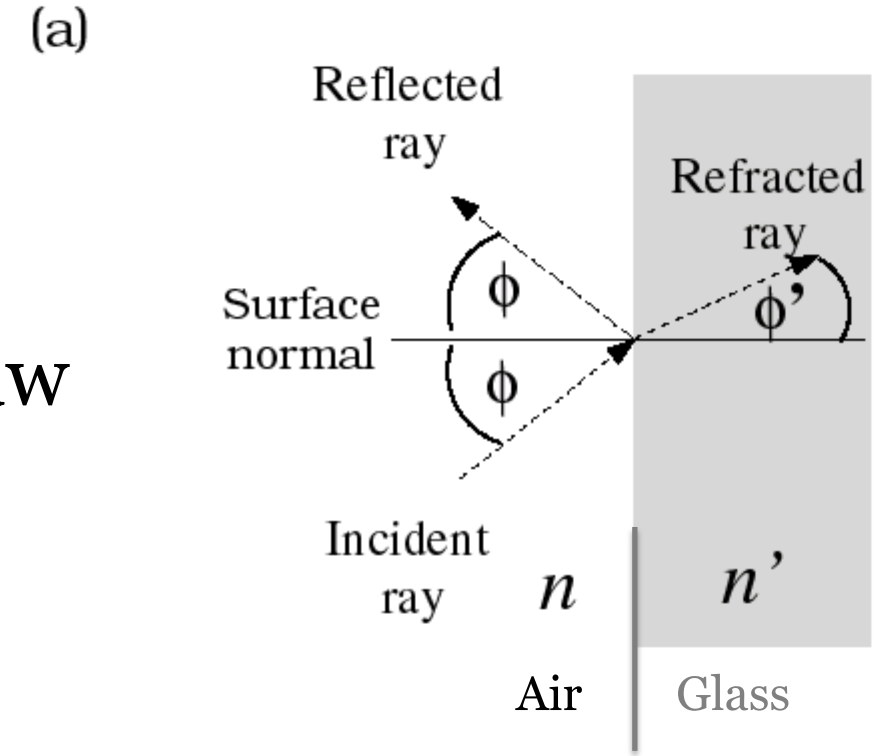


The speed of light changes at the material interface

Parts of the wavefront reach the interface earlier than other parts

Hence, the wave changes direction

Lens Design: Snell's Law



$$\frac{\sin(\phi)}{\sin(\phi')} = \frac{n'}{n}$$

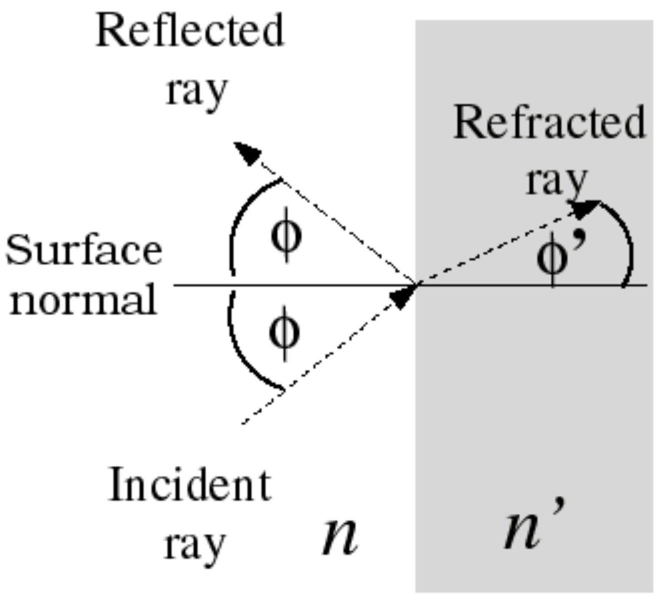
<u>n is the index of refraction</u>

$$n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in the medium}}$$

n is about 1 in air, and about 1.5 in glass
The value is wavelength dependent

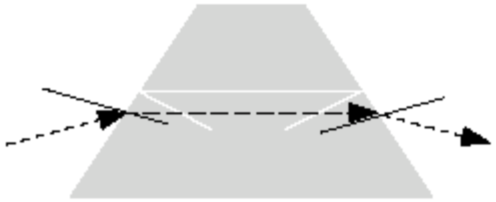
Lens Design: Snell's Law

(a)

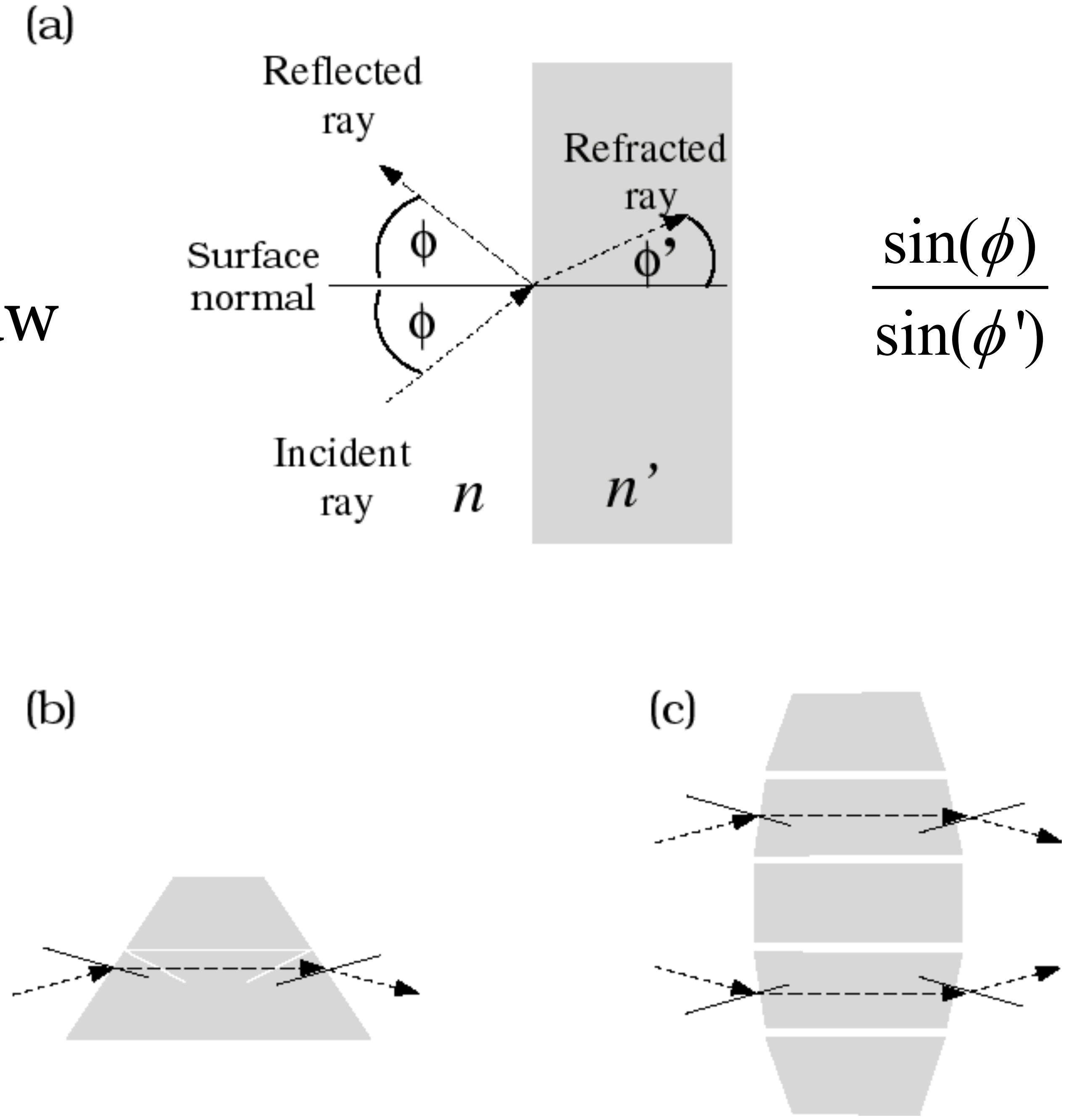


$$\frac{\sin(\phi)}{\sin(\phi')} = \frac{n'}{n}$$

(b)



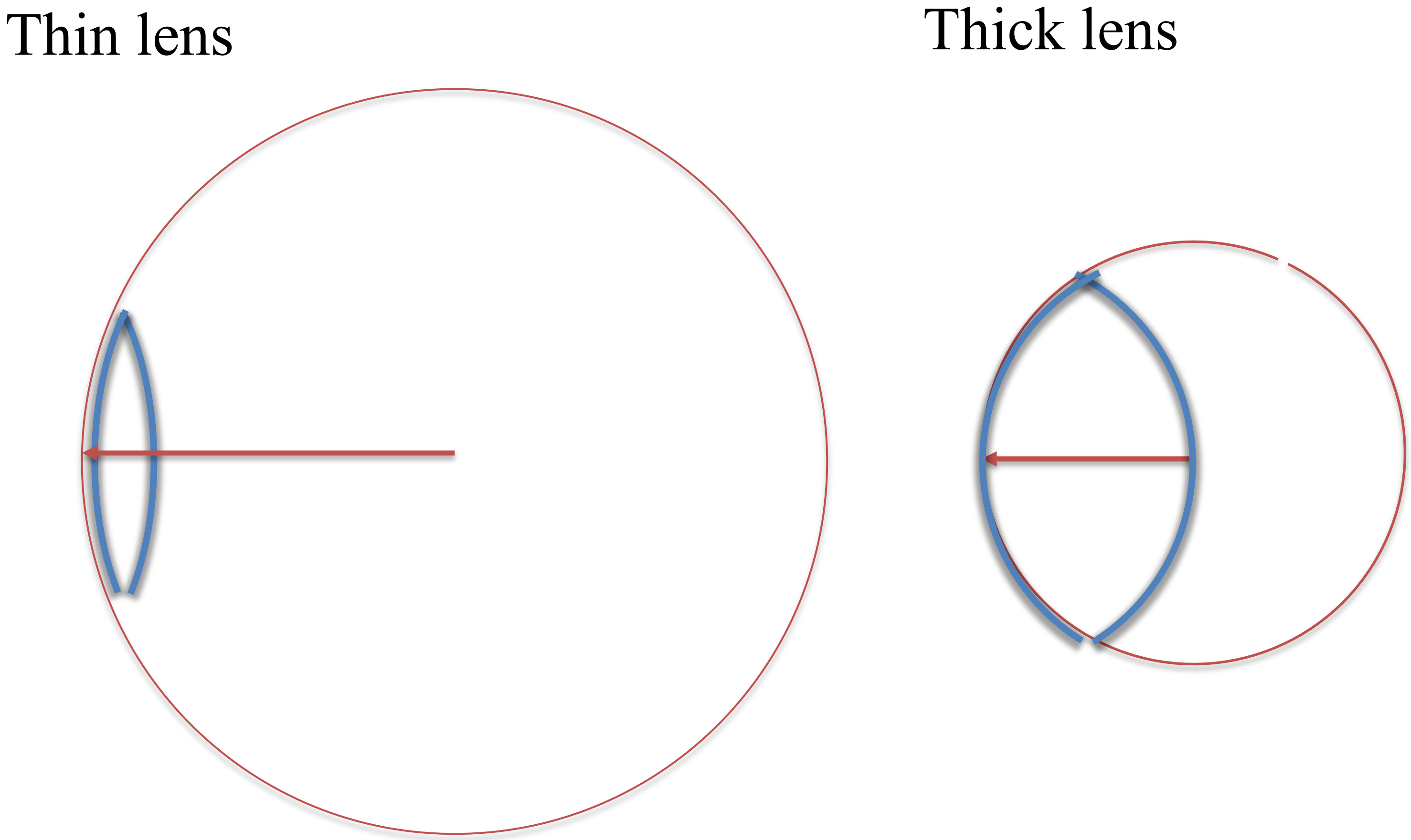
Lens Design: Snell's Law



$$\frac{\sin(\phi)}{\sin(\phi')} = \frac{n'}{n}$$

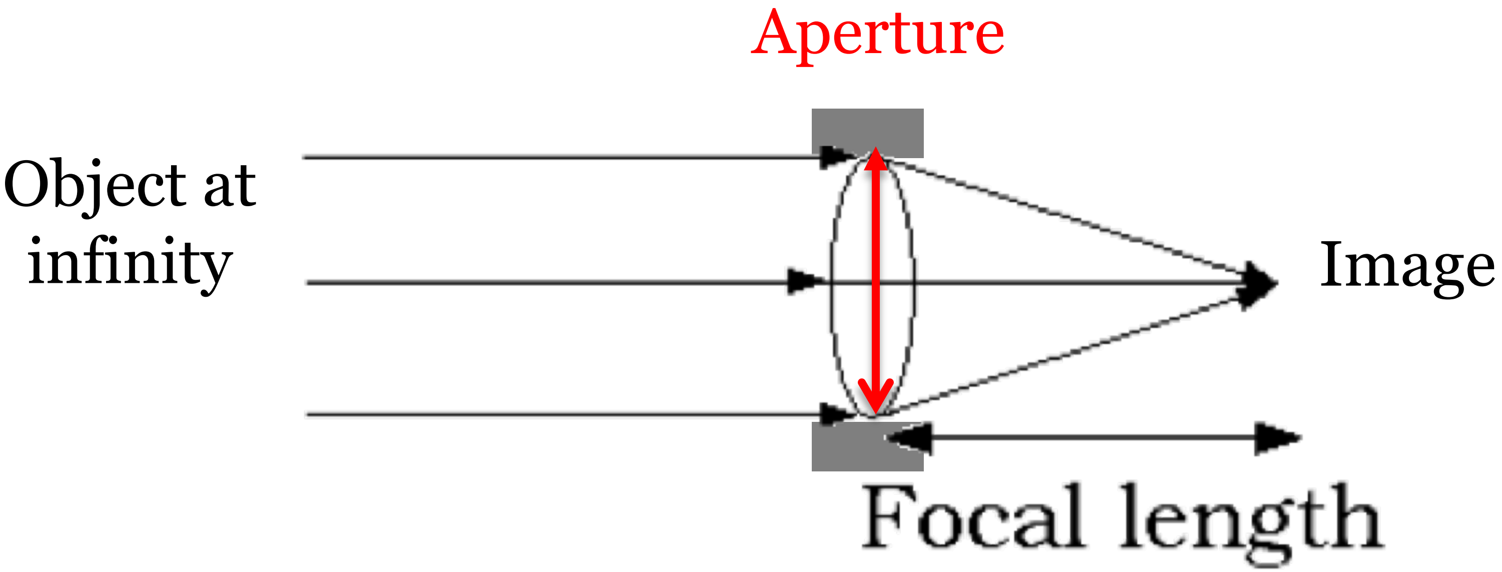
Thin lens and radius of curvature

Thin lens: a lens with a thickness (distance along the optical axis between the two surfaces of the lens) that is small compared to the radii of curvature of the lens surfaces.



Thin lens characterization

- A perfect perfect thin lens can be characterized by diameter of its aperture and its focal length



Lens Power

Lens power (diopters) = 1/(Focal length(m))

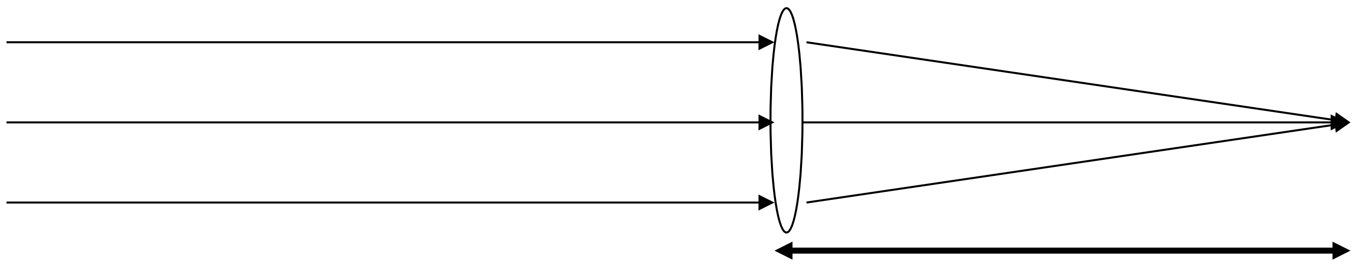
Source at infinity

High power

Lens Power

Lens power (diopters) = 1/(Focal length(m))

Source at
infinity



Focal length

Low power

F/# summarizes light sensitivity (speed)

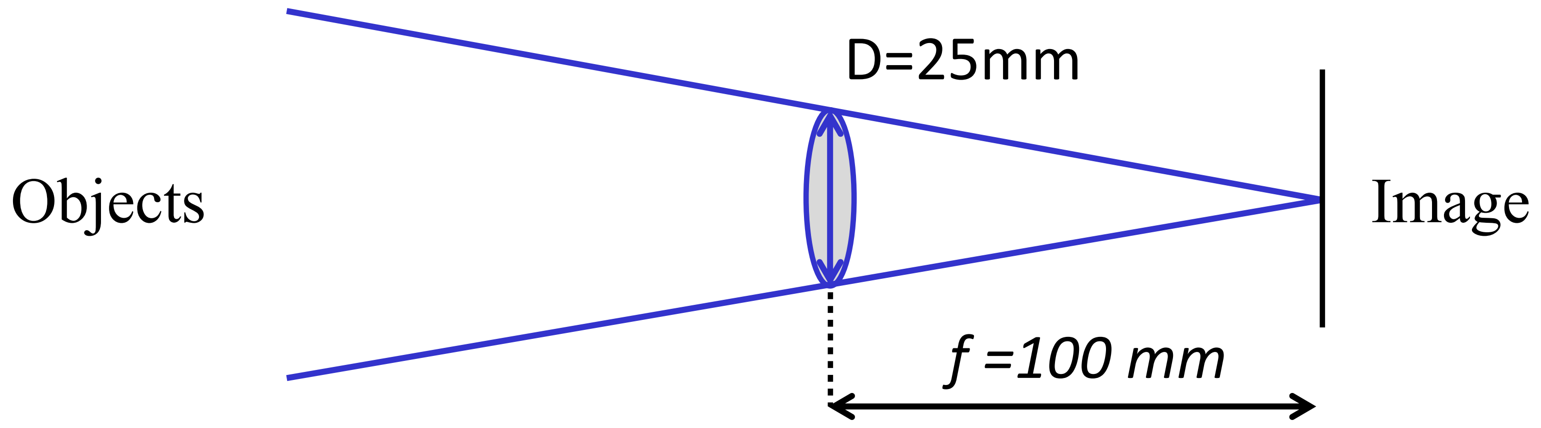
- For many purposes, a thin lens can be summarized by a single number, the ratio of the aperture diameter to focal length, or **F-number** (f/#)

- A value of f/4, for example, means the aperture diameter is 1/4 the focal length of the lens

$$f/\# = \frac{focal\ length}{aperture\ diameter}$$

The geometry of the f/# and angle of the rays (numerical aperture)

From an image point, the angle of the rays captured by two lenses with the same f/# is the same.



The geometry of the f/# and angle of the rays (numerical aperture)

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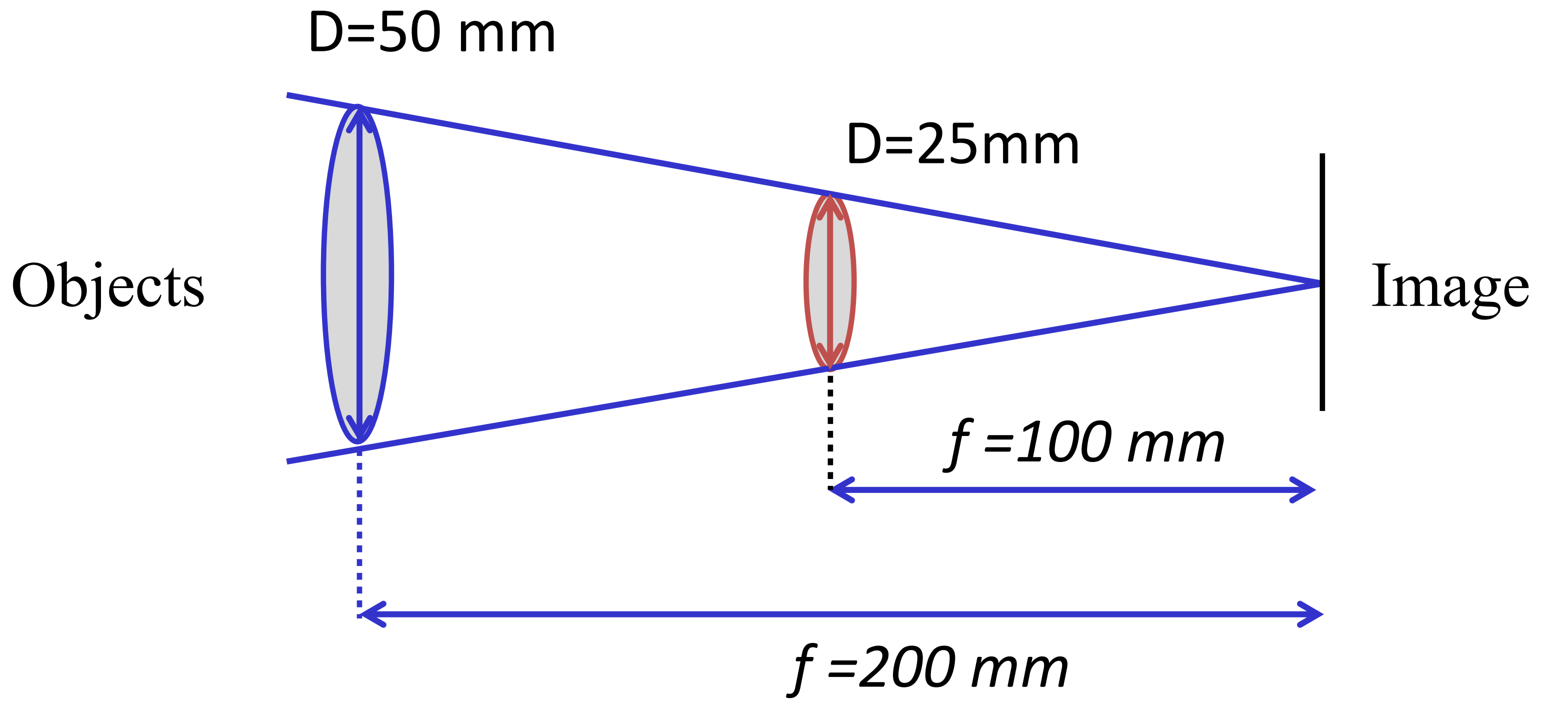
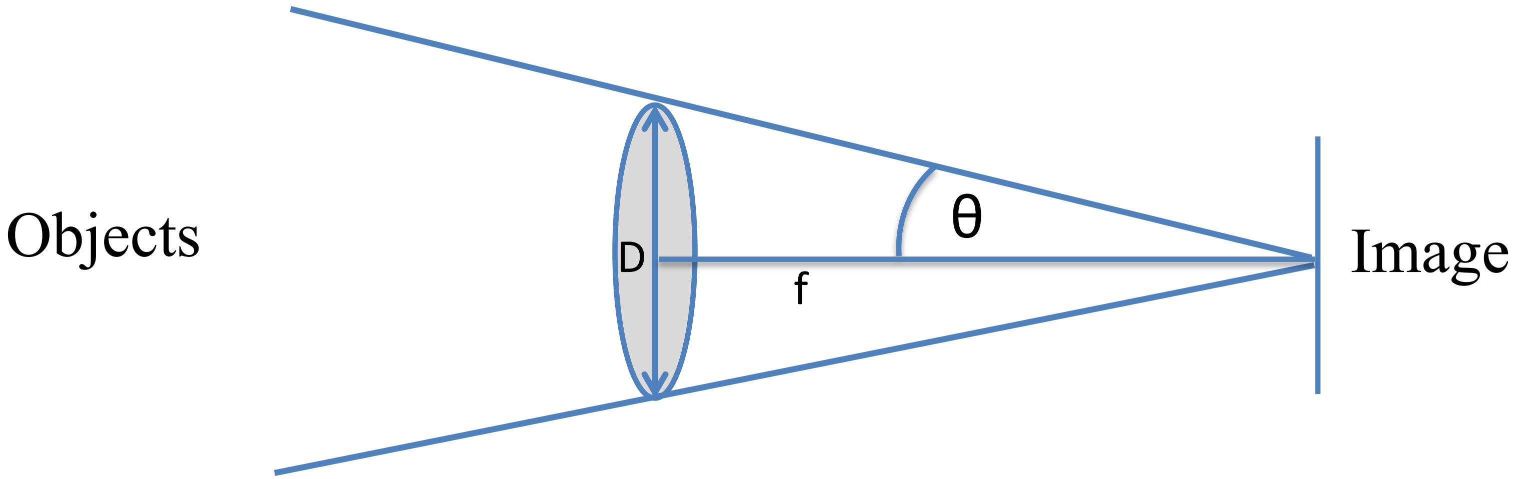


An alternative measure: Numerical aperture (NA)

The angle of the rays captured by two lenses is also used directly as the numerical aperture

$$NA = n\sin(\theta)$$



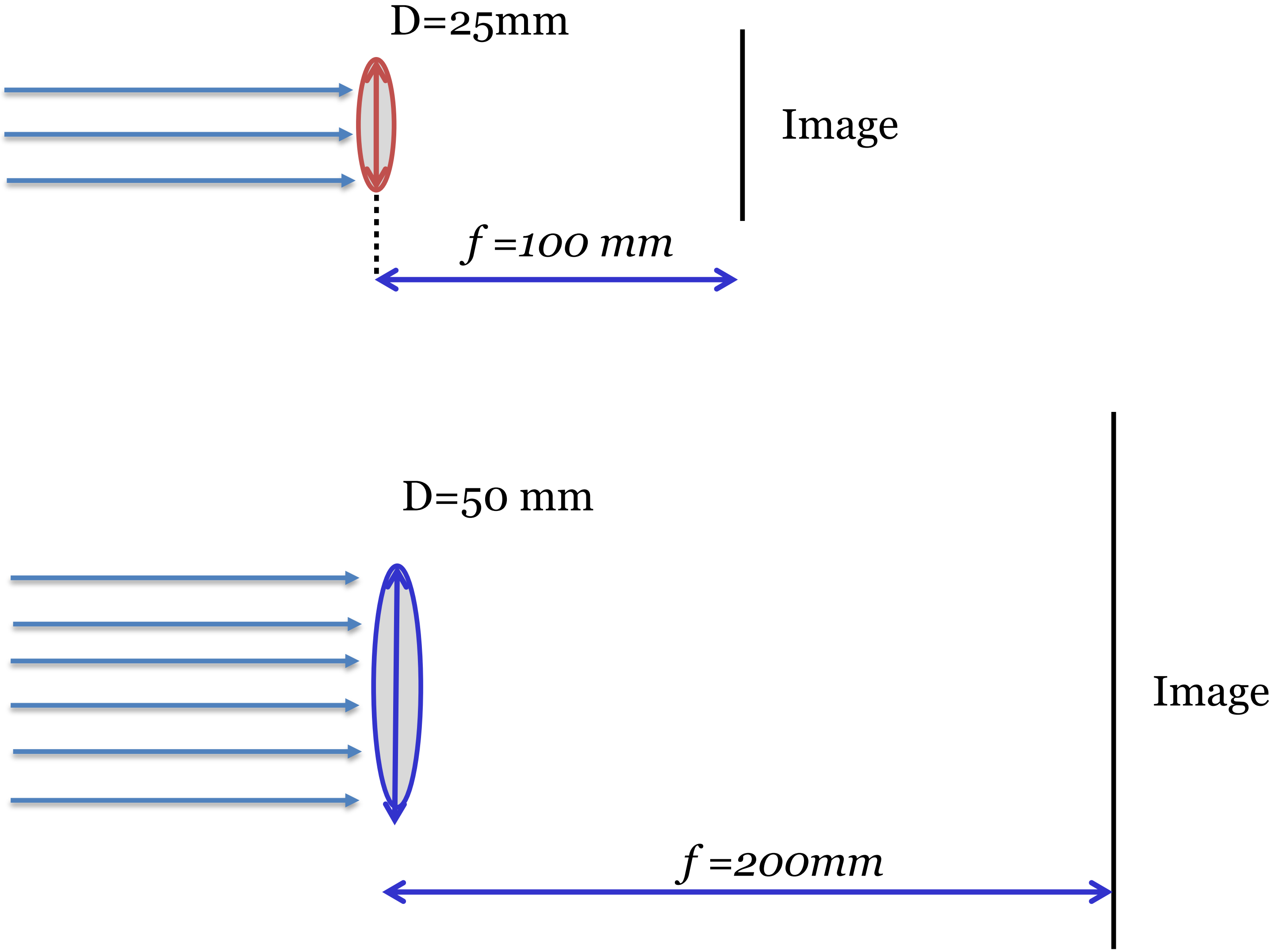
- NA is a dimensionless value
- Used commonly in microscopy
- Like f/# it measures sensitivity
  (the acceptance angle)
- NA accounts for the index of refraction

To good approximation NA is half the inverse of the f/#

$$NA \approx \frac{D}{2f} = \frac{1}{2}\frac{D}{f} = \frac{1}{2(f/\#)}$$

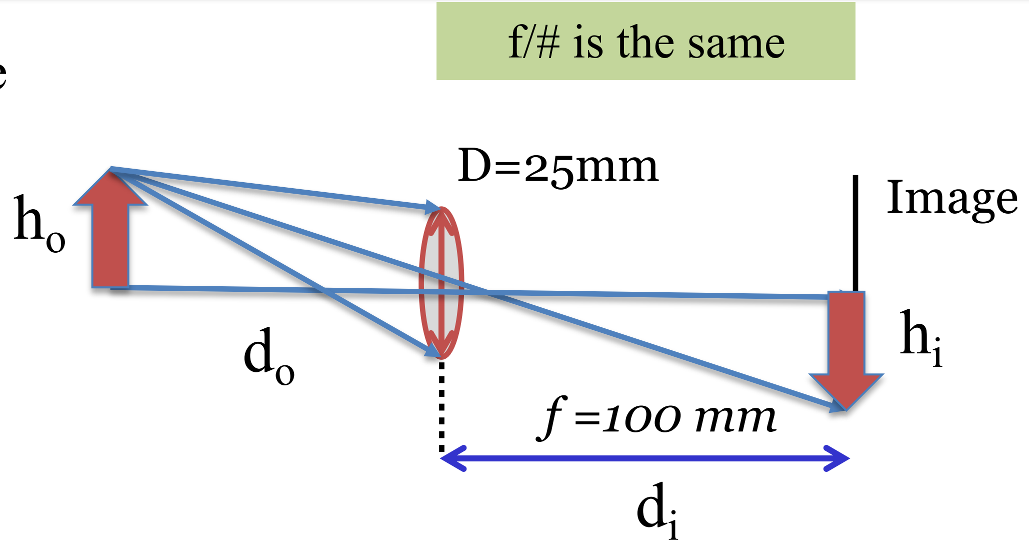
F/# summarizes light sensitivity (speed)

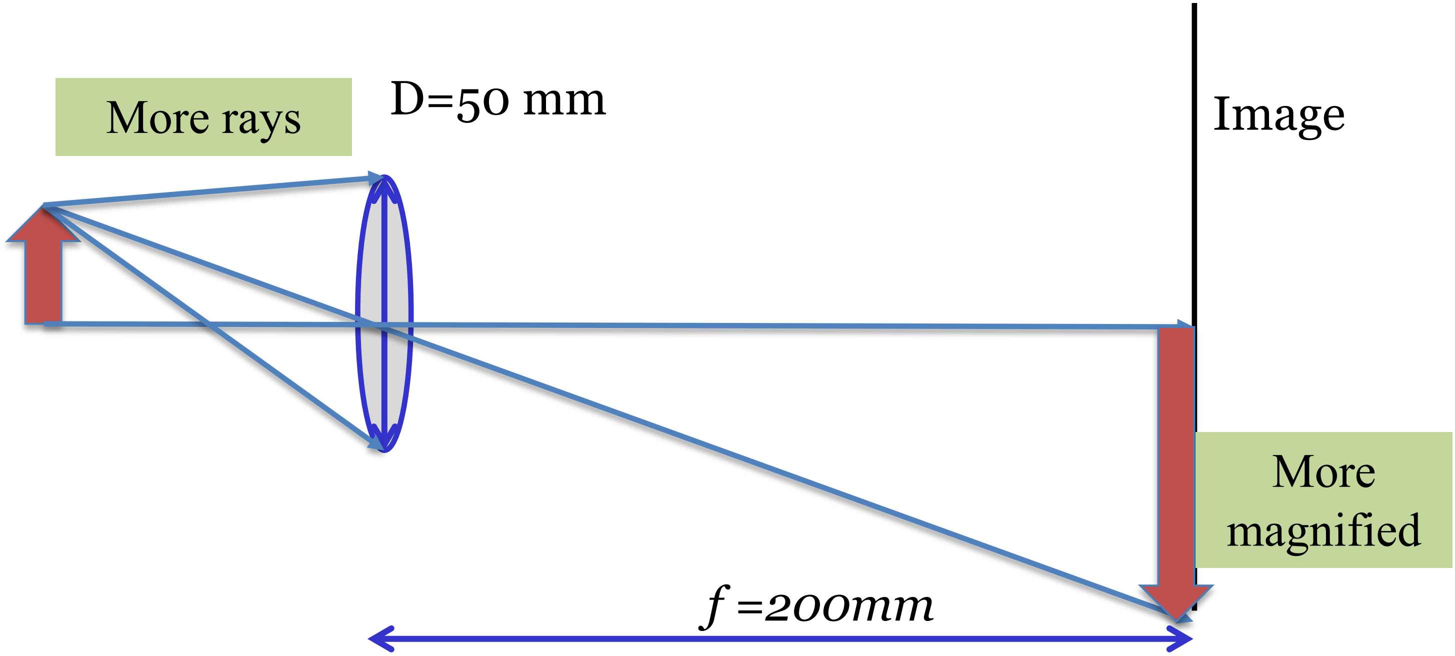
- The increase in the aperture captures more rays from a point at infinity (or closer)



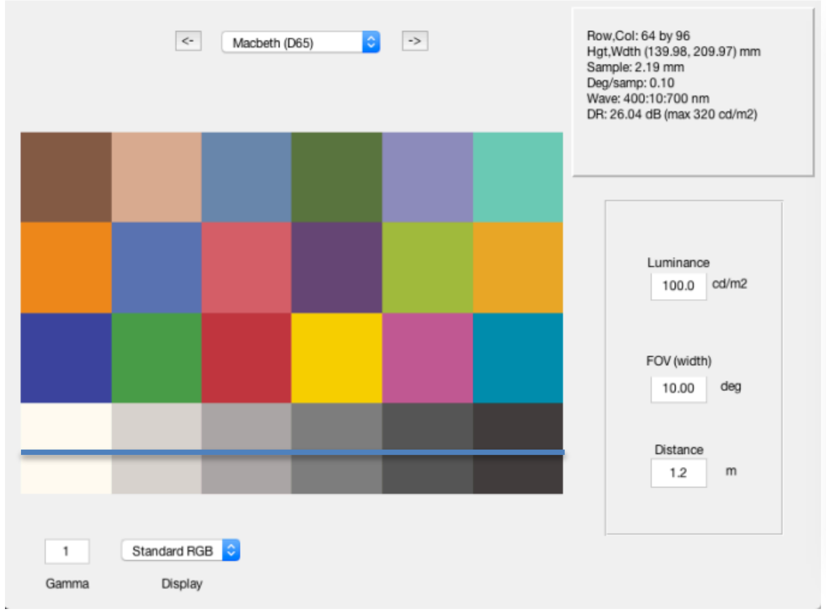
Magnification and aperture size effects compensate yielding equal irradiance

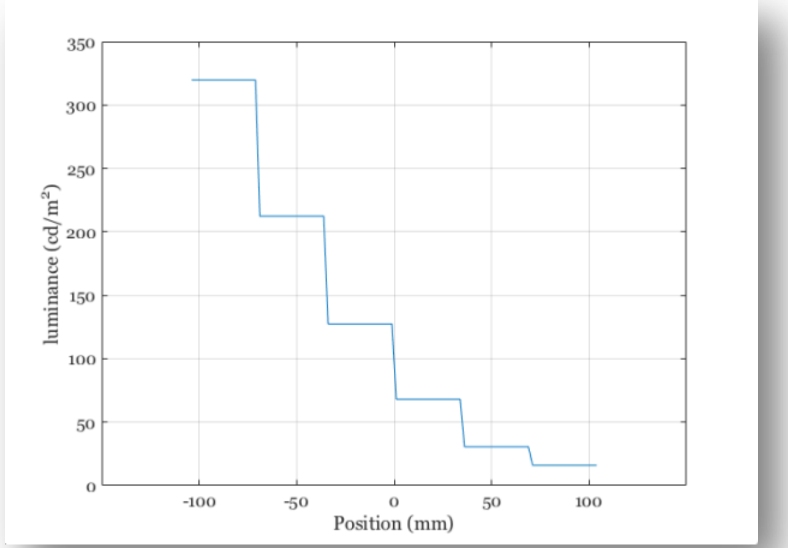
- The increase in the aperture captures more rays from a point at infinity (or closer)
- The magnification $(h_i/h_o) = (-d_i/d_o))$ depends on the focal length, $f / (f - d_0)$. A longer focal length has a larger magnification
- The magnification and aperture factors cancel (more rays for larger aperture, higher density of rays for smaller focal length)
- So, for fixed f/# the irradiance (photons/m^2/sec) in each small area within the image is unchanged.





Formula: Converting radiance to irradiance



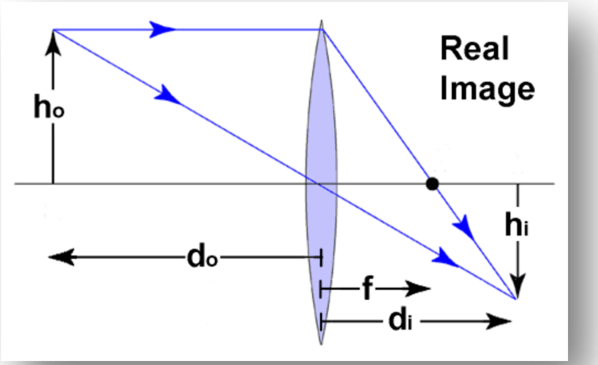


Irradiance formula

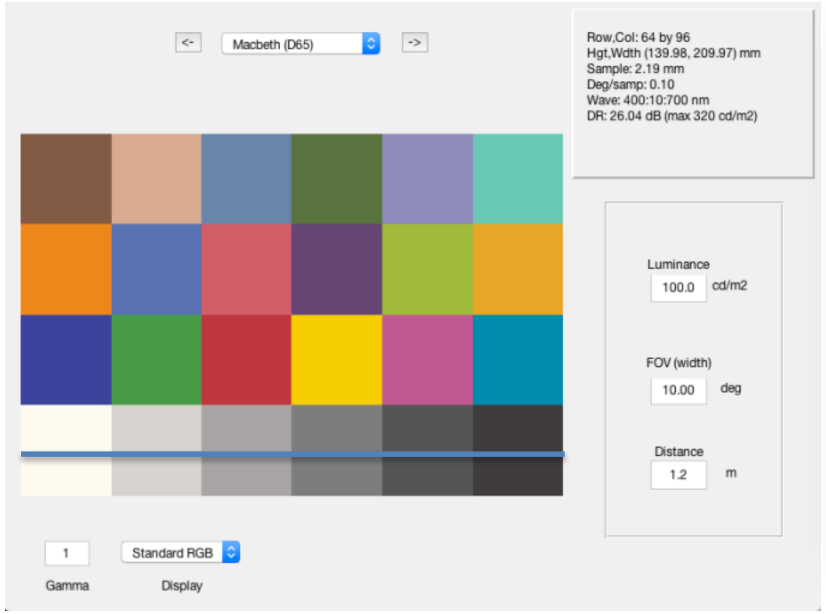
$$I = \frac{\pi}{1+4(f\,\#)^2(1+|m|)^2} R$$

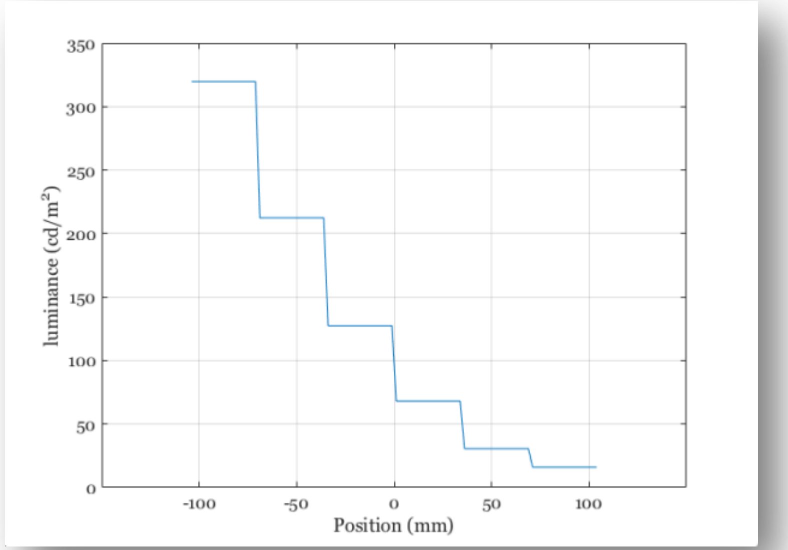
R = *radiance*
$f\#$ = *f-number*
m = *magnification*

$$m = \frac{f}{f - d_0}$$



Formula: Converting radiance to irradiance



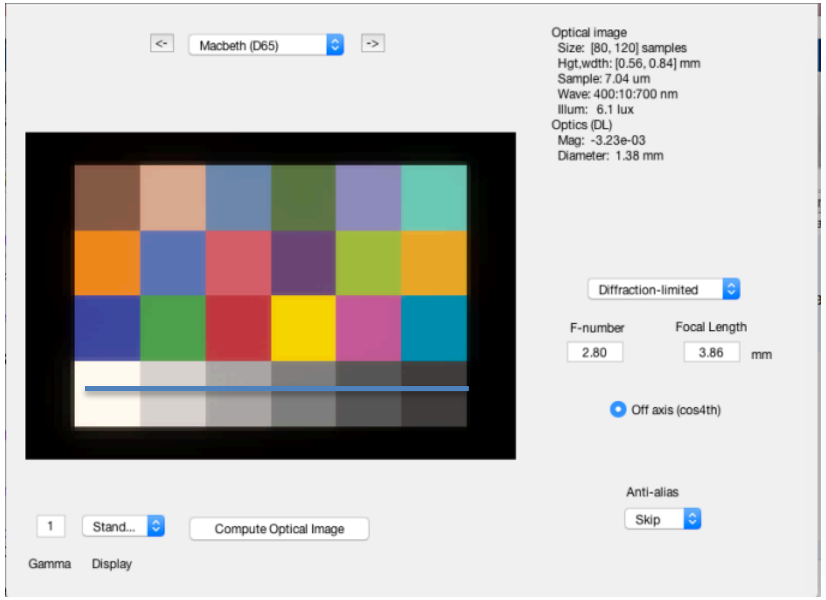


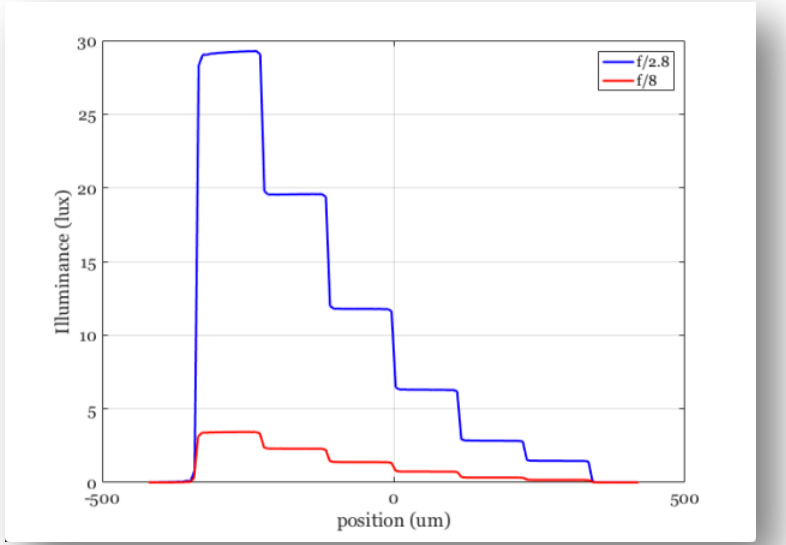
Irradiance formula

$$I = \frac{\pi}{1 + 4(f\#)^2(1+|m|)^2} R$$

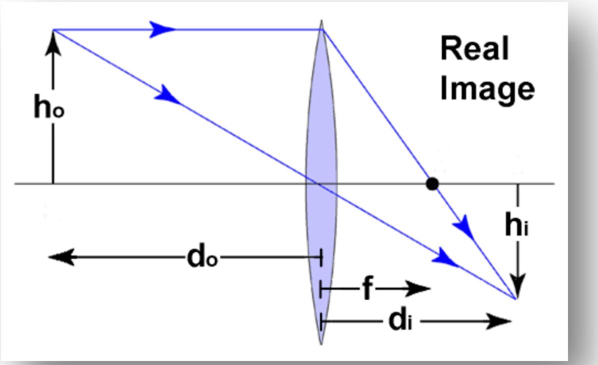
R = radiance
$f\#$ = f-number
m = magnification

Ideal thin lenses f/2.8 and f/8

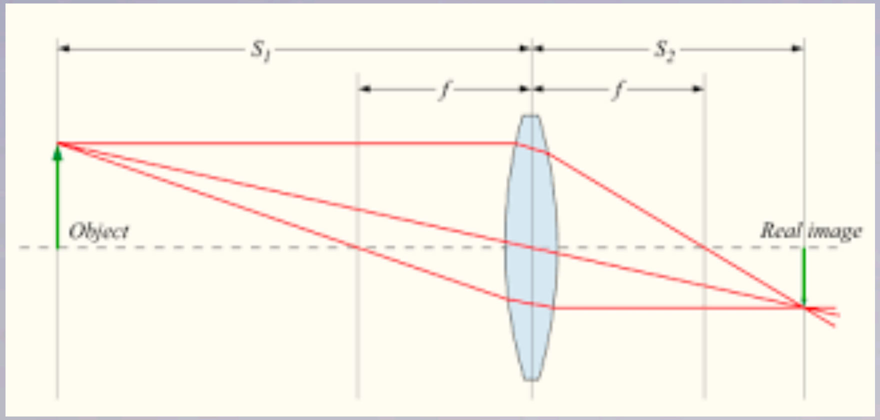


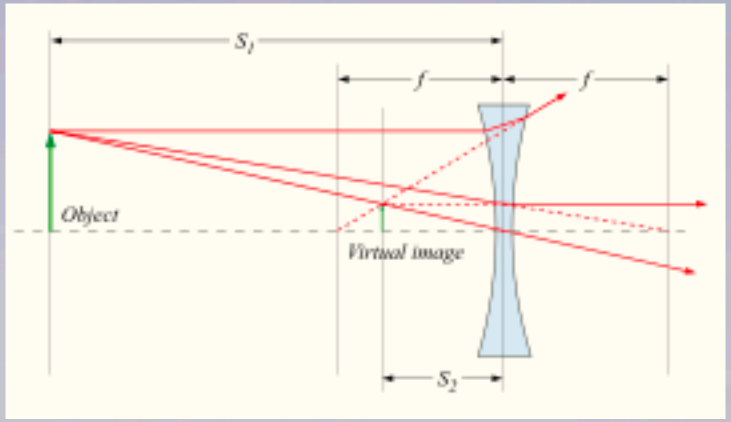


$$m = \frac{f}{f - d_0}$$



The lens maker's equation





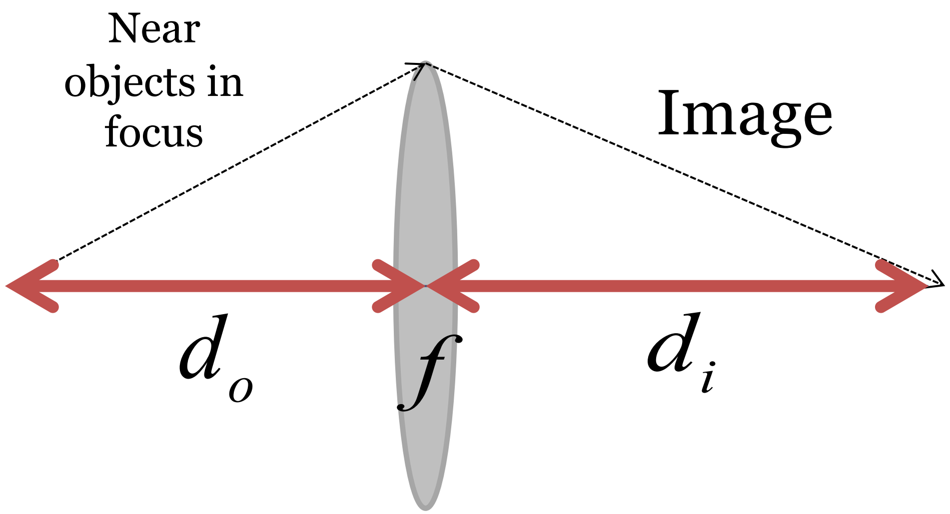
Lens maker's equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$d_o = object\ dist$

$d_i = image\ dist$

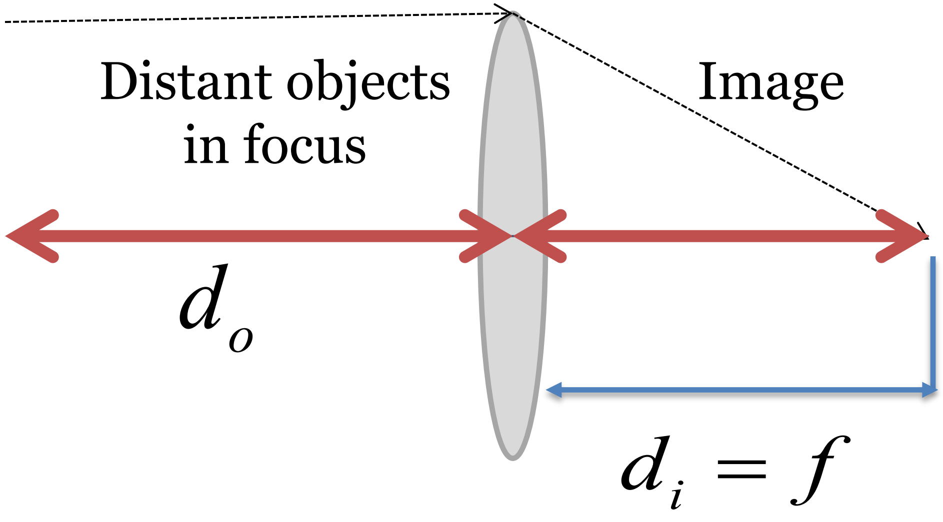
$f = focal\ length$



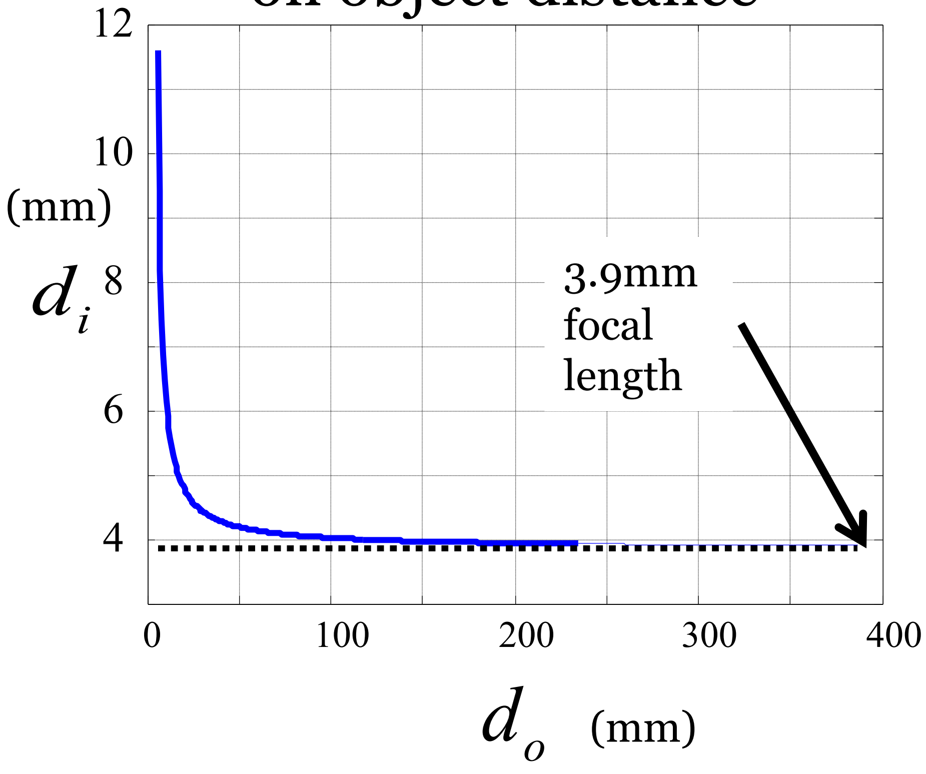
Depth and defocus

Lensmaker's equation

$$\frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f}$$



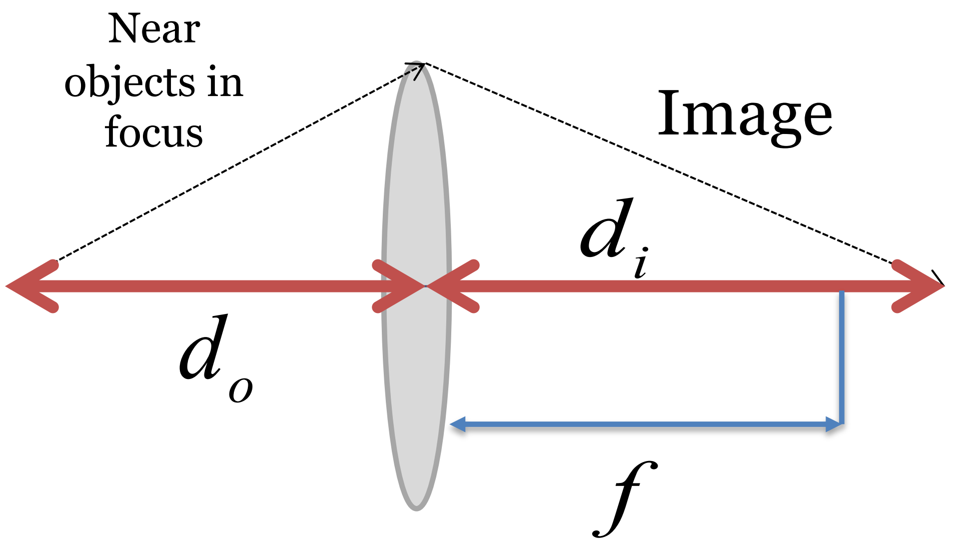
Distance to in-focus image plane depends on object distance



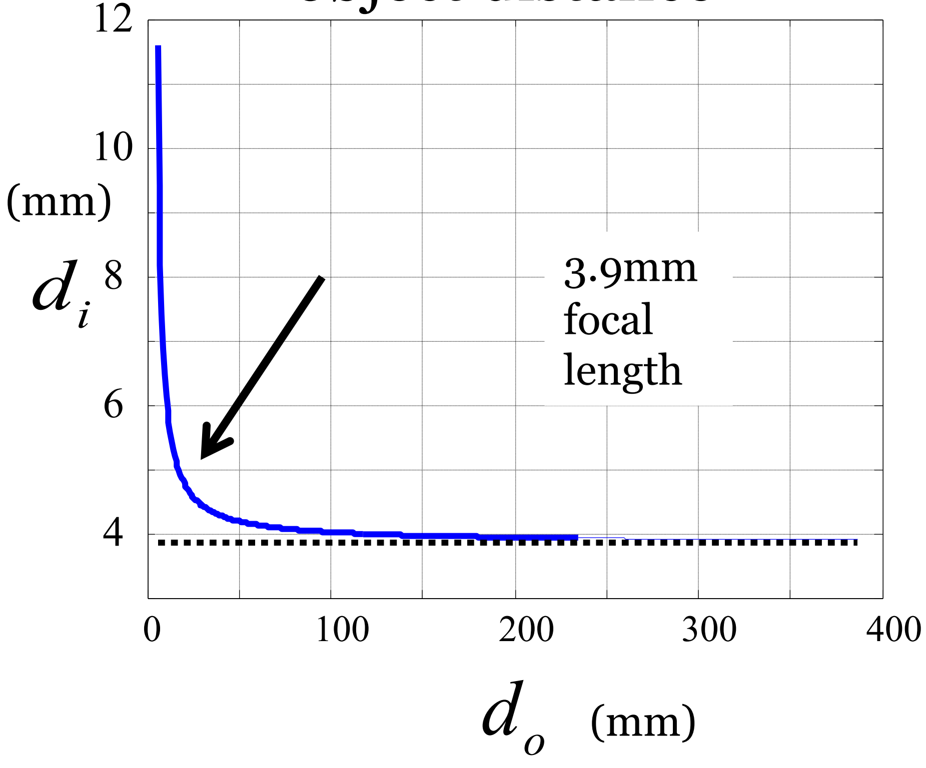
3D Scenes: Depth and Defocus

Lensmaker's equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$



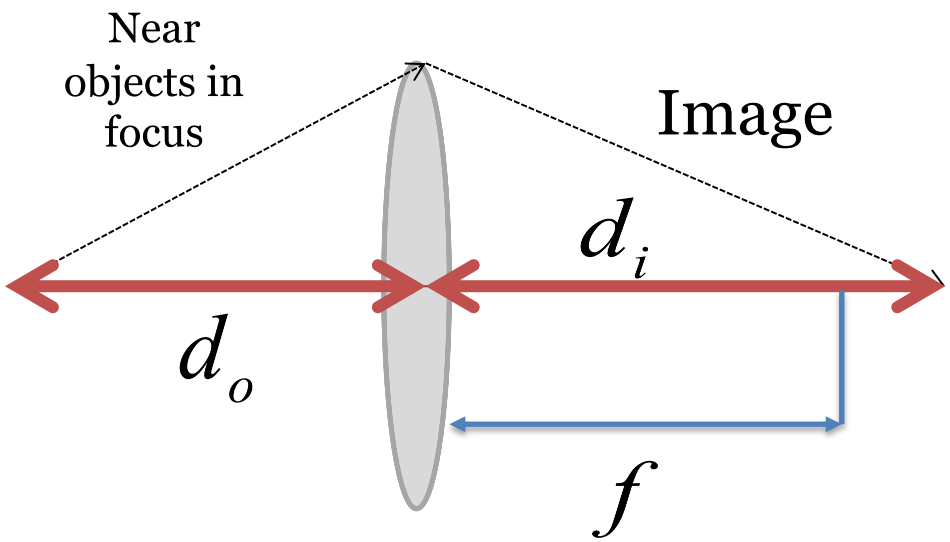
Distance to image focal plane depends on object distance



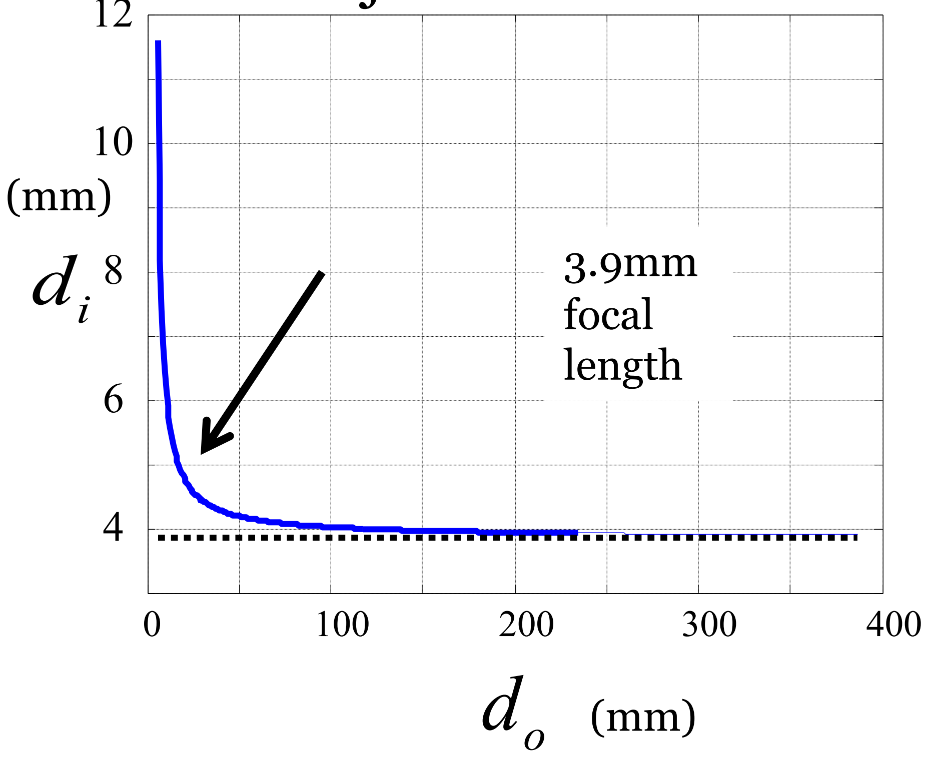
Lensmaker's equation

Re-arranging the terms to solve for the in-focus image plane

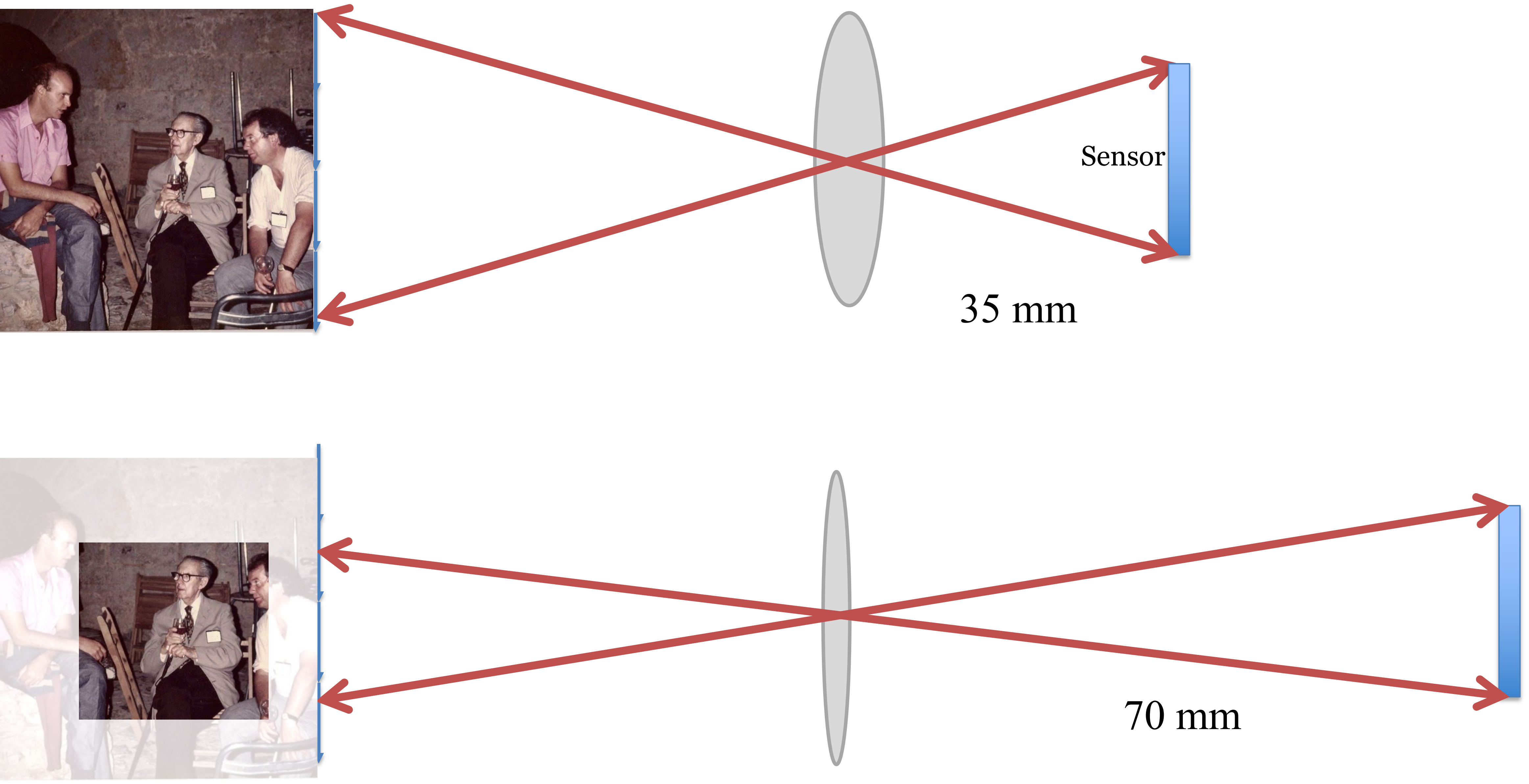
$$d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}}$$



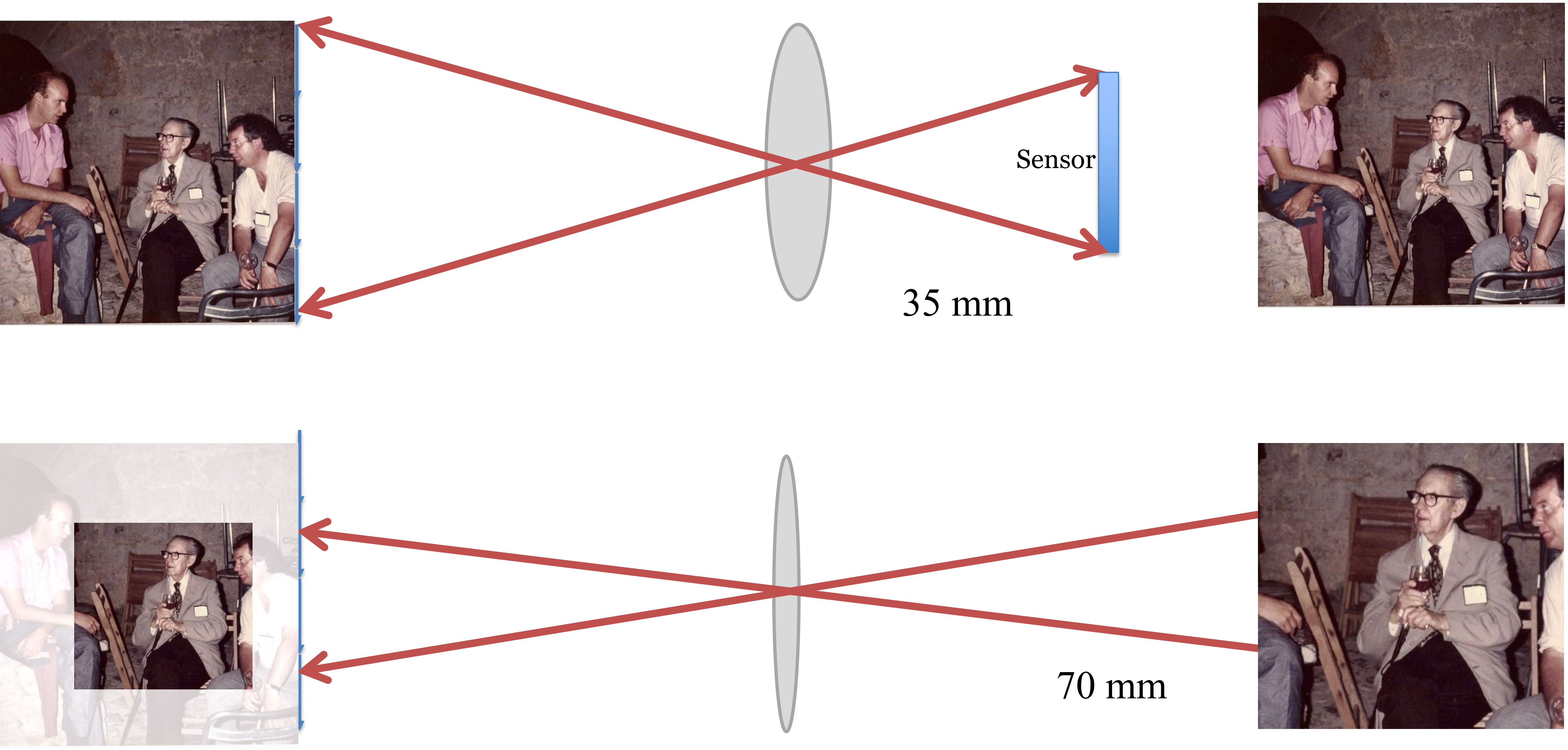
Distance to image focal plane depends on object distance



For a fixed sensor size, increasing the focal distance changes the field of view



For a fixed sensor size, increasing the focal distance changes the field of view



Depth of field and the circle of confusion

s_opticsCoC

s_opticsDepthDefocus

Depth of field (DOF)

Bokeh[1]

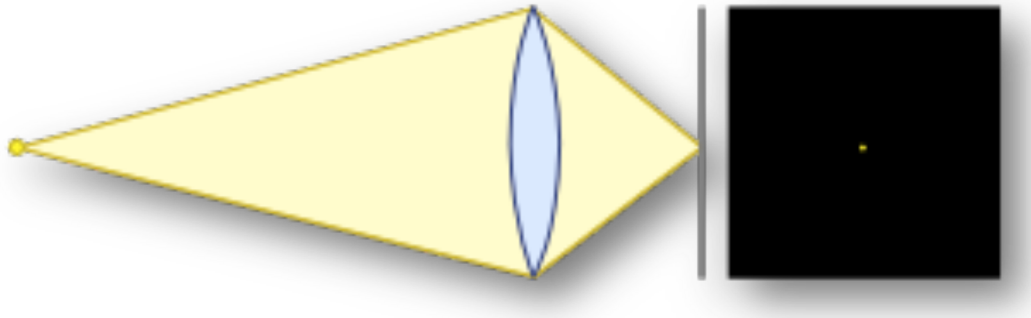
1. Bokeh: The visual quality of the out-of-focus areas of a photographic image, especially as rendered by a particular lens (pronounced both ways, in practice).

Beyond the focal plane: Depth of field

Depth of field is the amount of distance between the nearest and farthest objects that appear in acceptably sharp focus in a photograph. A preferred selection Depth of field ("DOF") in a focused subject is subjective.

Circle of confusion (CoC)

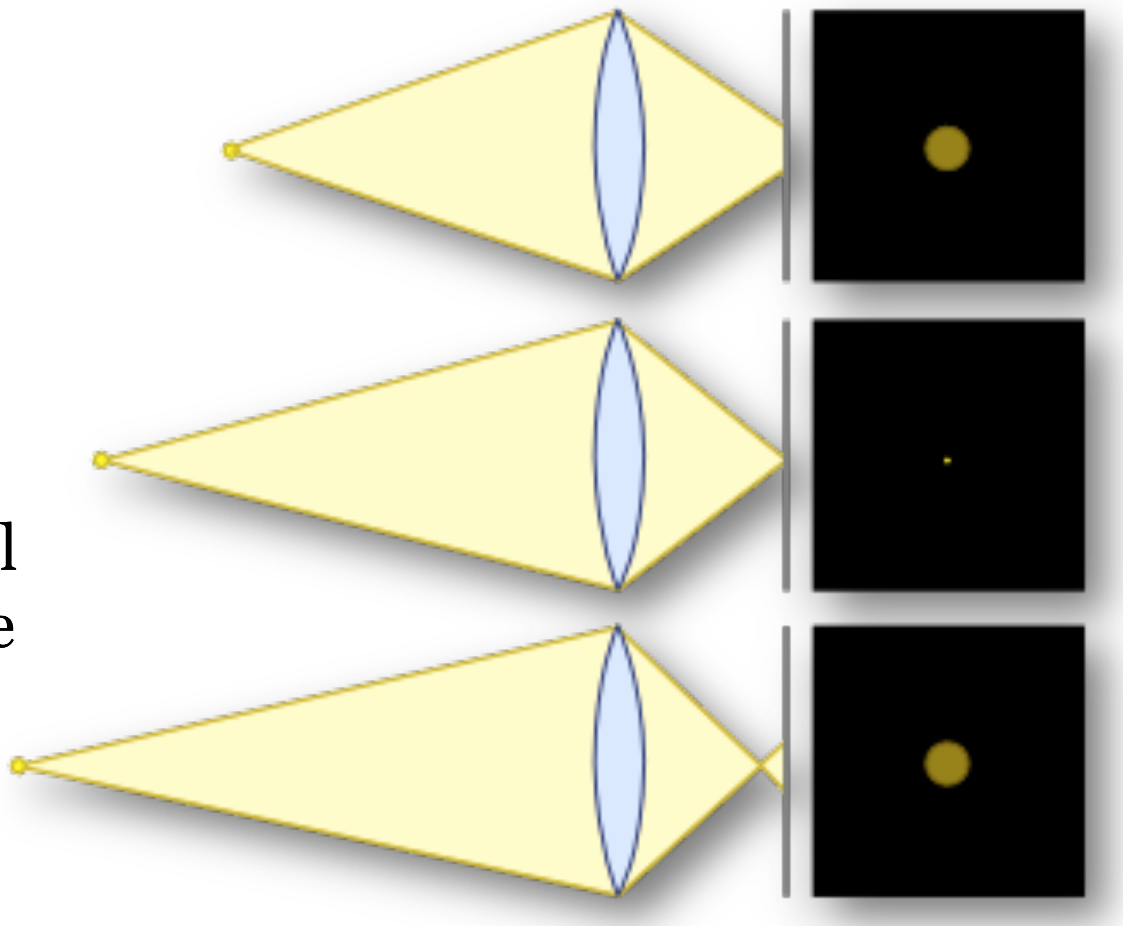
"In focus" means a match between point distance and sensor plane



Circle of confusion (CoC)

“In focus” means a match between point distance and sensor plane

Out of focal plane blurring (geometric defocus) is often the largest defocus in natural scenes. The CoC is a valuable figure of merit for this defocus

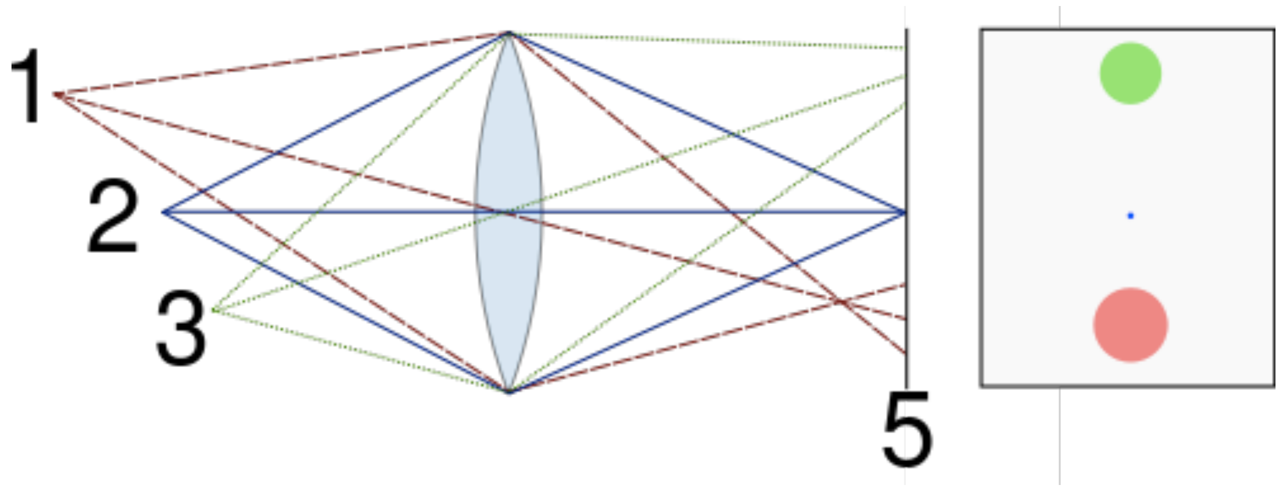


s_opticsCoC

DOF and circle of confusion

Focal distance (2)

The points in focus (2) project points onto the image plane (5), but points at different distances (1 and 3) project blurred images, or circles of confusion.

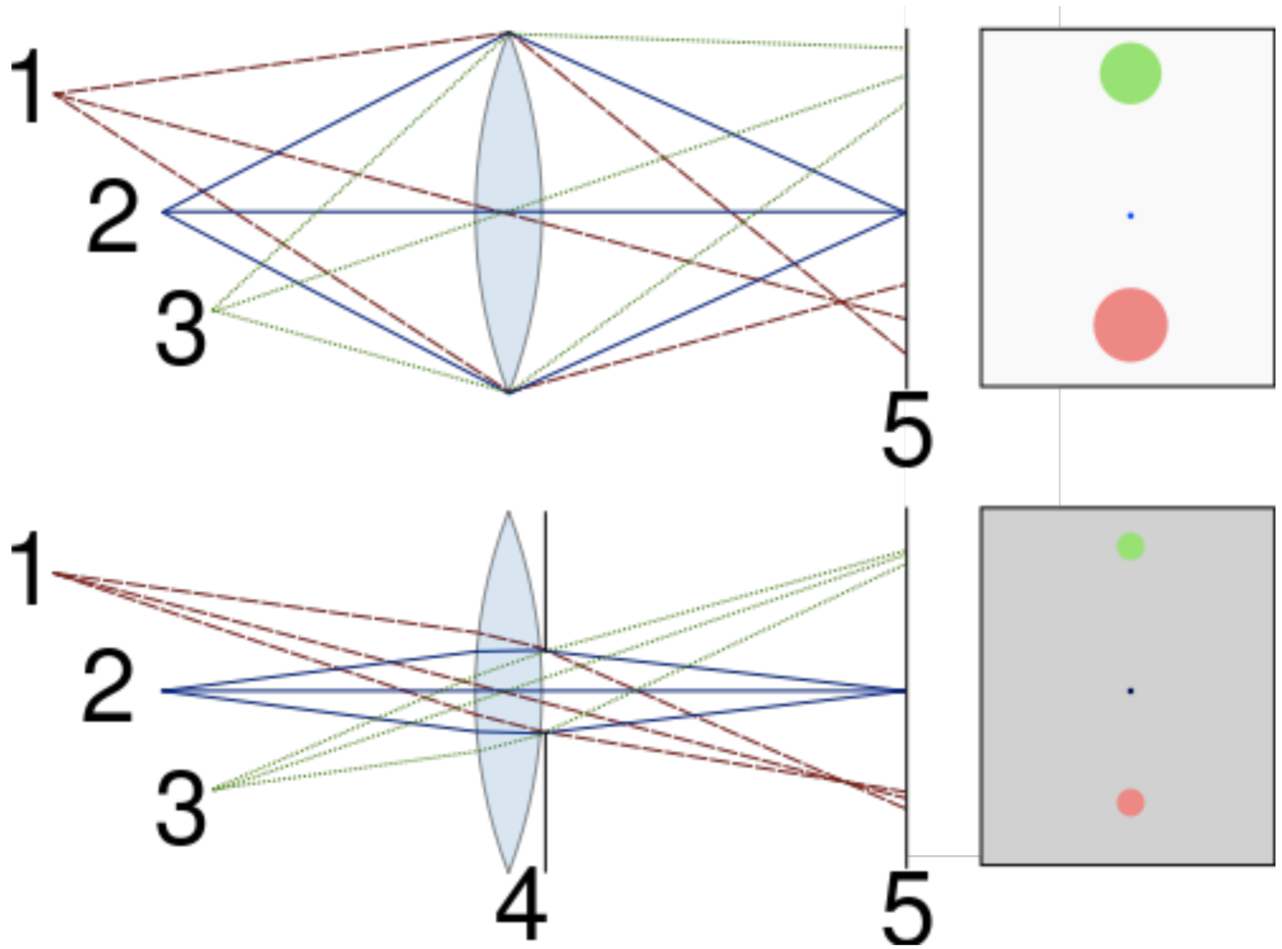


DOF and circle of confusion

The points in focus (2) project points onto the image plane (5), but points at different distances (1 and 3) project blurred images, or circles of confusion.

Decreasing the aperture size (4) reduces the size of the blur spots for points not in the focused plane, so that the blurring is imperceptible, and all points are within the DOF.

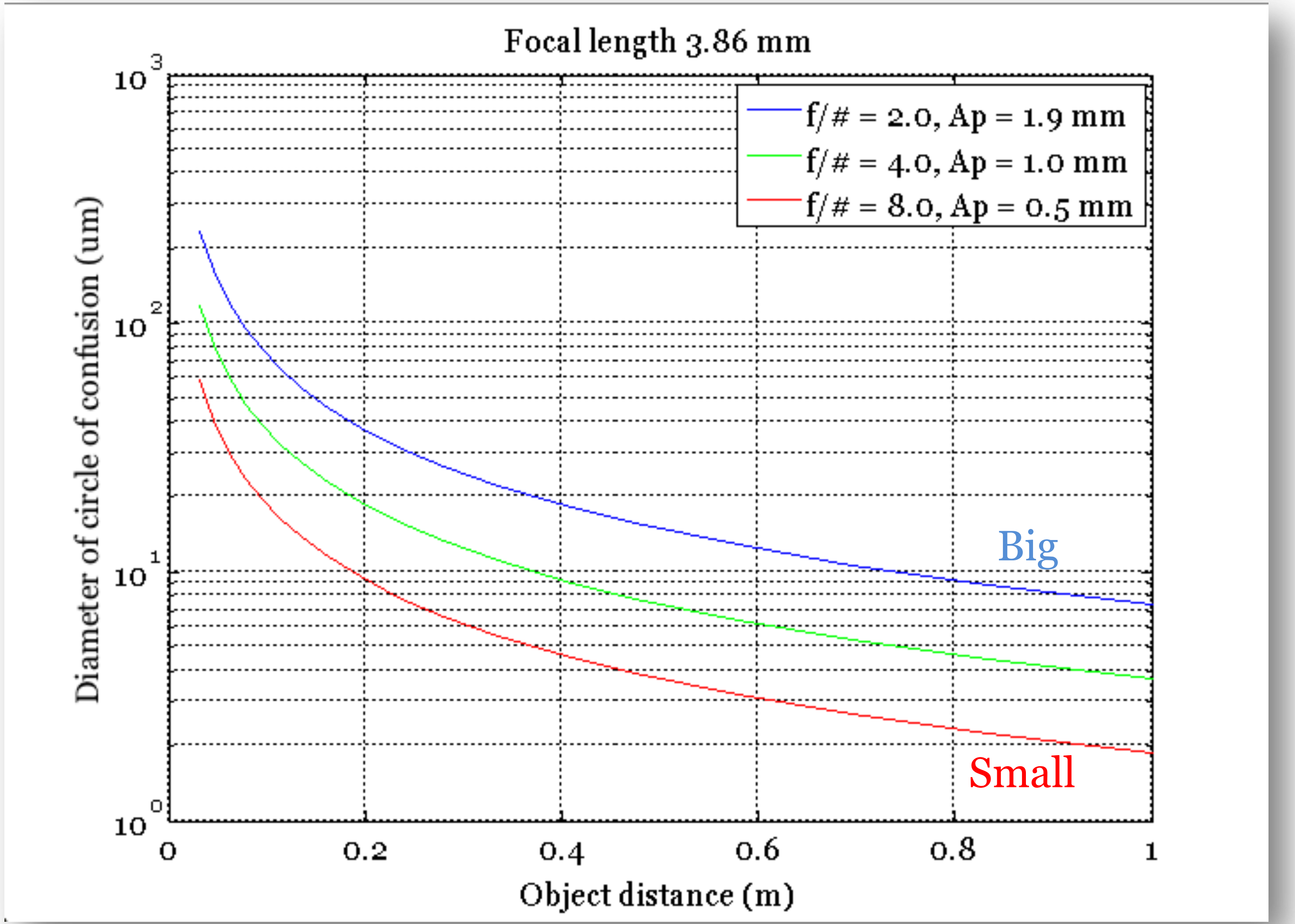
Focal distance (2)



Circle of confusion for geometric defocus

- The circle of confusion depends on the object distance and the aperture diameter
- Smaller aperture, smaller the CoC

Sensor at the focal length (3.86 mm)



s_opticsCoC

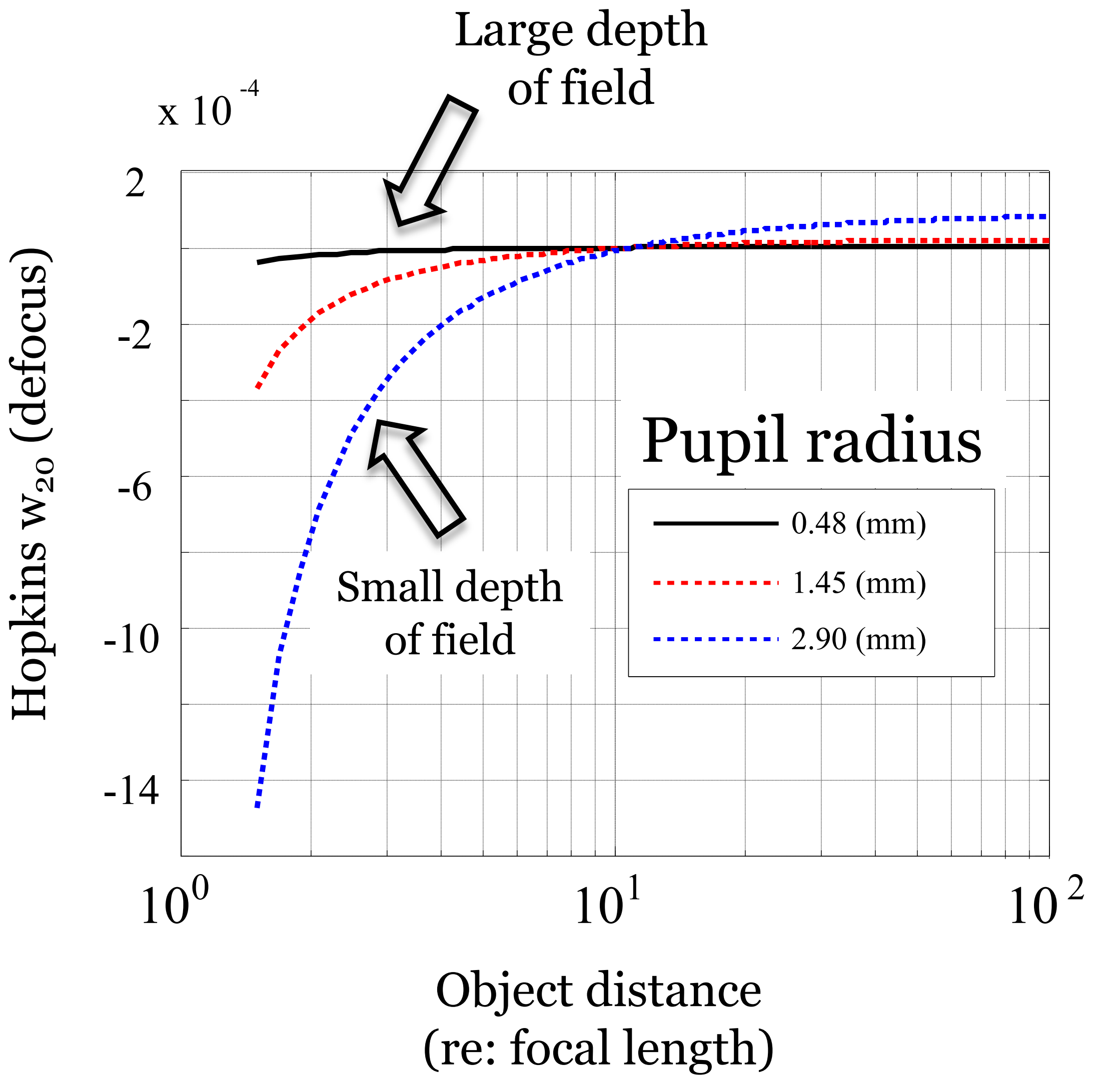
Depth of Field (DOF)

Depth of field is the range of distances with “good” focus

DOF depends on both

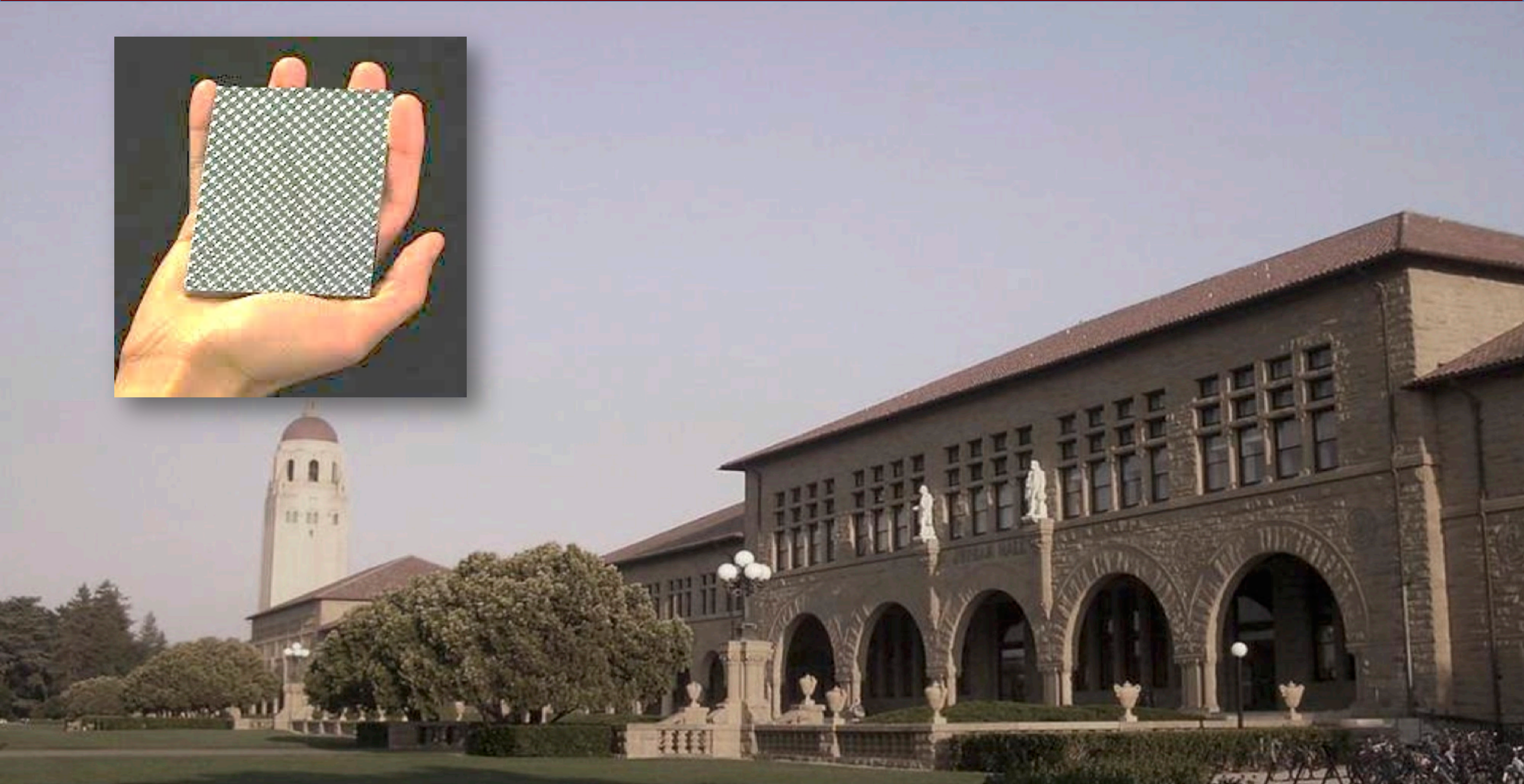
- Object distance
- Pupil radius

Small radius larger depth of field - and less light

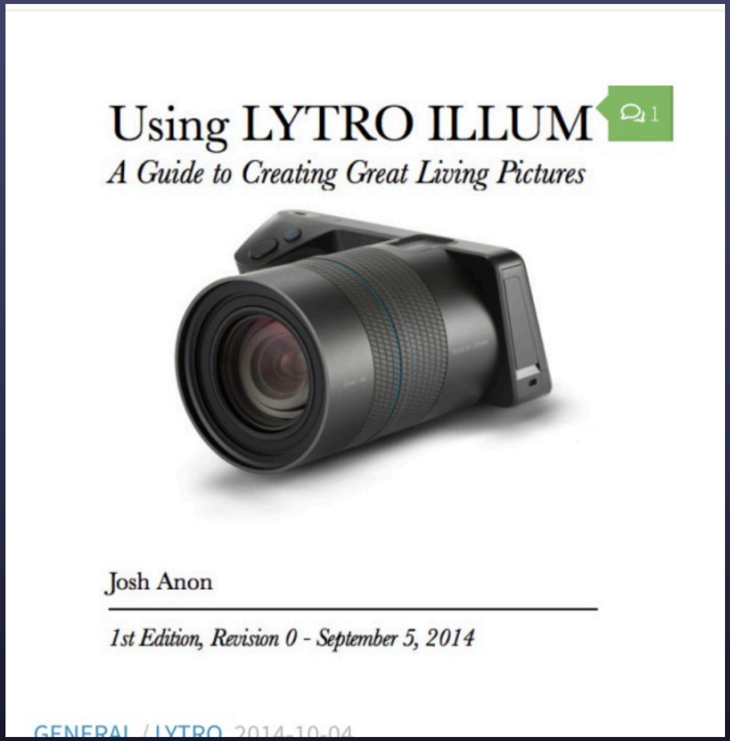


s_opticsDepthDefocus.m

Measuring light fields with a camera

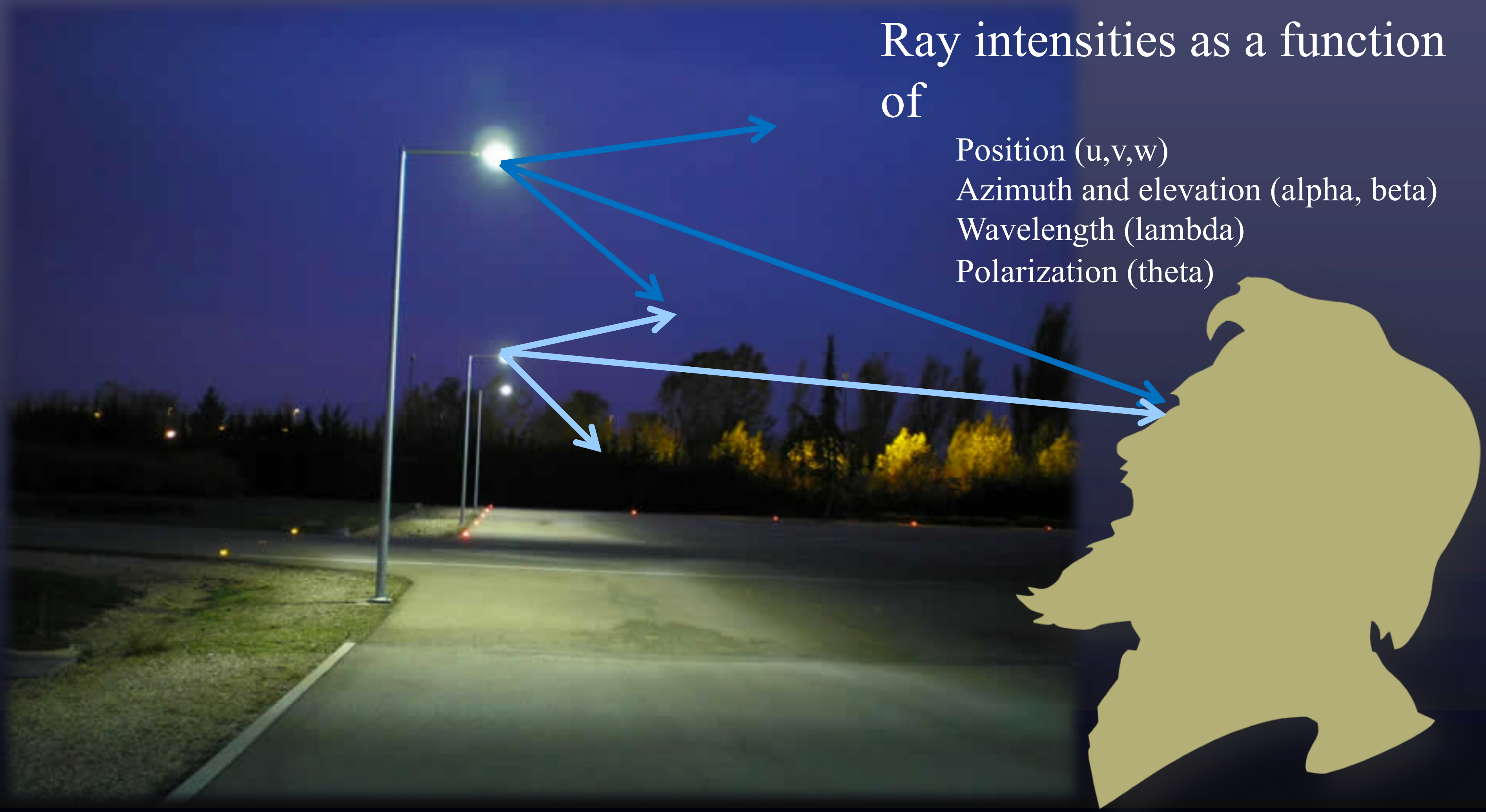


A goal of light field cameras: Digital refocusing



The scene spectral radiance

Arun Gershun (1936)



Ray intensities as a function of

- Position (u,v,w)
- Azimuth and elevation (alpha, beta)
- Wavelength (lambda)
- Polarization (theta)

Light field at the eye is the plenoptic function

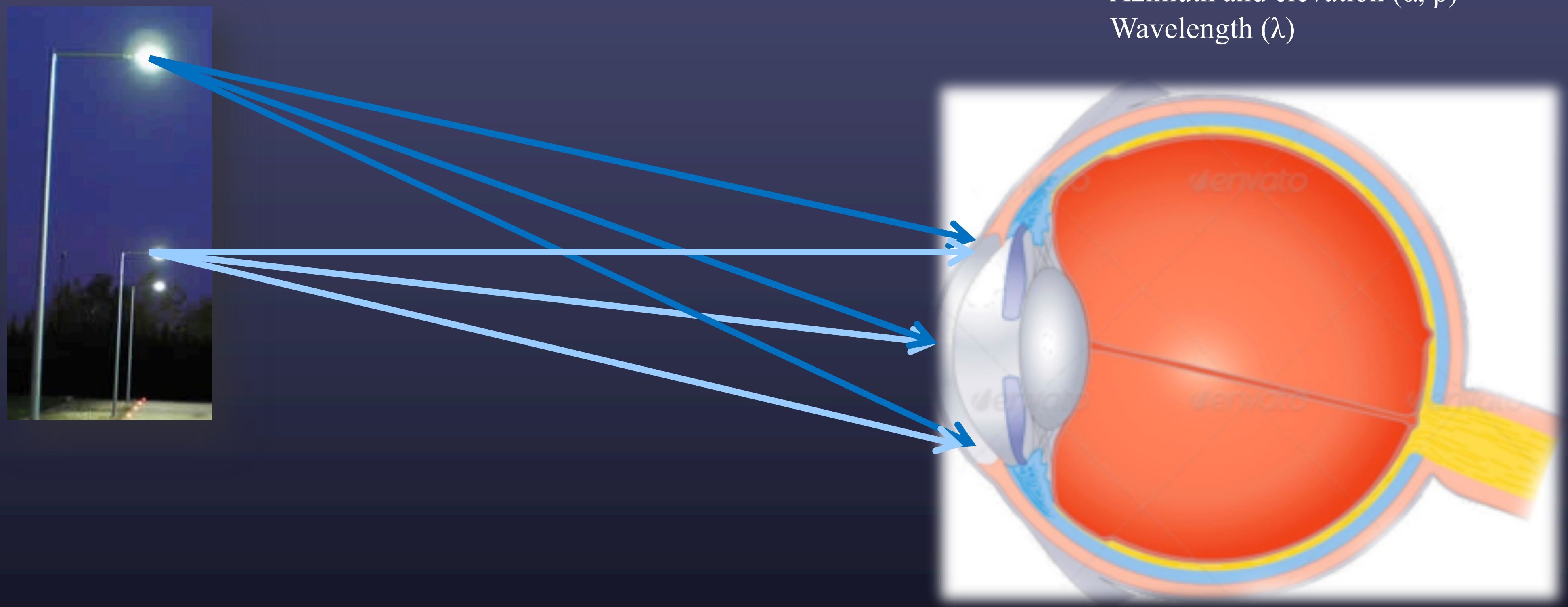
(Adelson and Bergen, 1991)

Ray intensities: $R(u,v,\alpha,\beta,\lambda)$

Position (u,v)

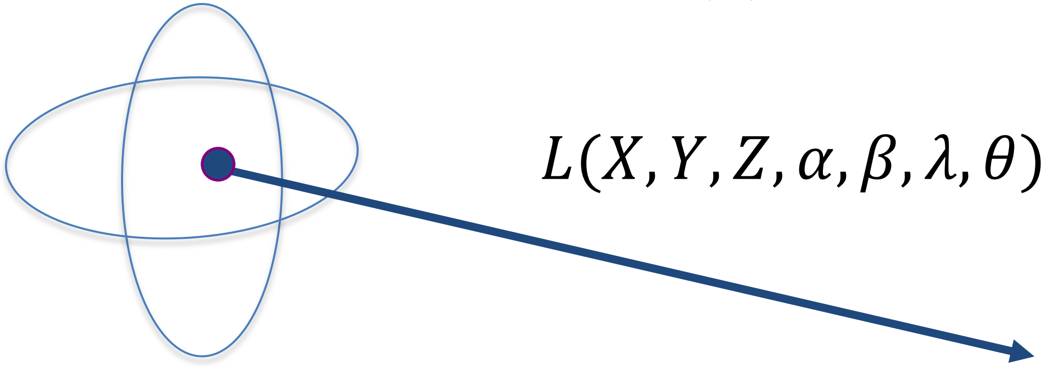
Azimuth and elevation (α, β)

Wavelength (λ)



Plenoptic function

Position (X, Y, Z)
Azimuth and elevation (α, β)
Wavelength (λ)
Polarization (θ)

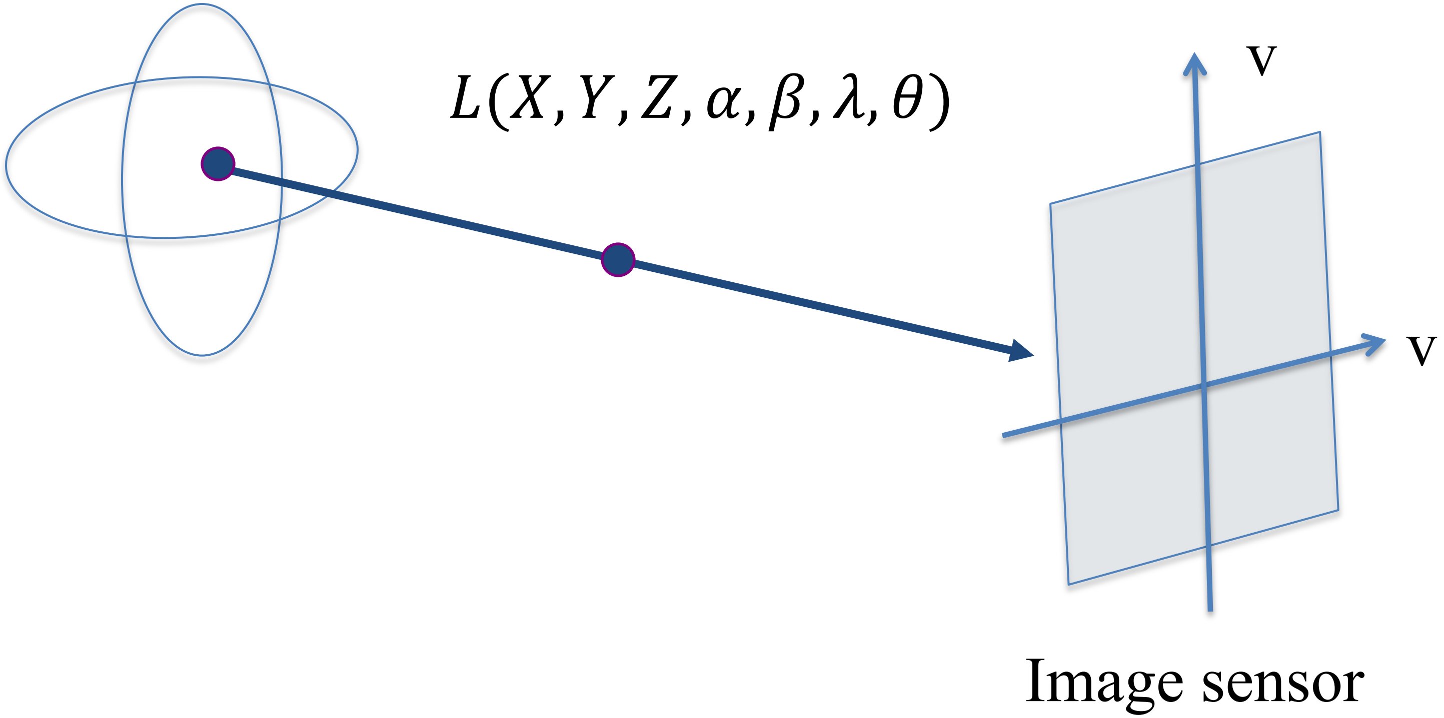


$L(X, Y, Z, \alpha, \beta, \lambda, \theta)$

Plenoptic function

(Adelson and Bergen, 1991)

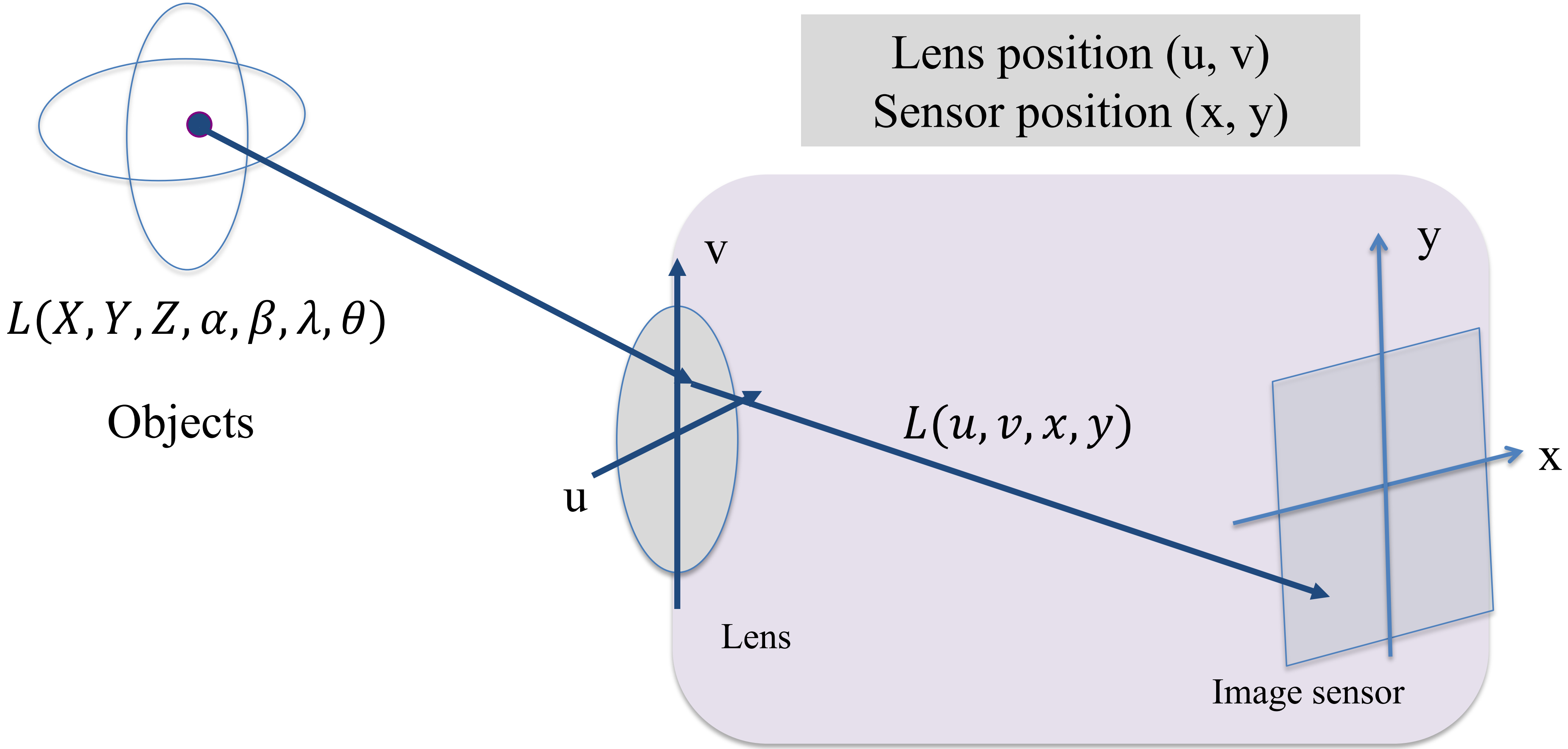
Signals along the ray all sum as they head towards the sensor



Light field imagers

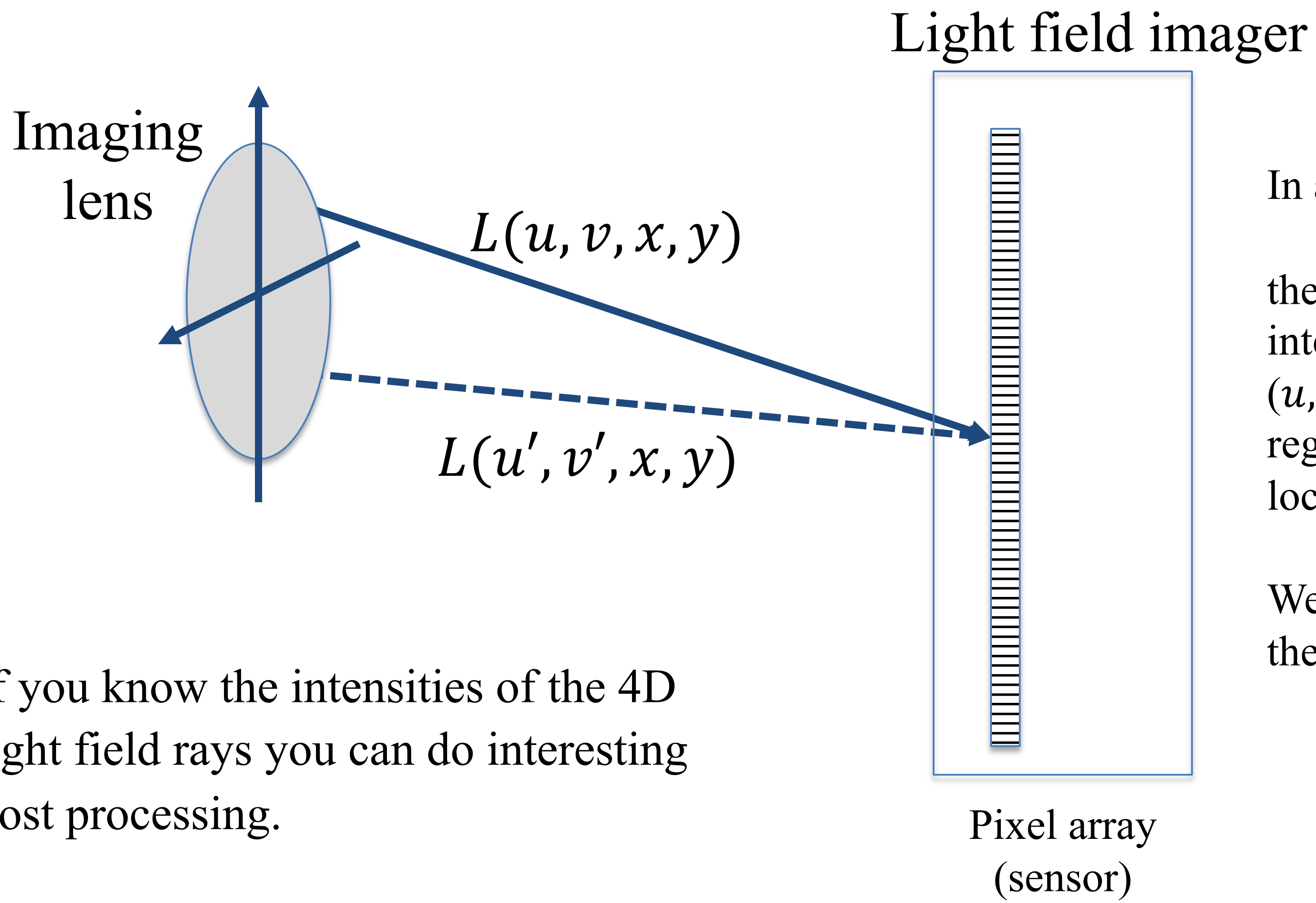
The light field rays in the camera are specified by 4 spatial coordinates

Lens position (u, v)
Sensor position (x, y)



Light field imager

Measures the intensity of each L(u,v,x,y) ray



In a conventional camera

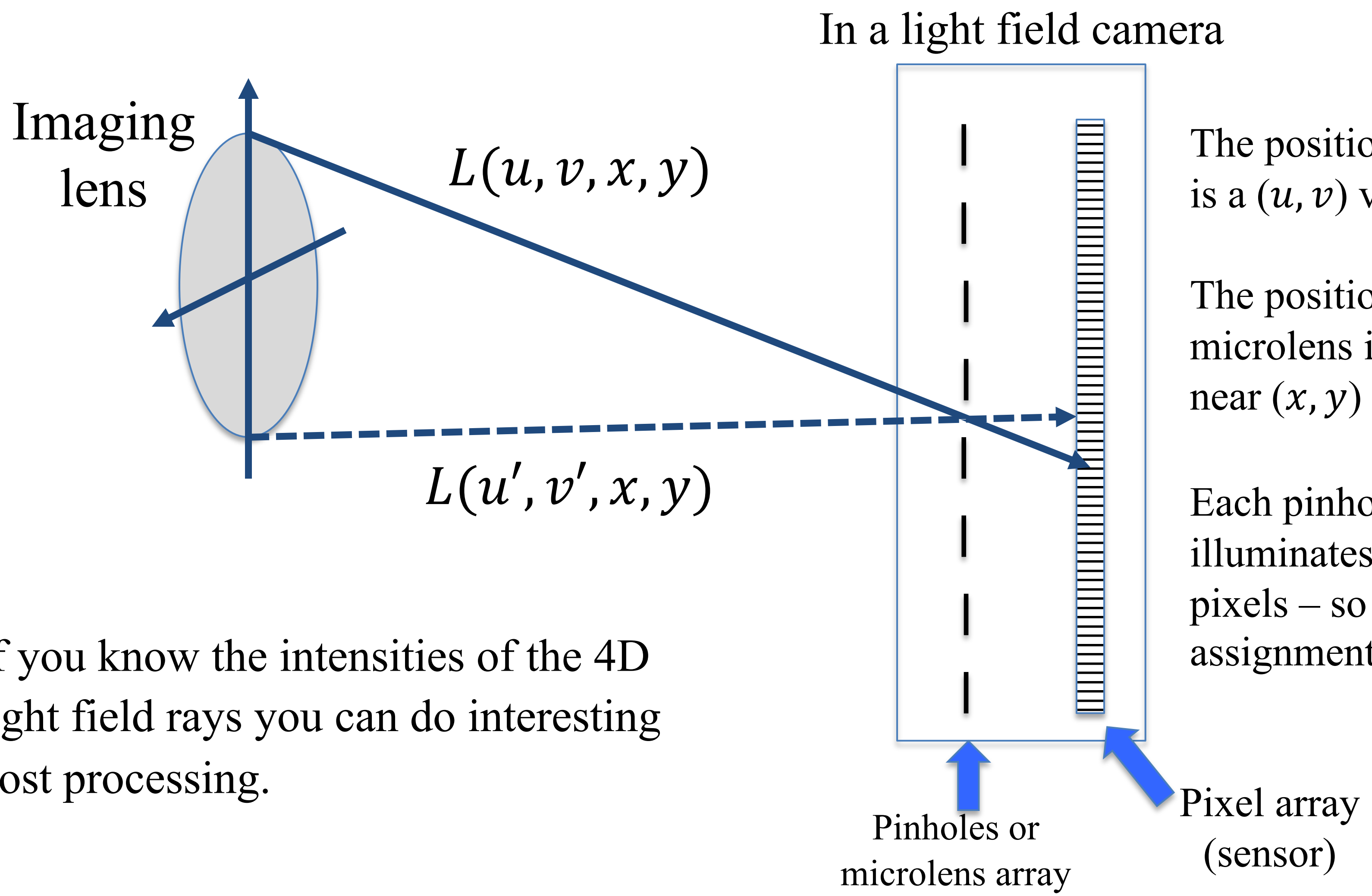
the pixel sums all the intensities from many (u, v) positions in the region near pixel location (x, y).

We do not separate out the two rays.

If you know the intensities of the 4D light field rays you can do interesting post processing.

Light field imager

s_lfIntro.m



The position in the lens is a (u, v) value

The position of the microlens is the region near (x, y)

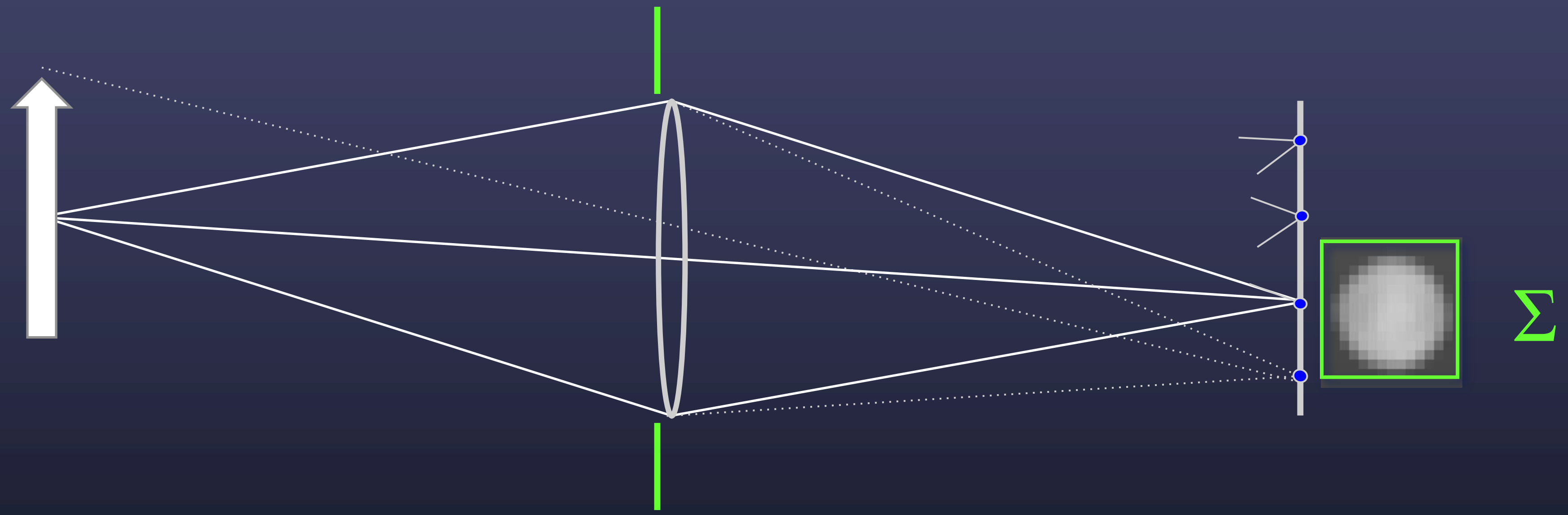
Each pinhole (microlens) illuminates a different set of pixels – so the (x, y, u, v) assignment is unambiguous

If you know the intensities of the 4D light field rays you can do interesting post processing.

Light field calculations: a reduced aperture

Reducing the aperture increases the f/#

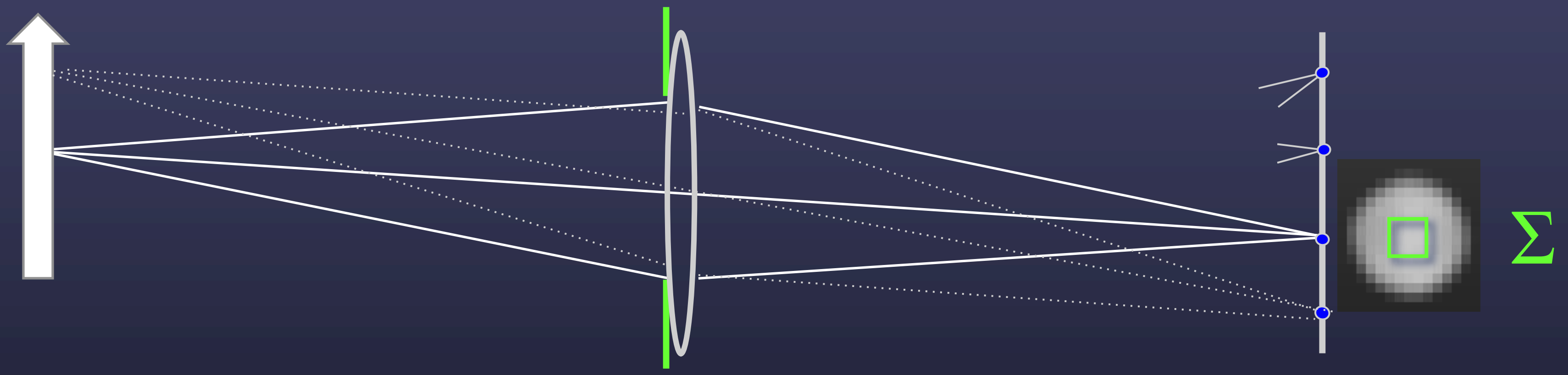
Summing all the rays is equivalent to have a large aperture



f/# = focal length / aperture

Light field calculations: a reduced aperture

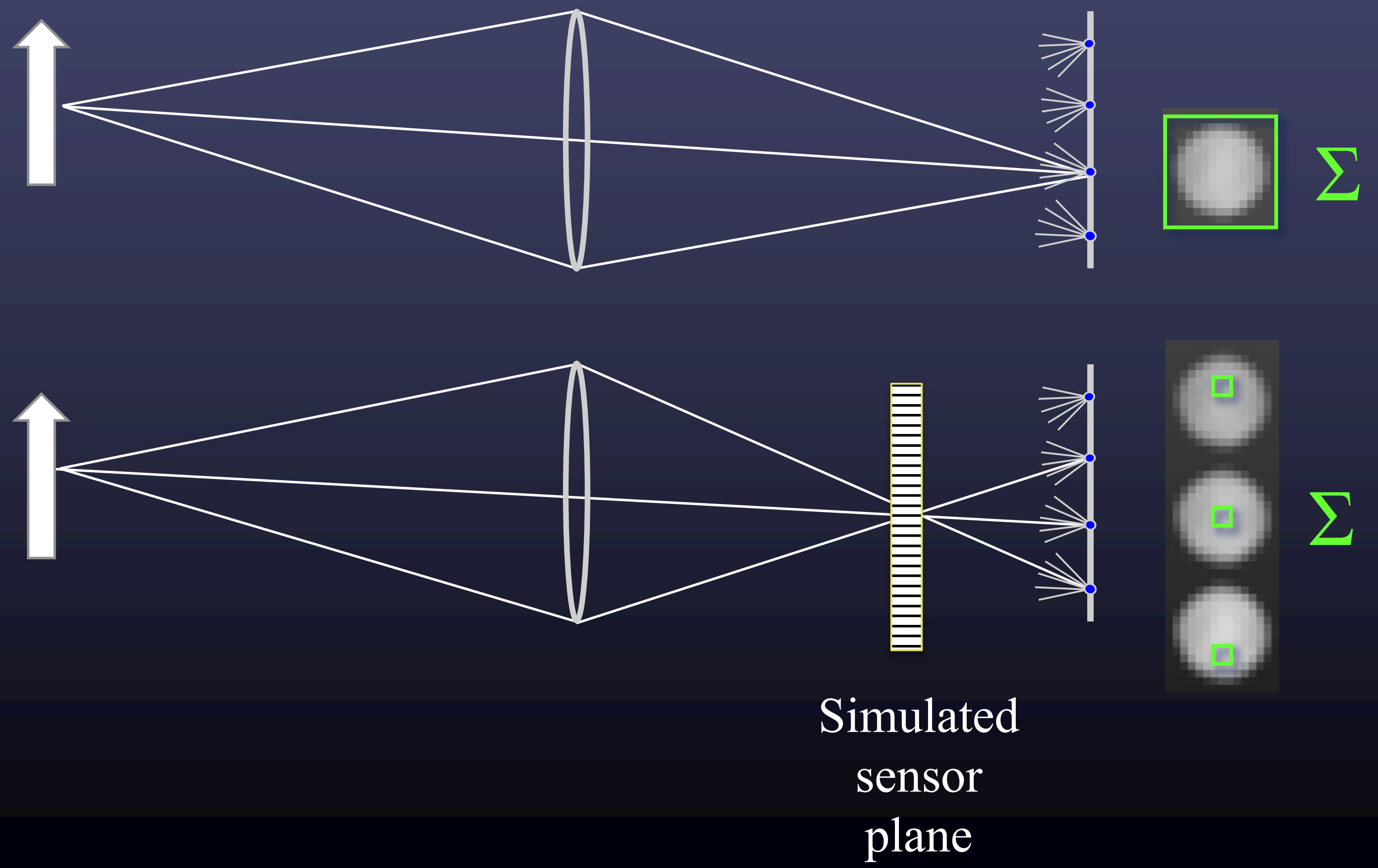
Reducing the aperture increases the f/#
Summing only the central rays behind each microlens reduces the aperture



f/# = focal length / aperture

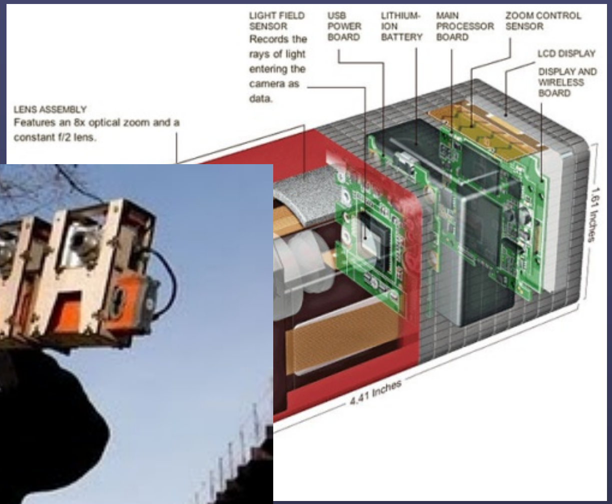
Light field calculations: changing the in-focus plane

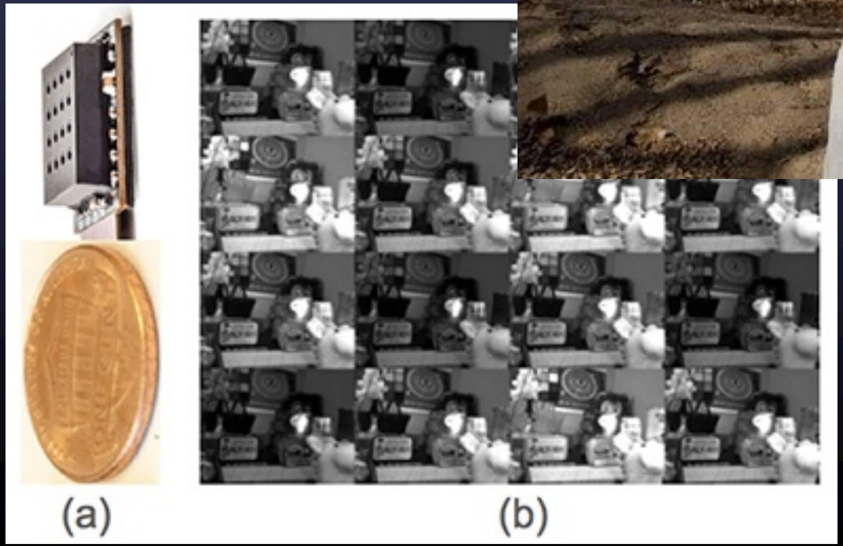
Summing rays from different parts of pixels behind several microlenses simulates moving the sensor position

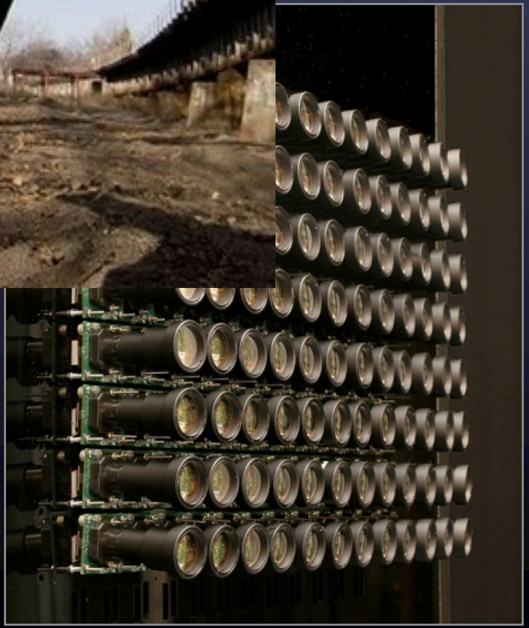


Many types of light-field cameras

(Lytro, Pelican, Stanford, MIT, Adobe, others)





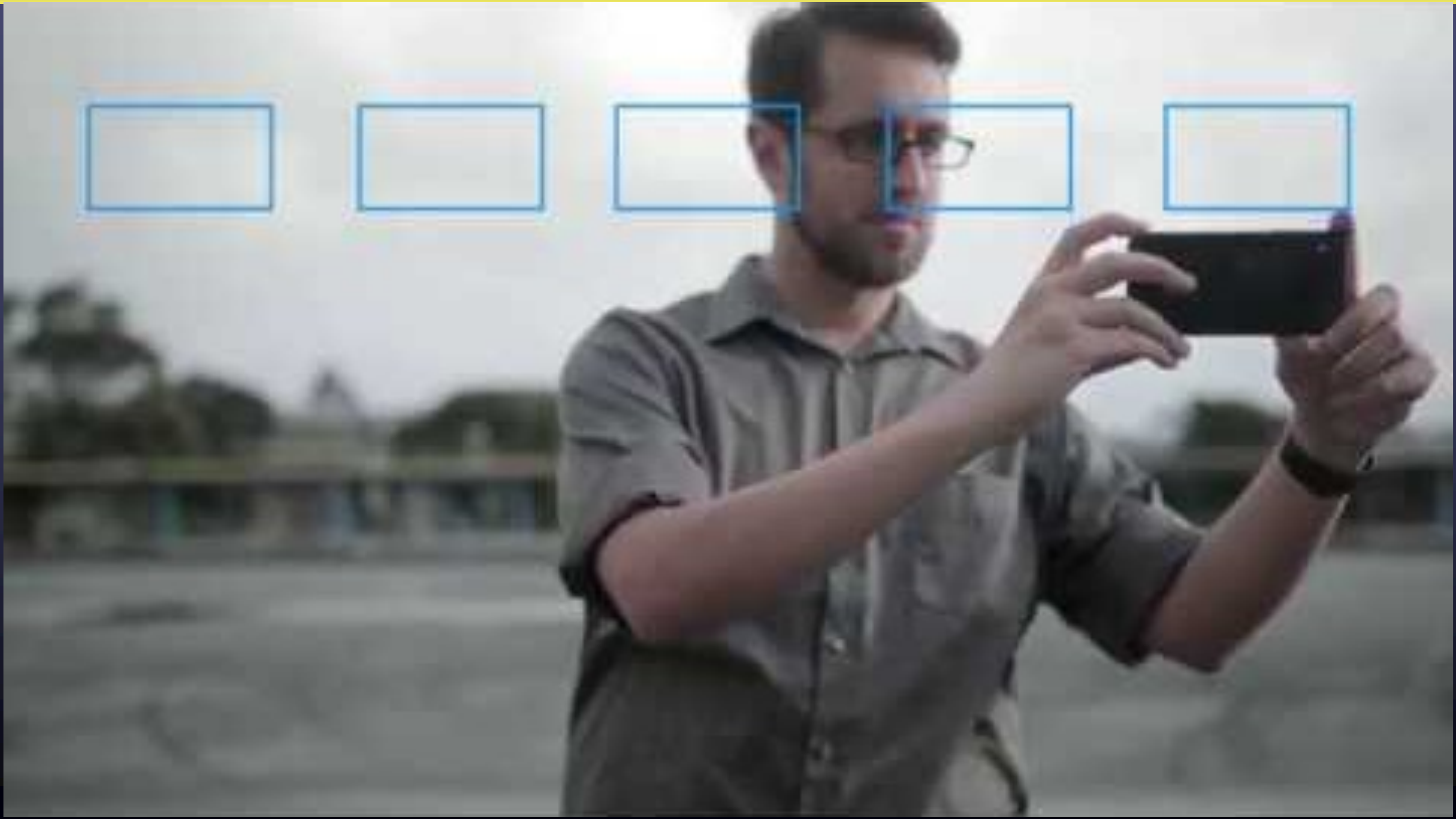


https://www.youtube.com/watch?v=4qXE4sA-hLQ

Lytro – what happened at the end stage

https://people.eecs.berkeley.edu/~bmild/llff/

Local Light Field Fusion (UCB Siggraph 2019)



Interactive online demonstration and video tutorial on ISET light field camera simulation

Captured light field data and interactive demonstration

http://lightfield.stanford.edu/lfs.html

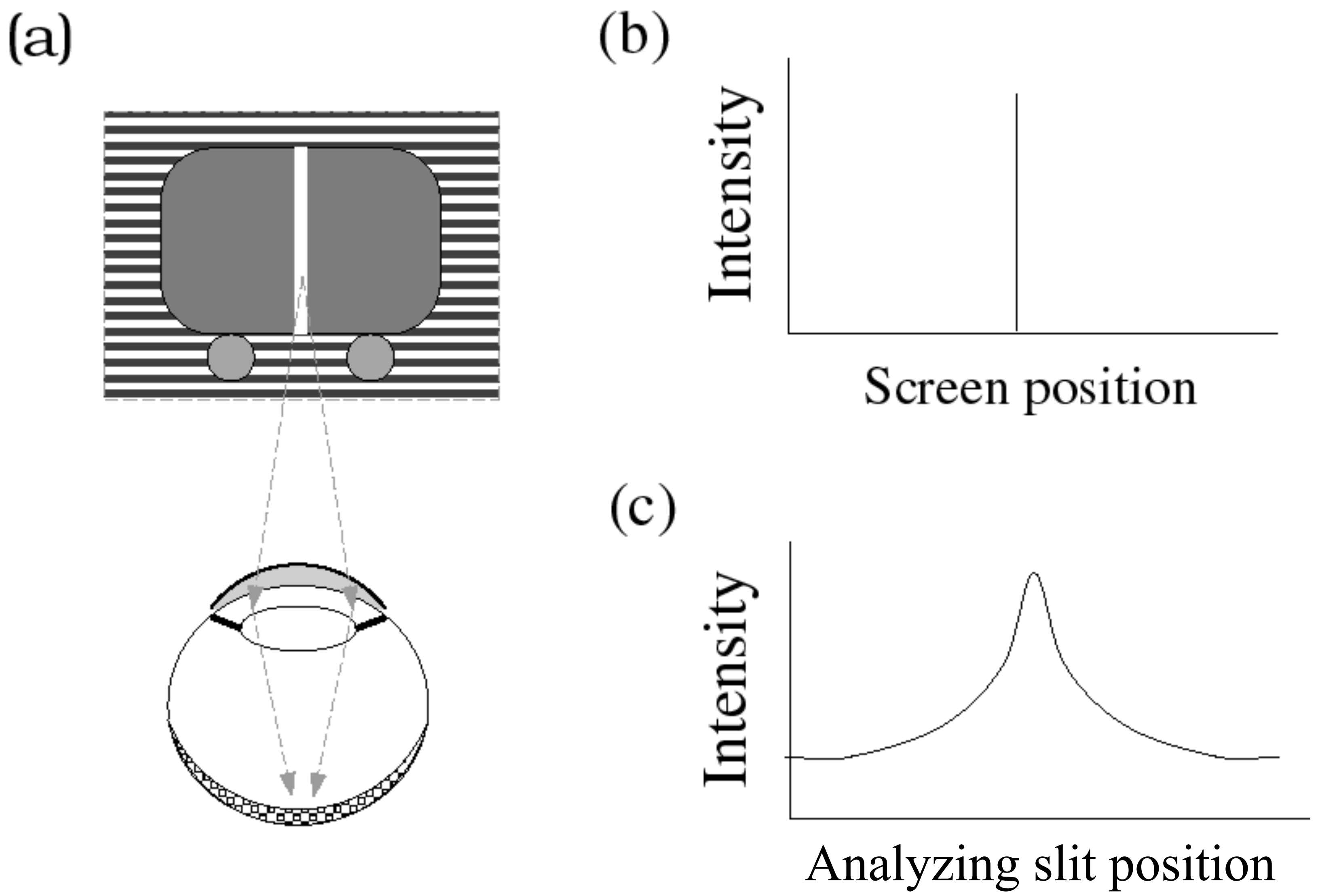
ISET simulation of light field camera design

https://www.youtube.com/watch?v=hZCd2m9TVBo

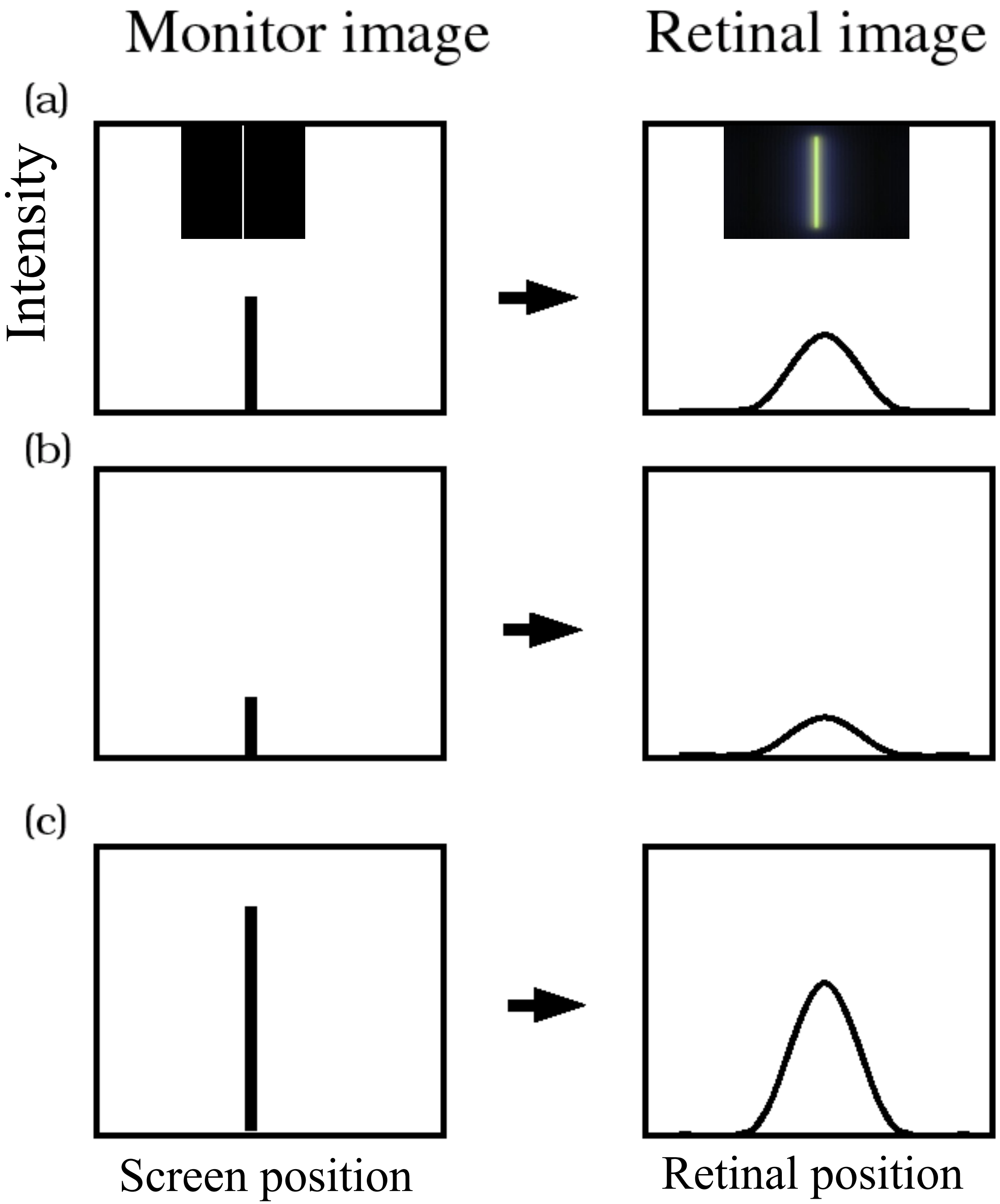
Linear systems and optics

- Linespread function
- Shift-invariant linear systems
- Harmonics and the modulation transfer function

The line spread Function: A summary of optical quality

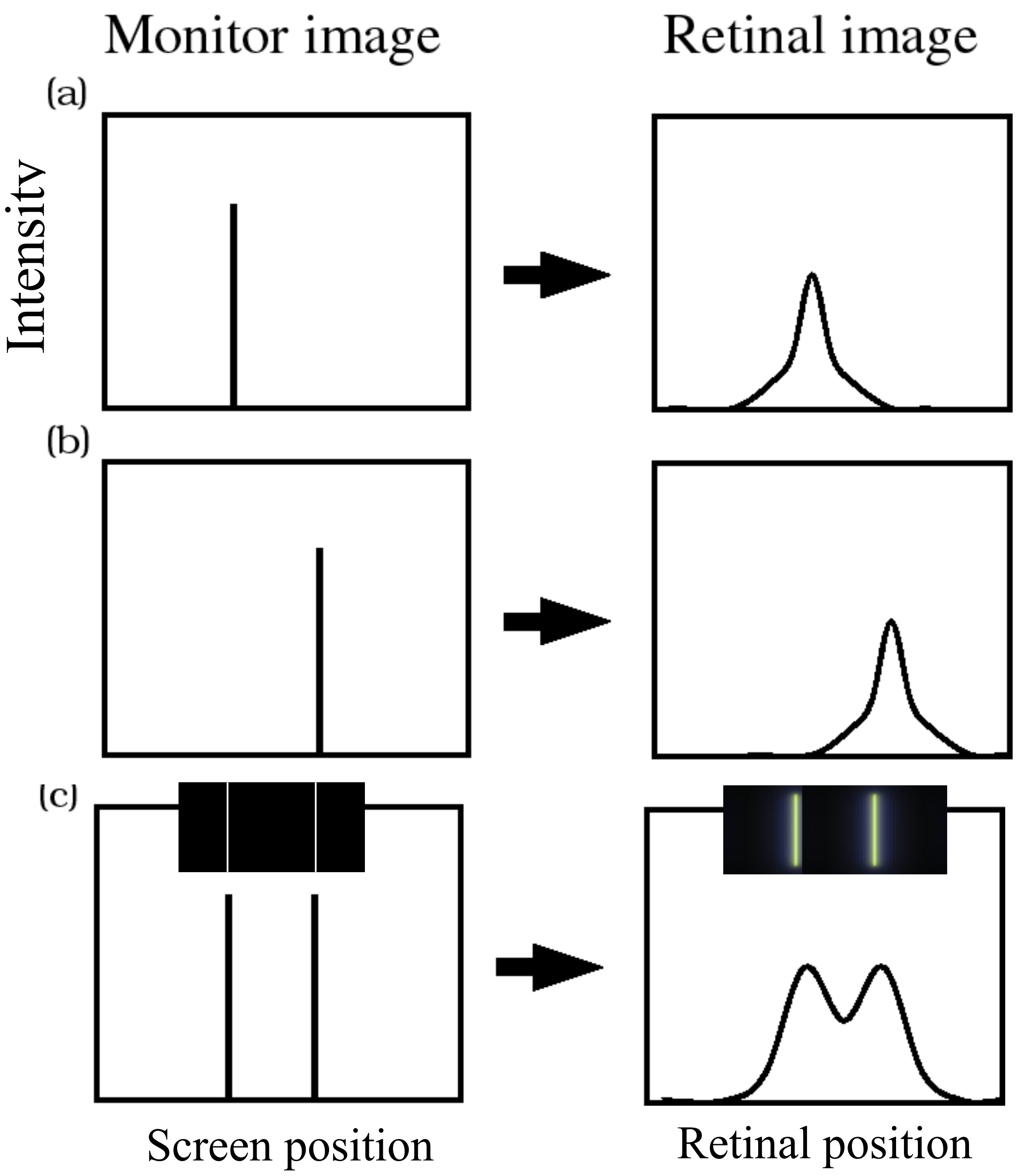


Thin lenses and many typical optical systems satisfy the linear system principle of homogeneity



Graphs for one wavelength

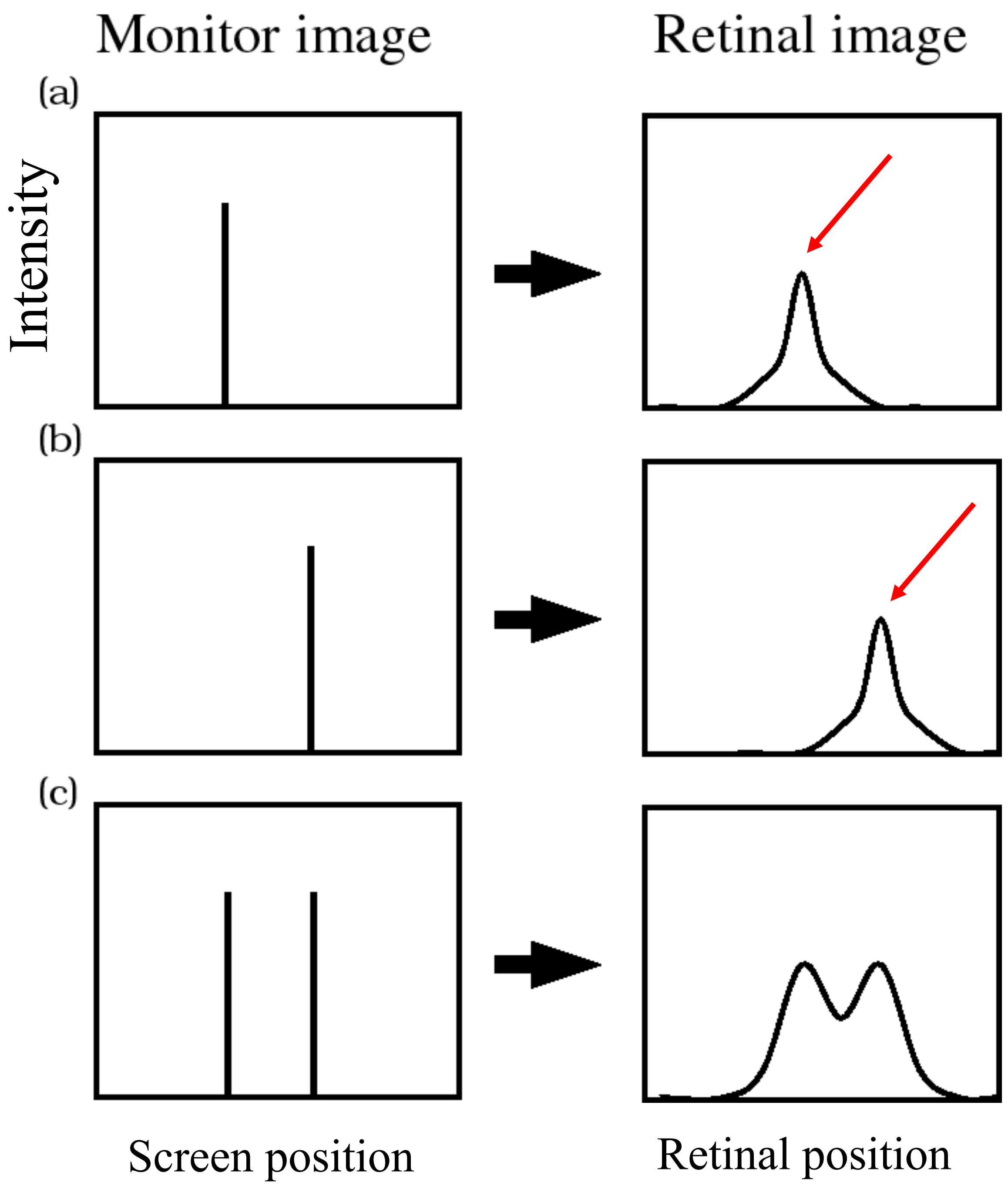
Many typical optical systems are linear systems and thus also satisfy superposition



Graphs for one wavelength

Some linear systems also satisfy shift-invariance; Many satisfy it locally

Isoplanatic *is the term used in optics for a shift-invariant region of the lens; most lenses are only locally shift-invariant*



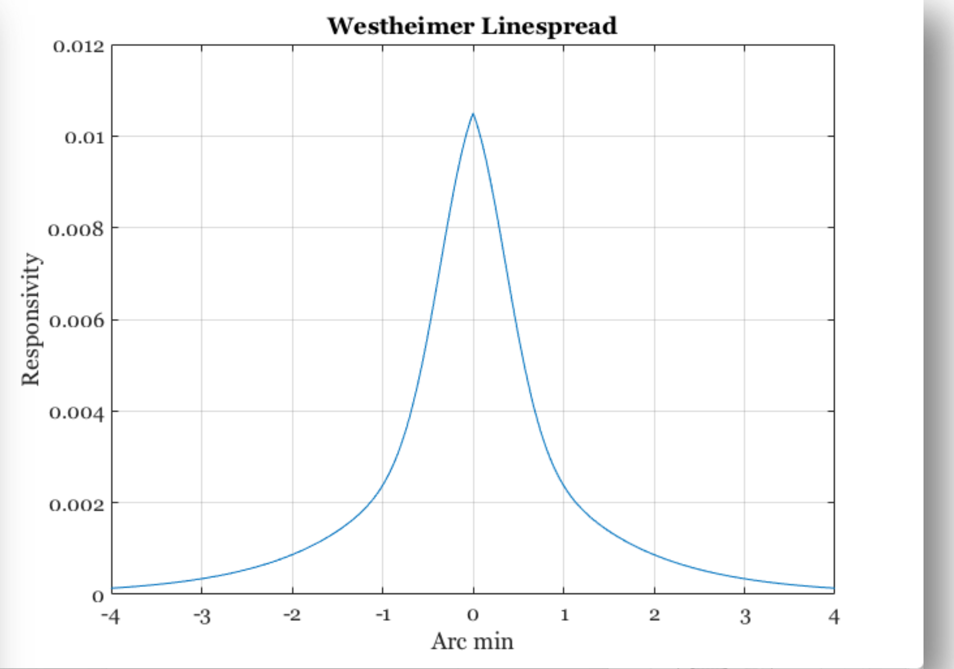
Westheimer line spread (mlx_hwImageFormation.mlx)

The Westheimer linespread function

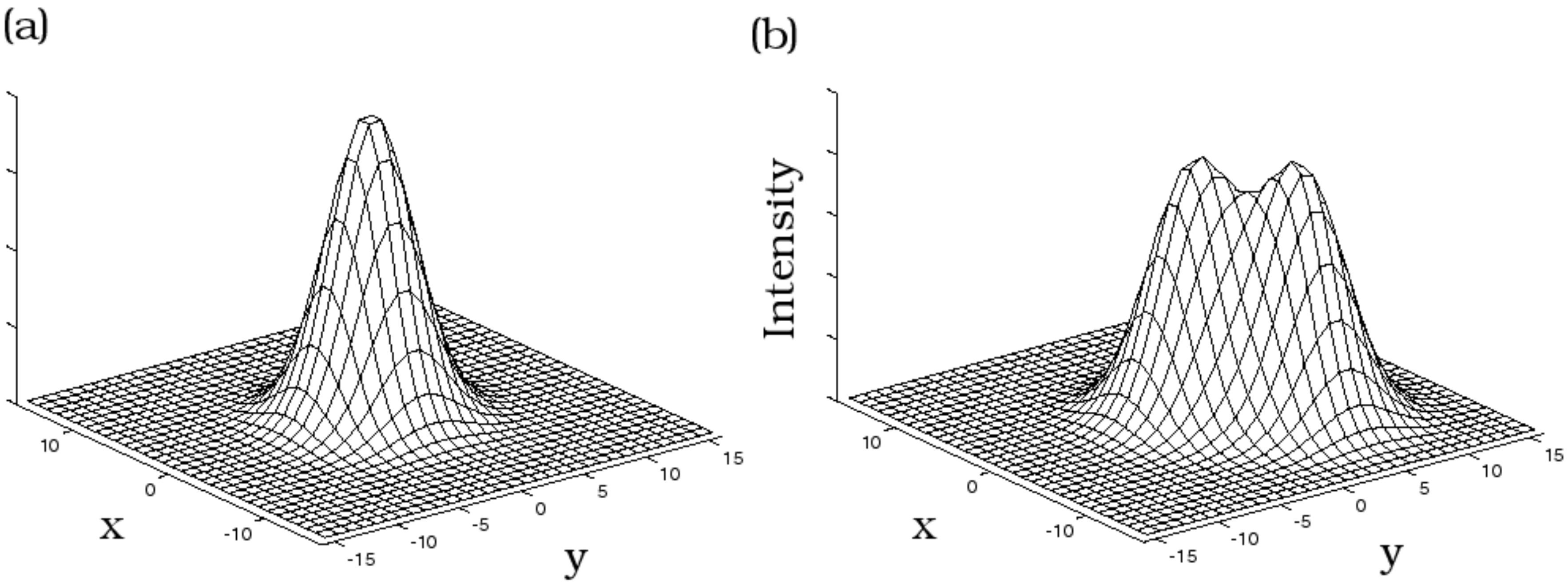
Westheimer was one of the first to estimate the linespread function of the human optics. He specified the spread of light at the back of the eye when the input is a line. He specified the line spread in terms of a variable, x, in minutes of arc. For a 3mm pupil size (and the pupil size matters) he proposed the formula

$$\mathrm{ls}(x) = 0.47 * \exp\left(-3.3 * x^2\right) + 0.53 * \exp\left(-0.93 * \mathrm{abs}(x)\right)$$

```
% Suppose we plot the linespread function at a slightly finer spatial resolution,
% in seconds of arc.
xSec = -300:1:300;

% To use the functional form from Westheimer we convert the units to minutes of arc
xMin = xSec/60;
ls = 0.47*exp(-3.3 *(xMin.^2)) + 0.53*exp(-0.93*abs(xMin));

% And we normalize the ls assuming that no light is lost.
ls = ls / sum(ls);

vcNewGraphWin;
plot(xSec/60,ls)
set(gca,'xlim',[-240 240]/60,'xtick',(-240:60:240)/60), grid  on
xlabel('Arc min'), ylabel('Responsivity'), title('Westheimer Linespread')
```

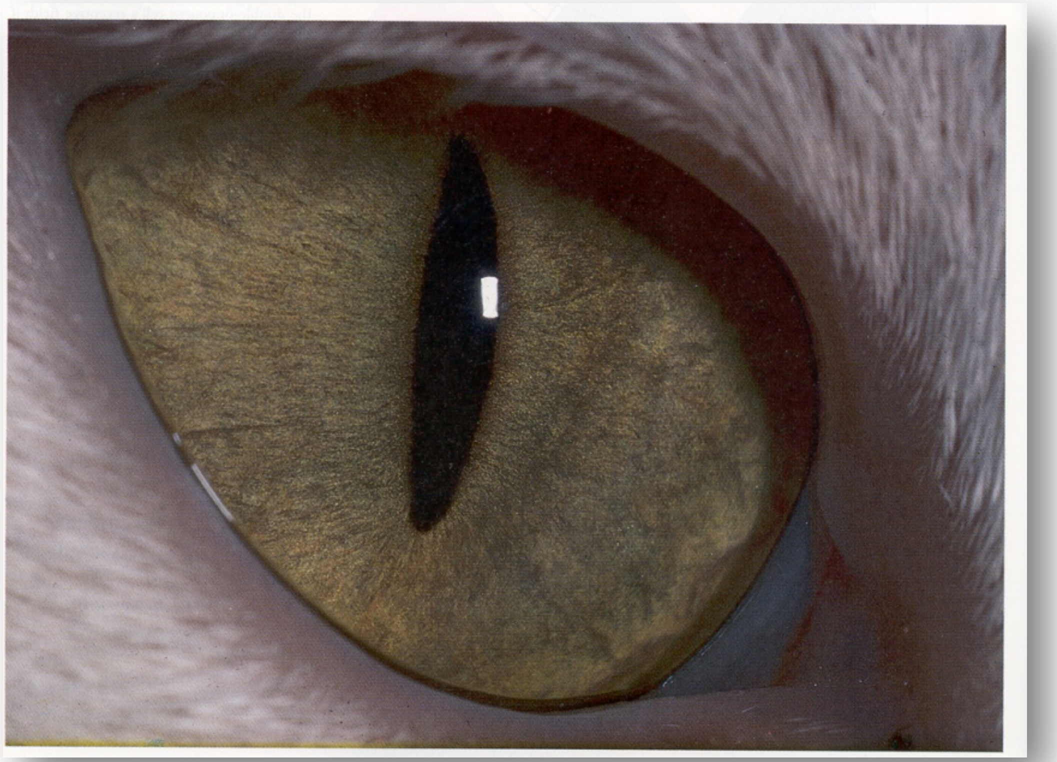


The point spread function generalizes the line spread

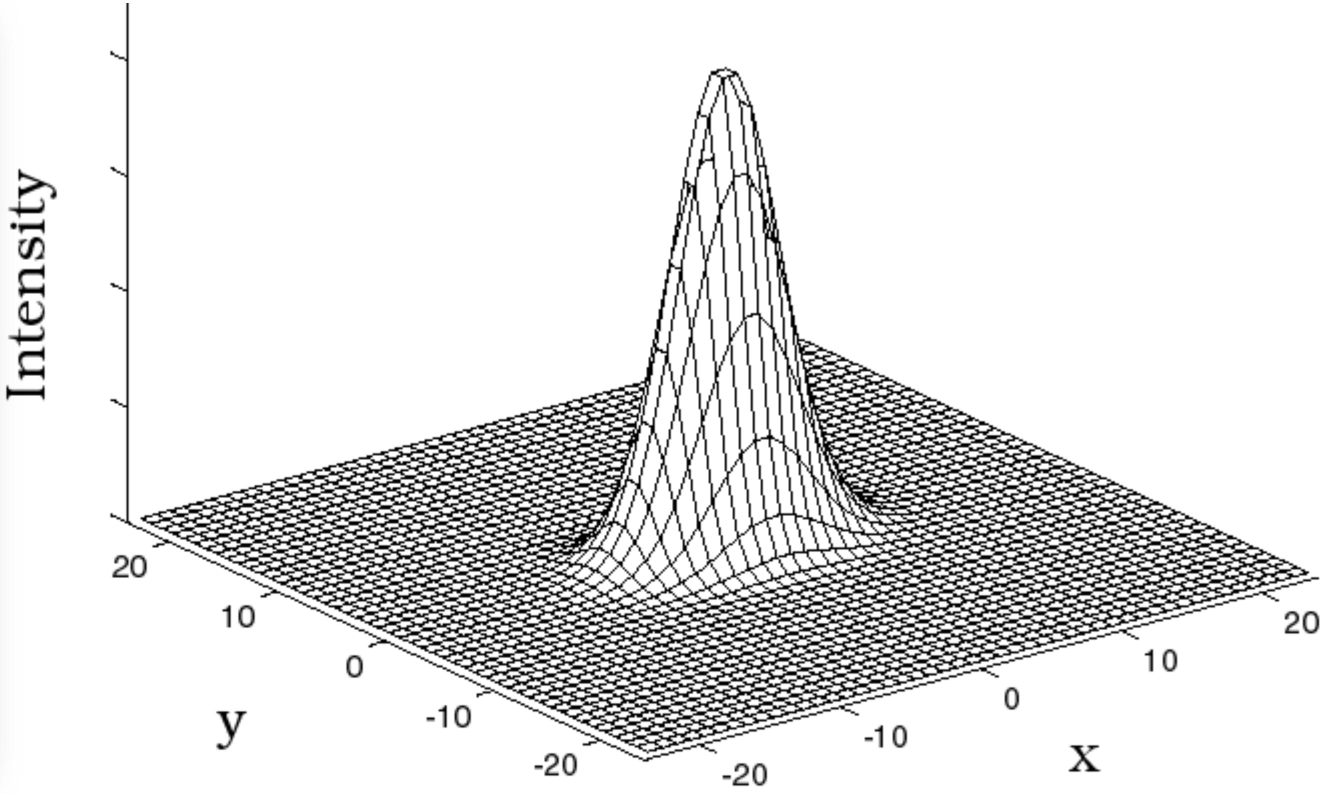


Astigmatism measures the orientation of the point spread function

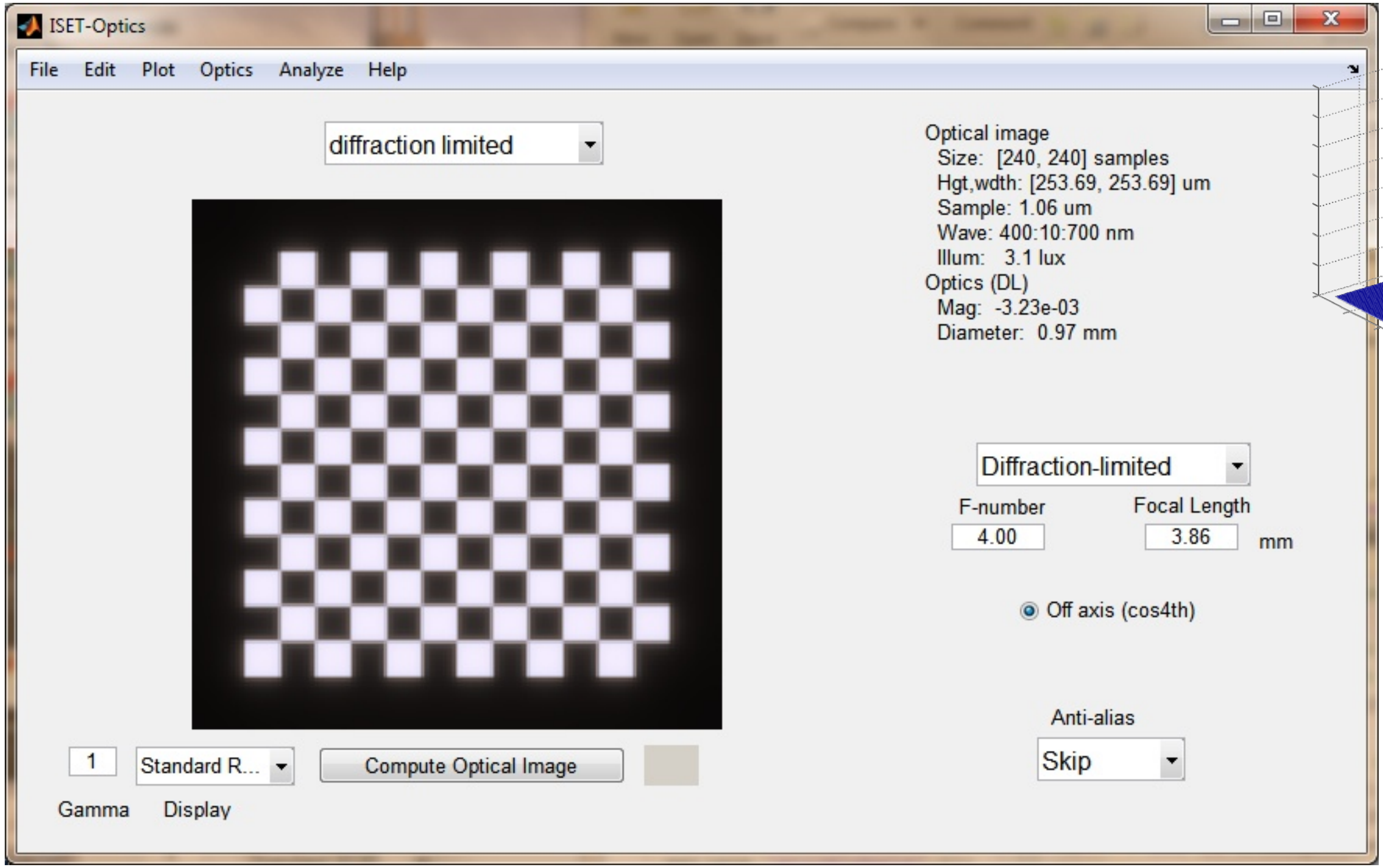
A cat's pupil closes as a slit, not circular

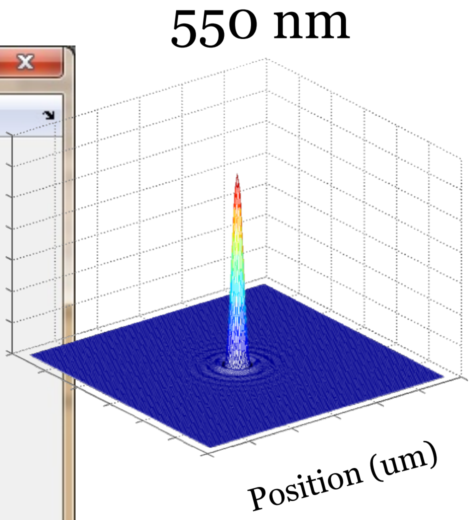


The blurring must differ between the two directions



Diffraction-limited image

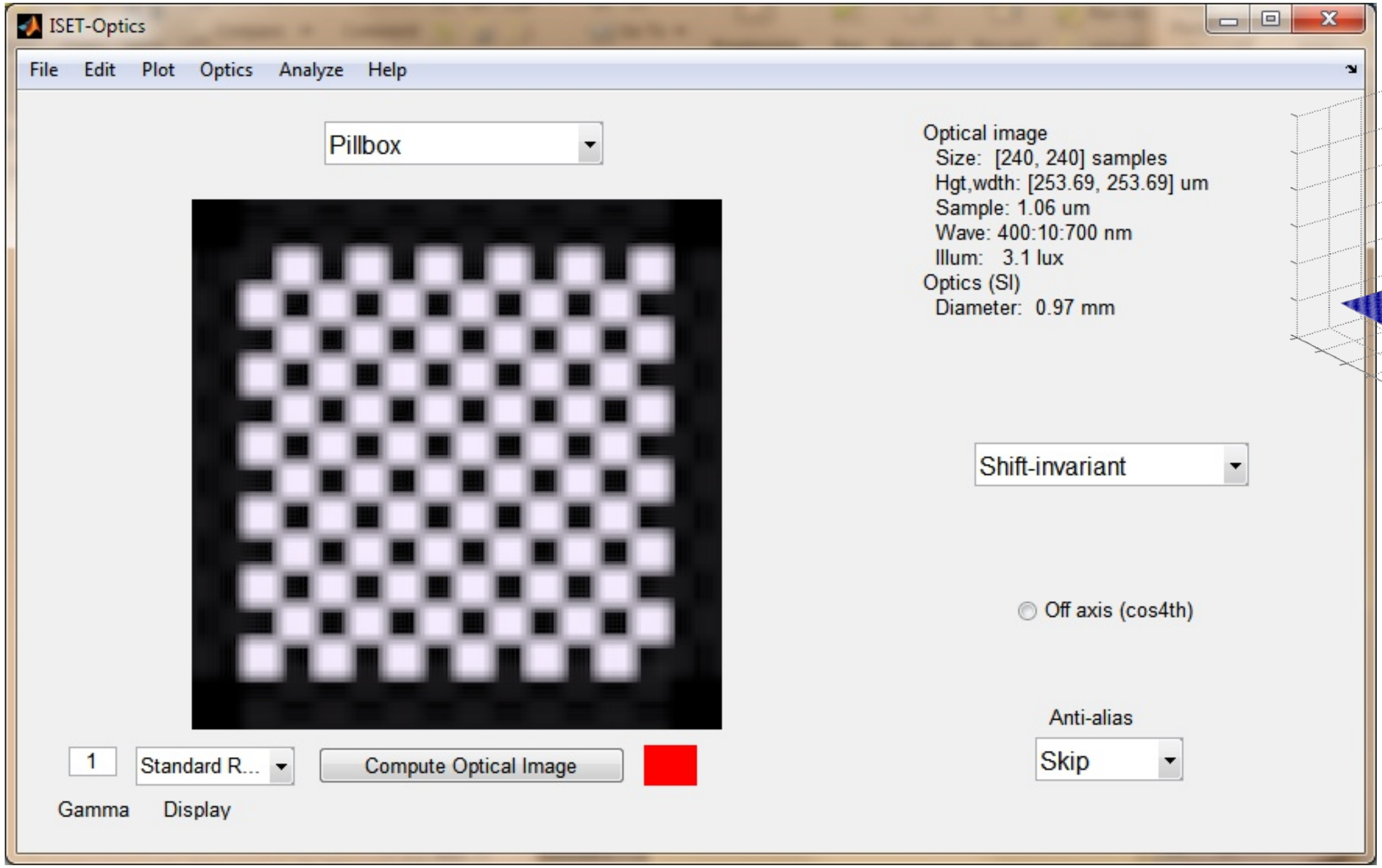




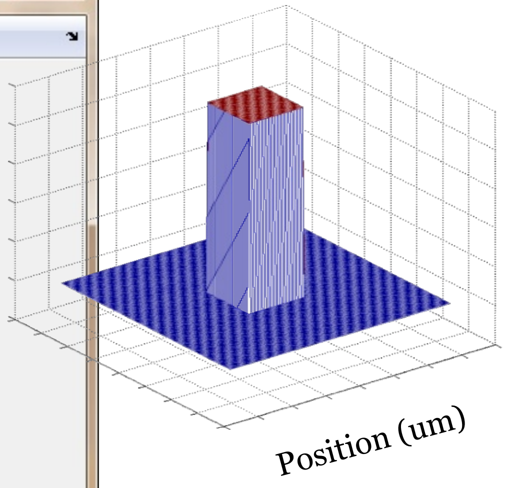
$$irrad(x,\lambda) = S \times \int rad(z,\lambda)\ psf(x-z,\lambda)\,dz$$

Shift-invariant pillbox blur

s_opticsSIExamples.mlx



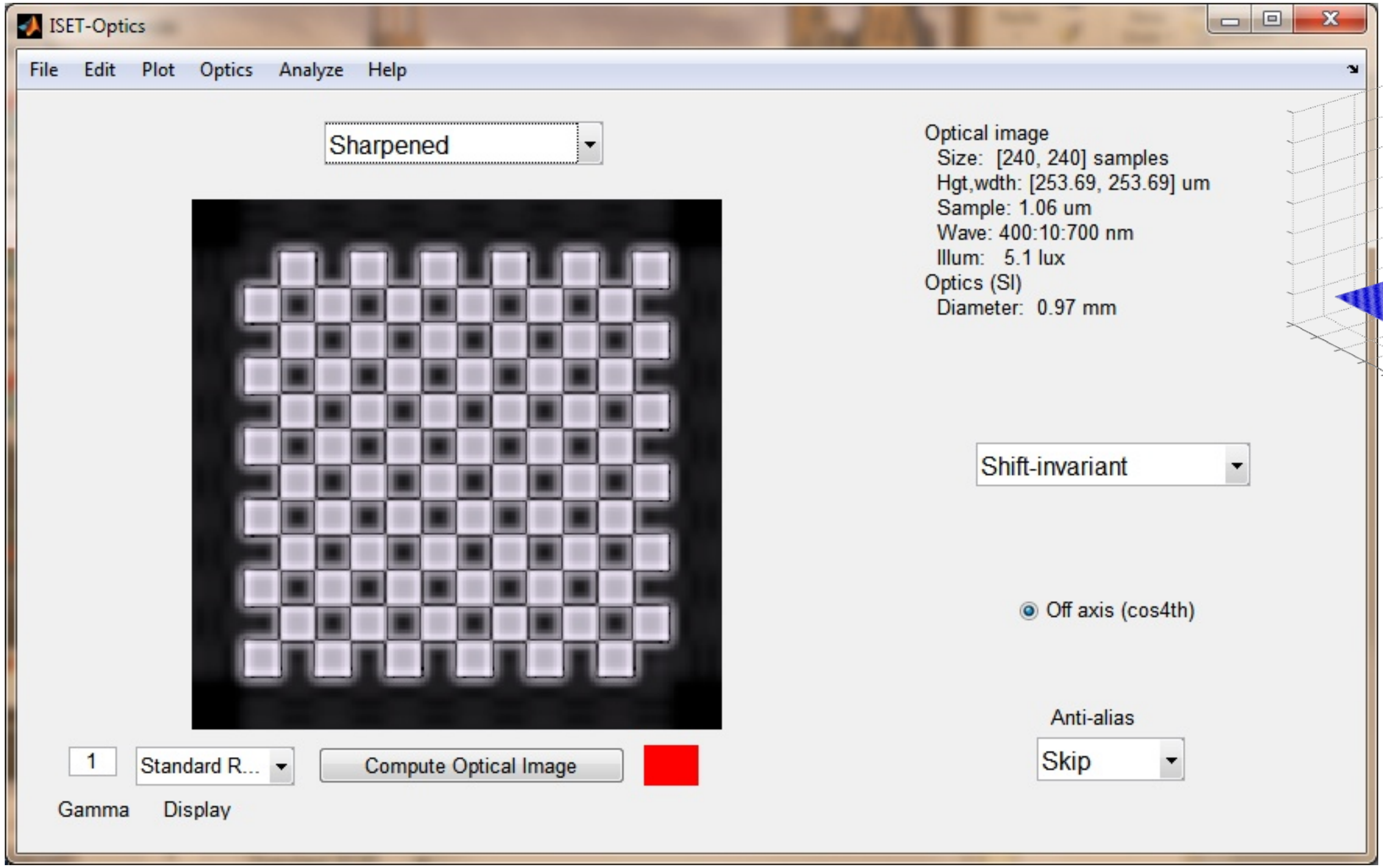
All wave



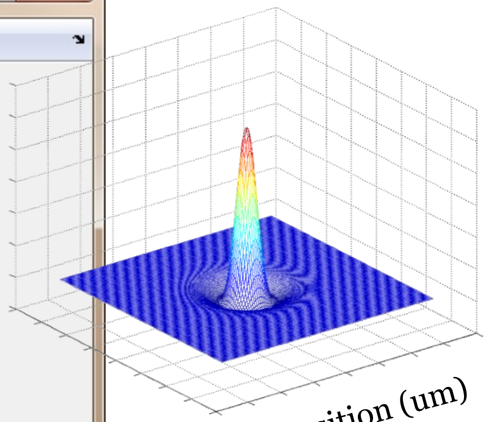
$$irrad(x,\lambda) = S \times \int rad(z,\lambda)\ psf(x-z,\lambda)\,dz$$

Shift-invariant sharpening

s_opticsSIExamples



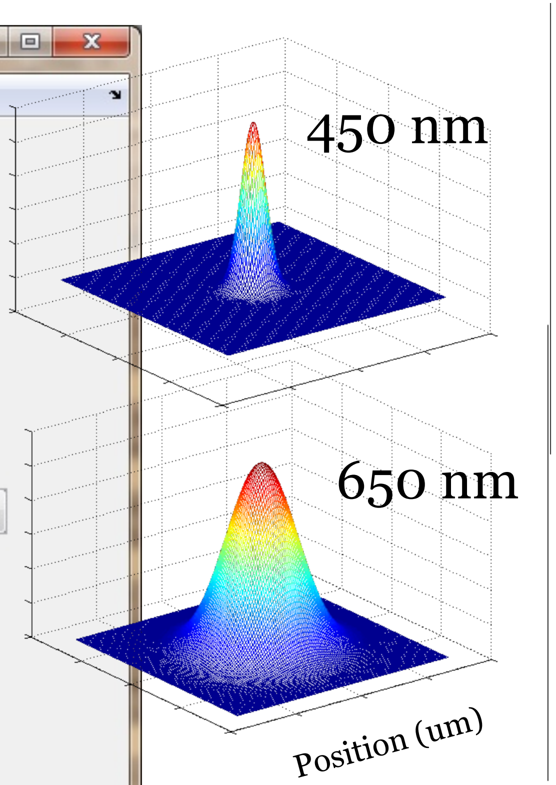
All wave



$$irrad(x,\lambda) = S \times \int rad(z,\lambda)\ psf(x-z,\lambda)\,dz$$

Wavelength-dependence

s_opticsSIExamples



$$irrad(x,\lambda) = S \times \int rad(z,\lambda)\ psf(x-z,\lambda)\,dz$$

Calculations in the transform domain: Fourier series

- Linear models
- 1D Fourier Series
- 2D Fourier Series

Linear models

Most branches of science use linear models to summarize and analyze their data

$$S(t) = \sum_{i=1}^{i=N} w_i B_i(t)$$

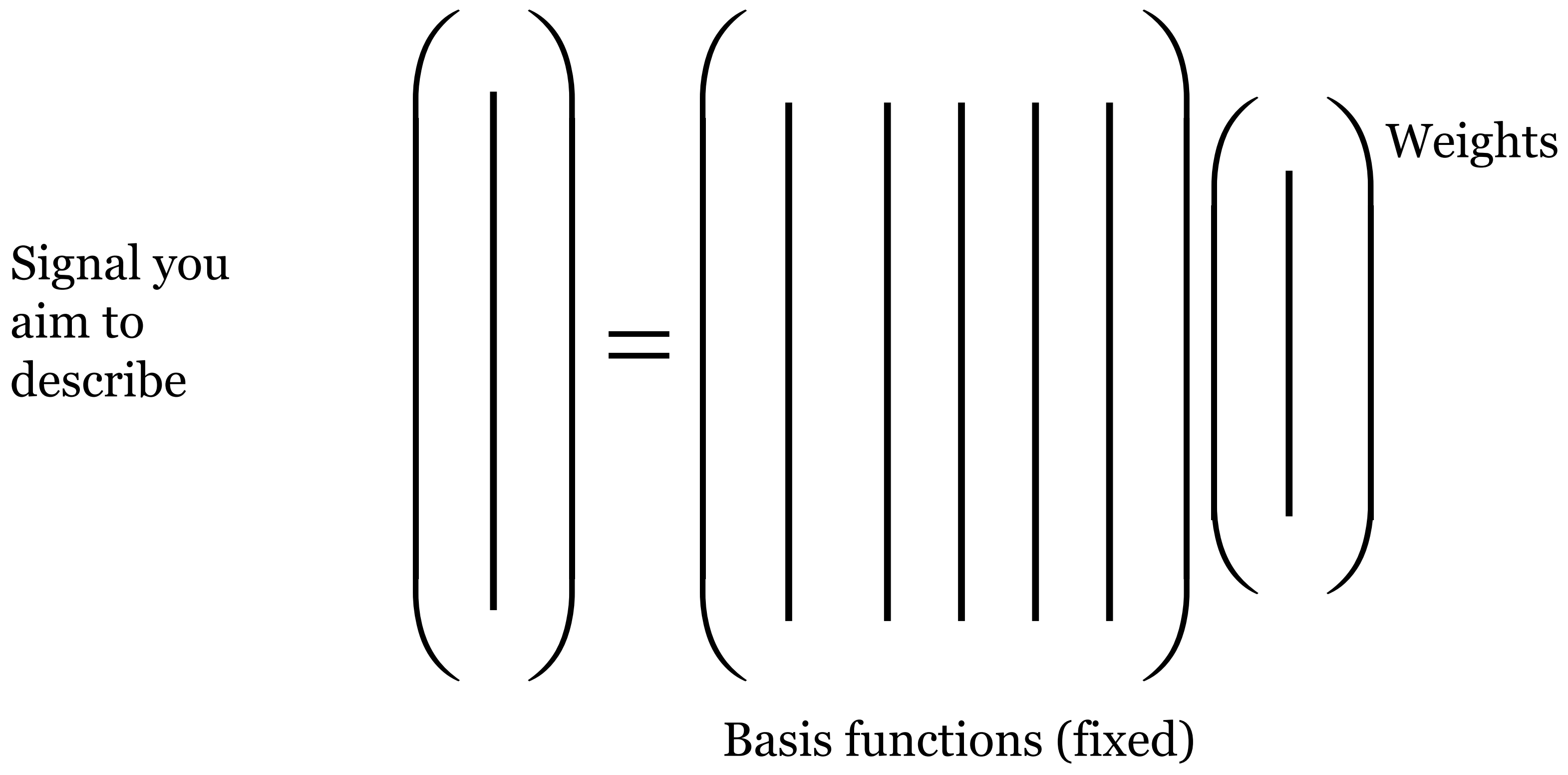
Signal you aim to describe

Weights

Basis functions

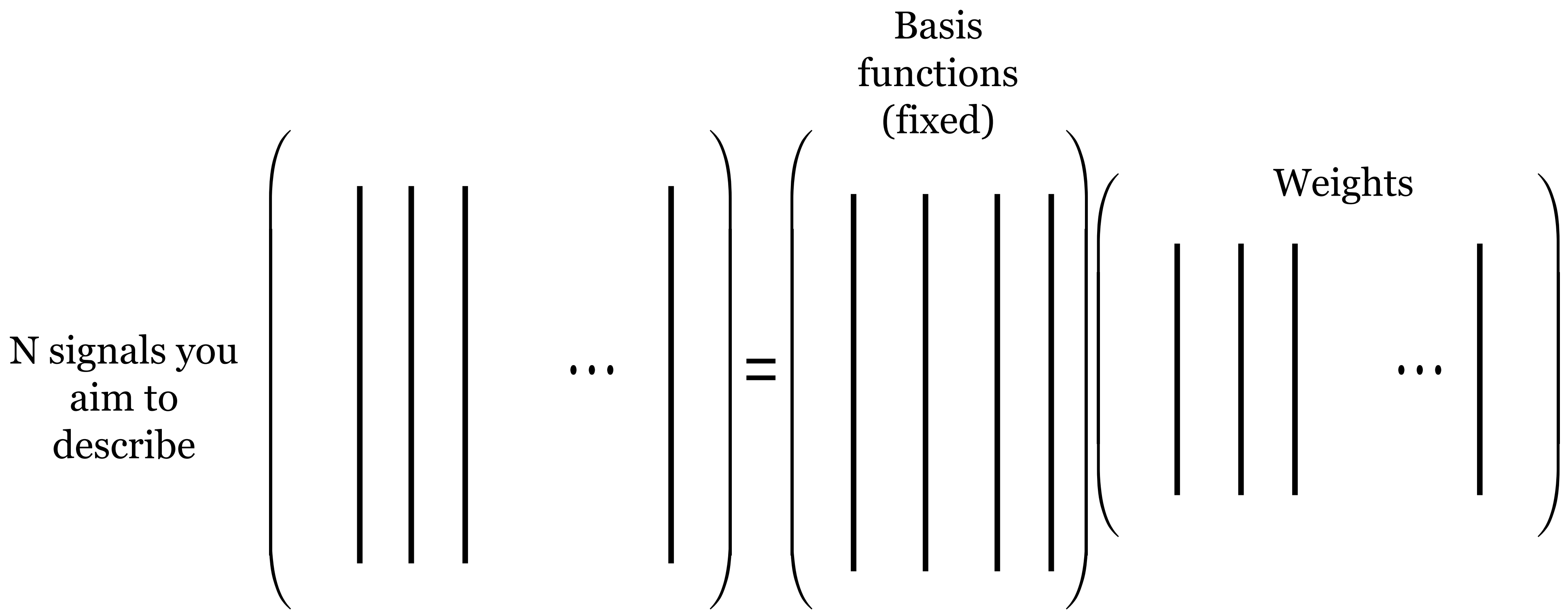
Linear models – matrix tableau

Most branches of science use linear models to summarize and analyze their data



Linear models – matrix tableau

Most branches of science use linear models to summarize and analyze their data



The Fourier series is a linear model

General linear model

$$S(t) = \sum_{i=1}^{i=N} w_i B_i(t)$$

Harmonic basis

$$S(t) = M + \sum_f u_f \sin(2\pi f t) + v_f \cos(2\pi f t)$$

Equivalent

$$S(t) = M + \sum_f a_f \sin(2\pi f t + \phi_f)$$

The Fourier series and linear systems

Fourier Series

$$S(x) = \frac{u_0}{2} + \sum_{f=1}^{N} \left(u_f \sin(2\pi f x) + v_f \cos(2\pi f x) \right)$$

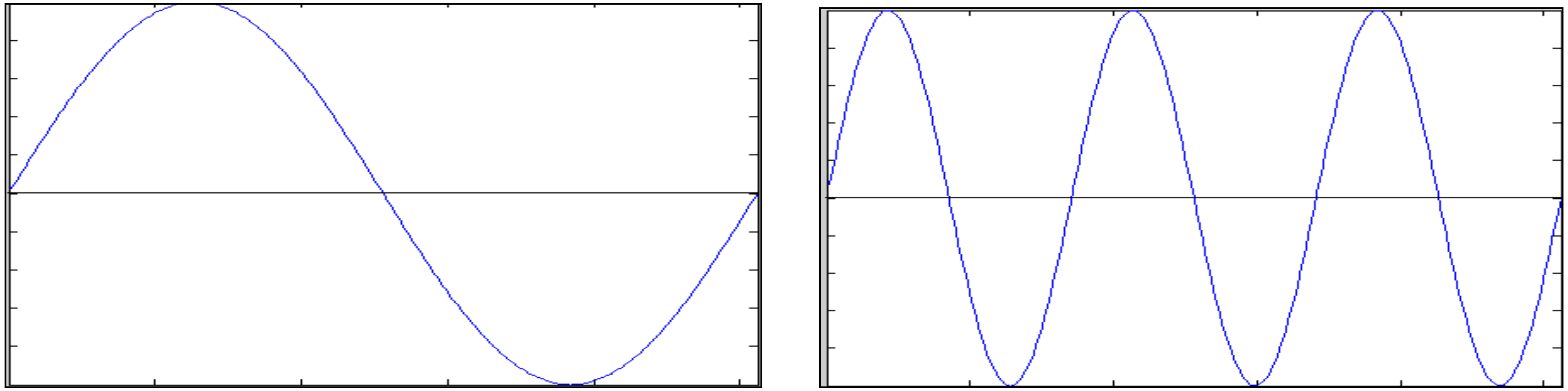
It is surprising that any periodic function, S(x), can be expressed as the sum of harmonics

Jean Baptiste Joseph Fourier

A French mathematician and physicist best known for describing the **Fourier series** and their application to problems of heat transfer. The **Fourier transform** and Fourier's Law are also named in his honor. Fourier is also generally credited with the discovery of the greenhouse effect [1]. (Wikipedia)

The Fourier series is a linear model

$$S(t) = M + \sum_f a_f \sin(2\pi f t + \phi_f)$$

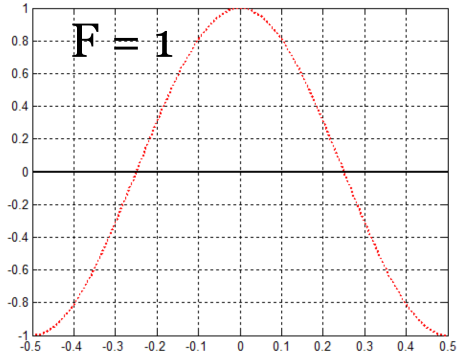


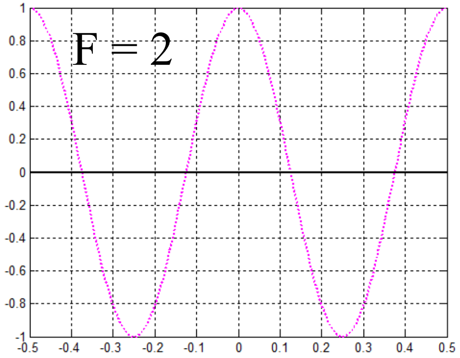
Space

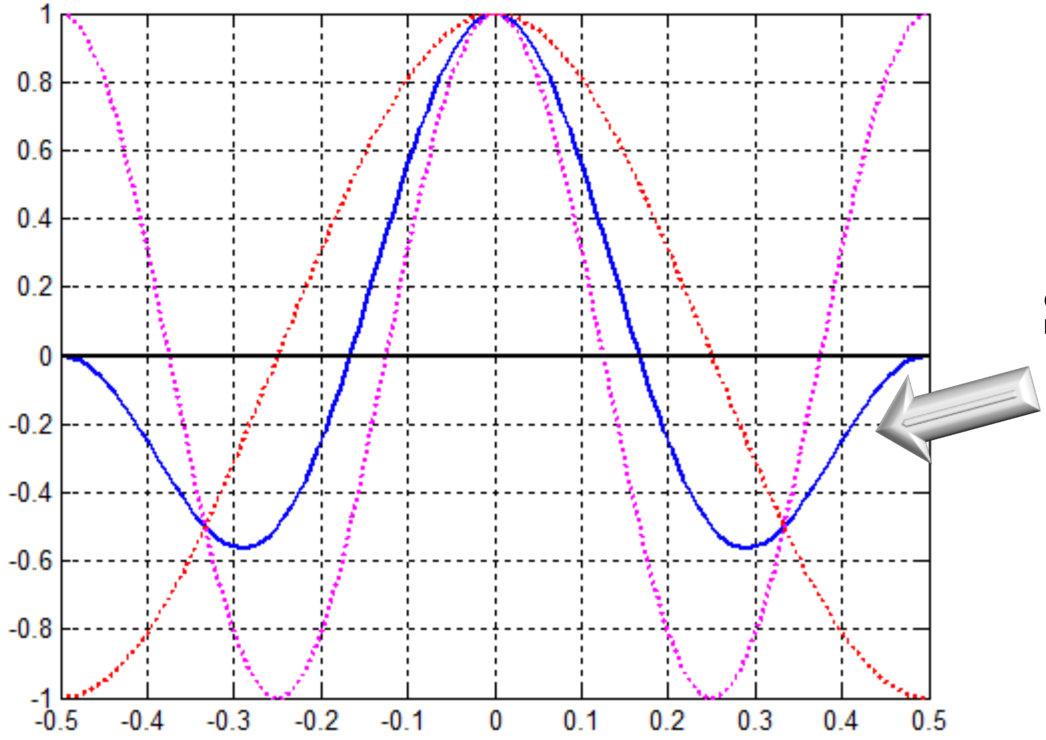
a – Amplitude of the harmonic
f – Frequency of the harmonic
ϕ – Phase of the harmonic

How can harmonics sum to an impulse?

Summing across frequencies,
the cosinusoids add at the origin
and cancel at all other positions





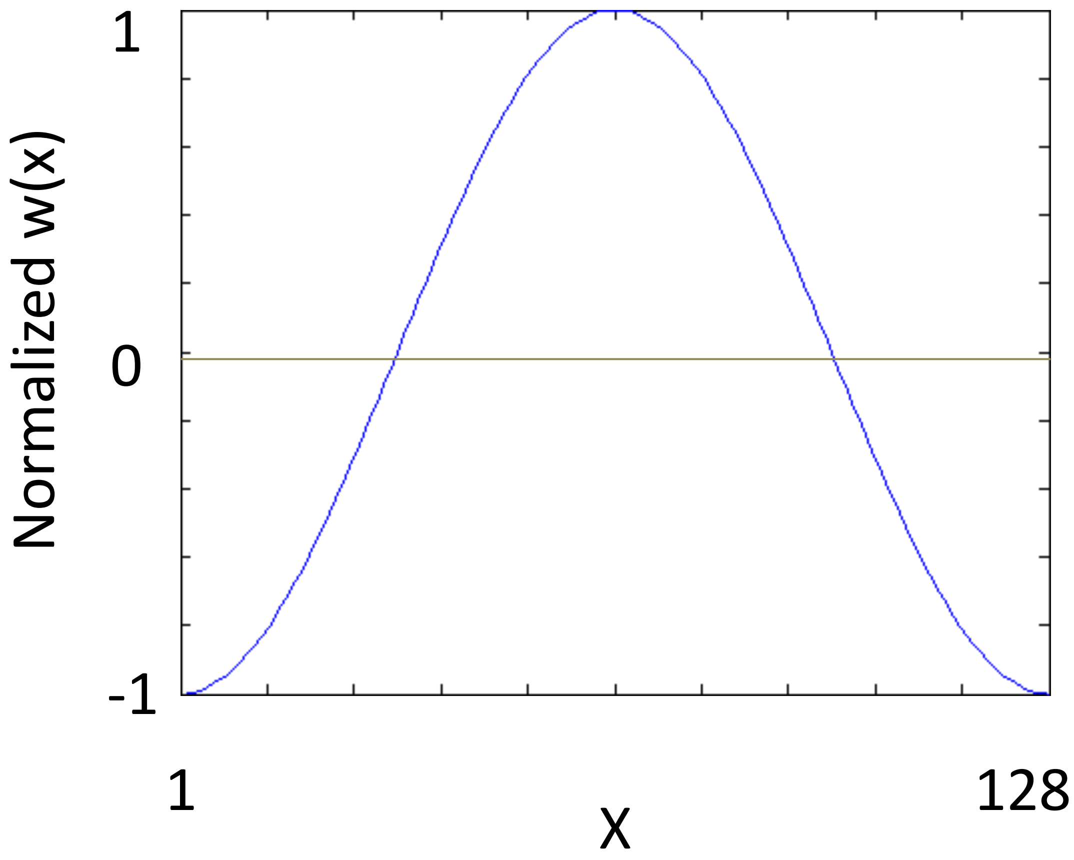


Sum of the
first two
harmonics

How can harmonics sum to an impulse?

$$w(x) = \left(\frac{1}{N_f}\right) \sum_{f=1}^{f=128} \cos(2\pi f x)$$

At each step, we normalize by the number of frequency terms, N_f



Images and Fourier series

- Image contrast
- Transform space

Images, harmonics, and contrast

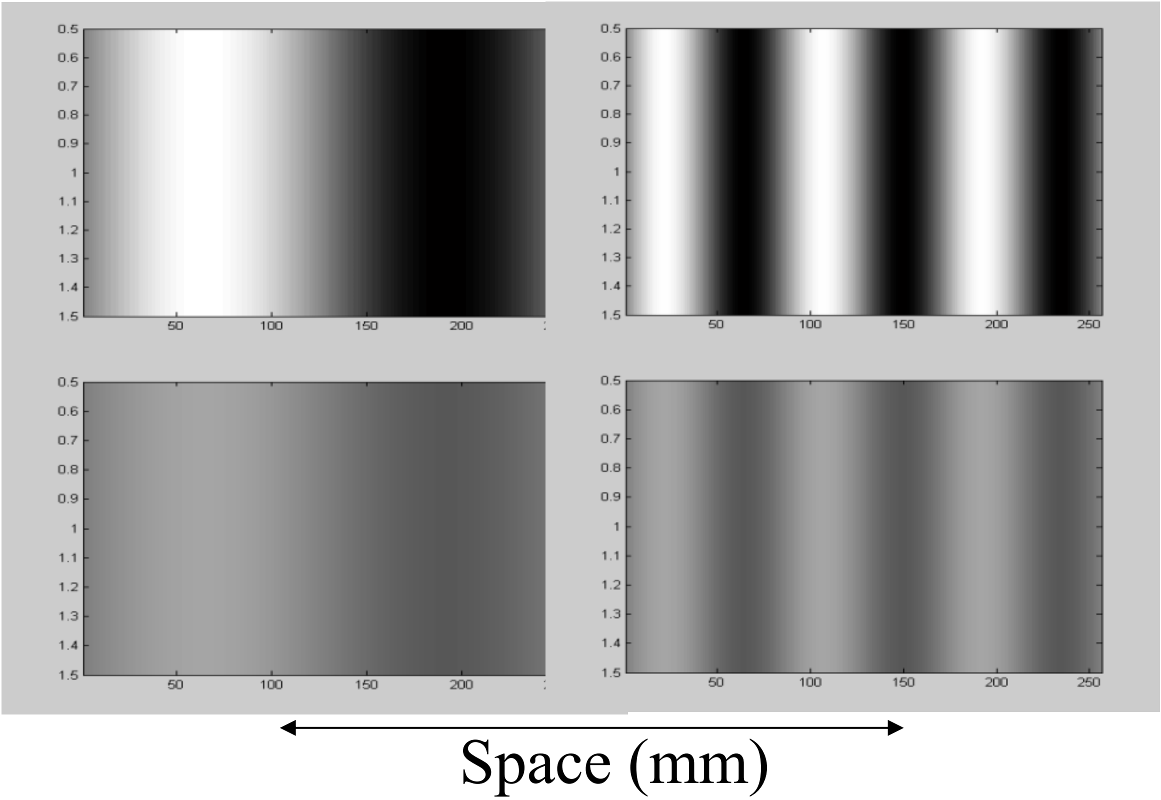
- Light intensity is always > 0, image harmonics are specified in contrast – variations around the mean

- The contrast harmonic parameters are
  contrast : -1 < a < 1
  (dimensionless)
  frequency: f
  (cycles per unit distance)
  phase:
  (shift in position) ϕ

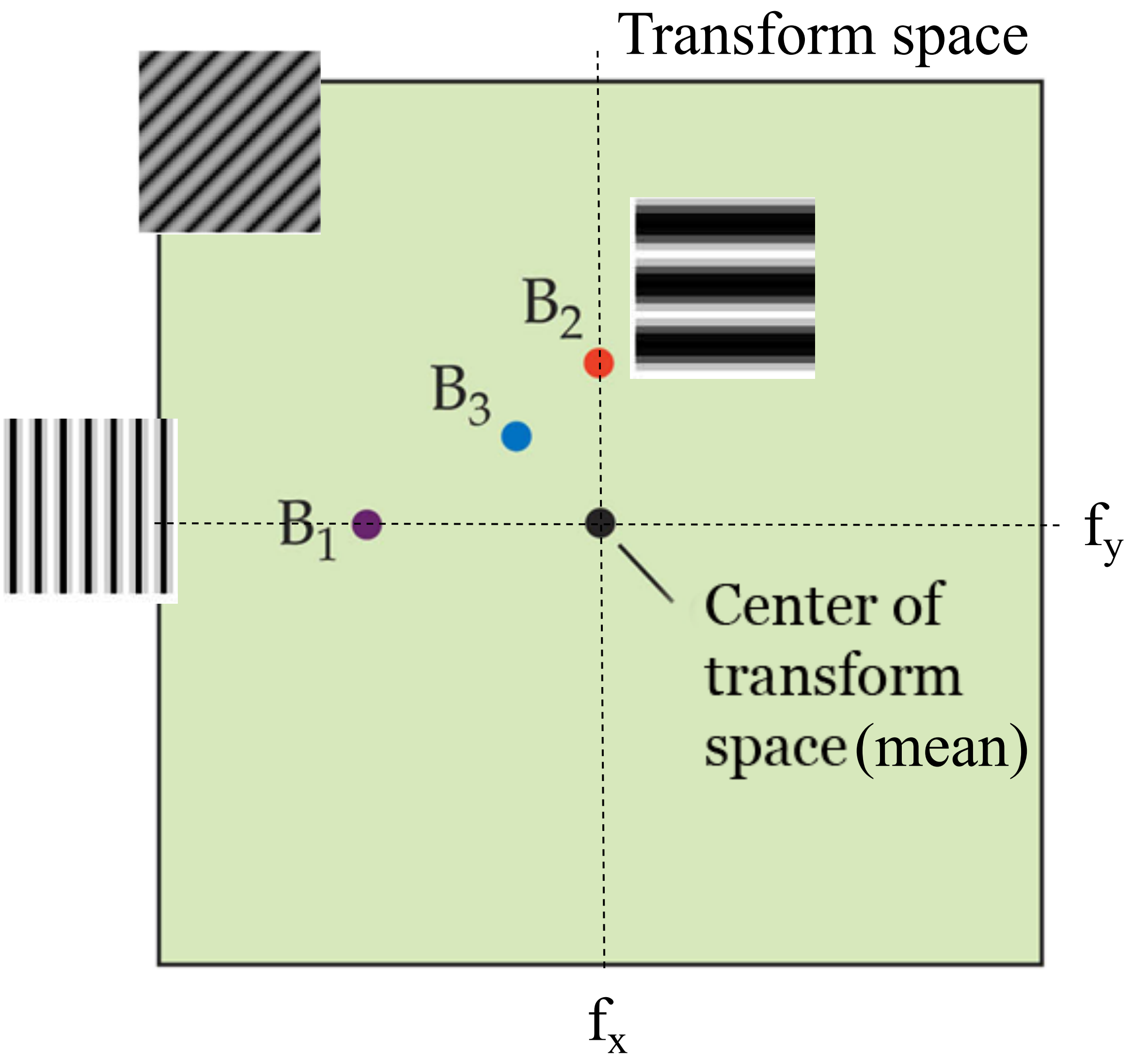
Fourier: Any image can be expressed as a weighted and shifted sum of harmonics

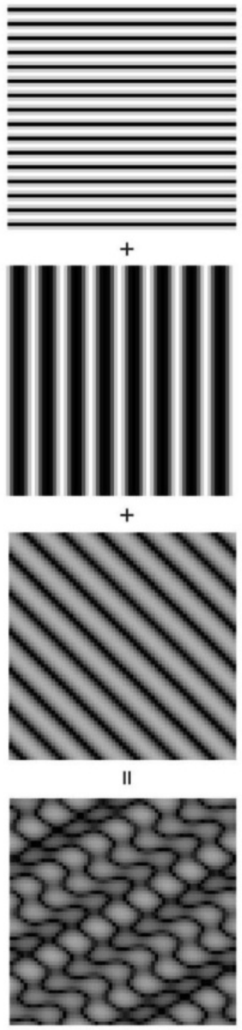
$$c(x) = M(1 + a\ \sin(2\pi fx - \phi))$$

M = mean level, a = contrast, f = spatial frequency

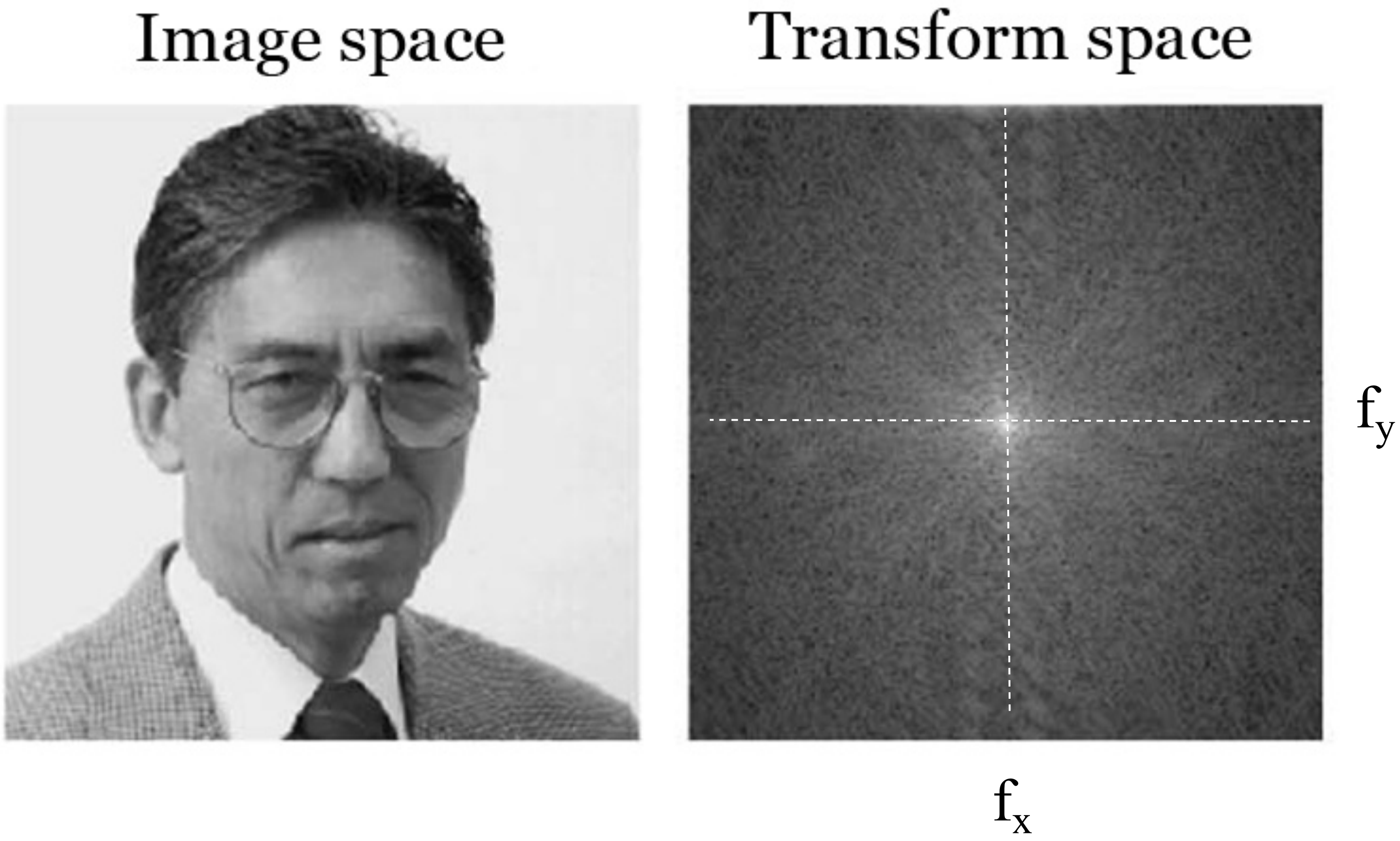


Images can be represented as Fourier series using 2D harmonics



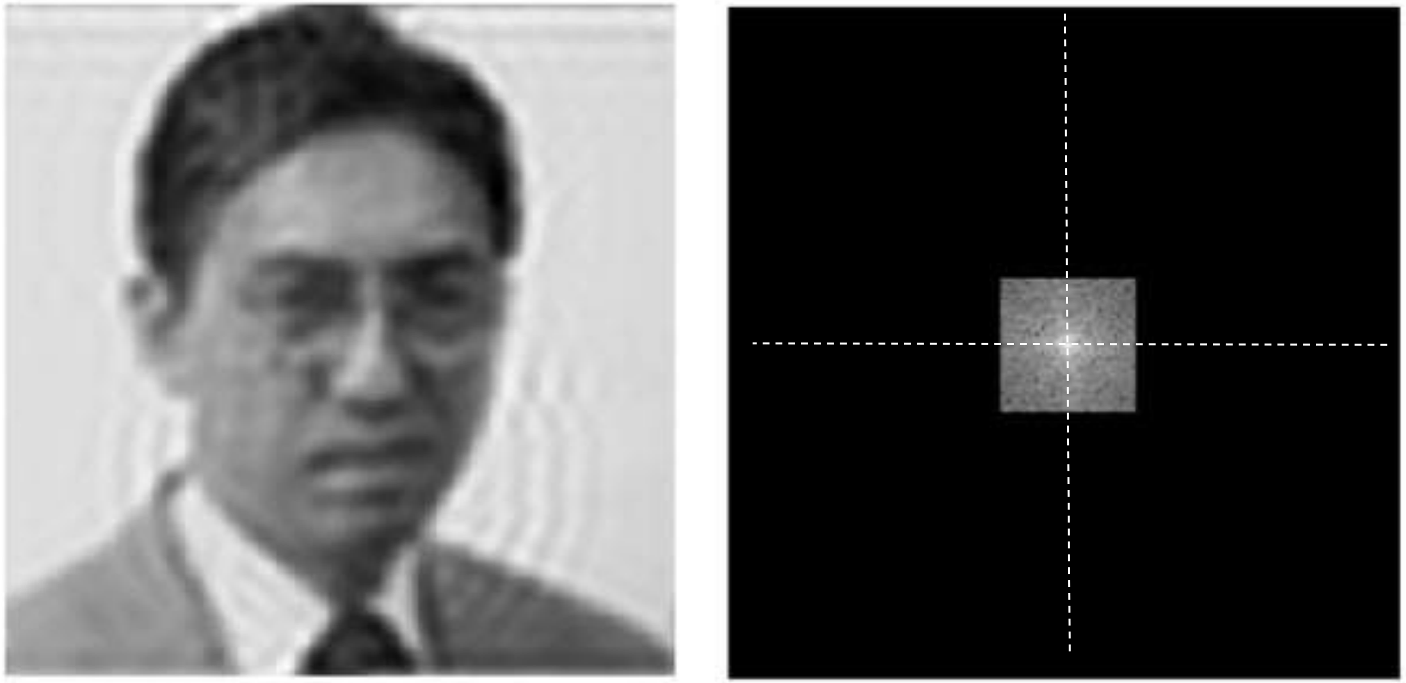


How different parts of transform space contribute to image space



Functional Magnetic Resonance Imaging, Huettel et al. © Sinauer, 2004, 2009

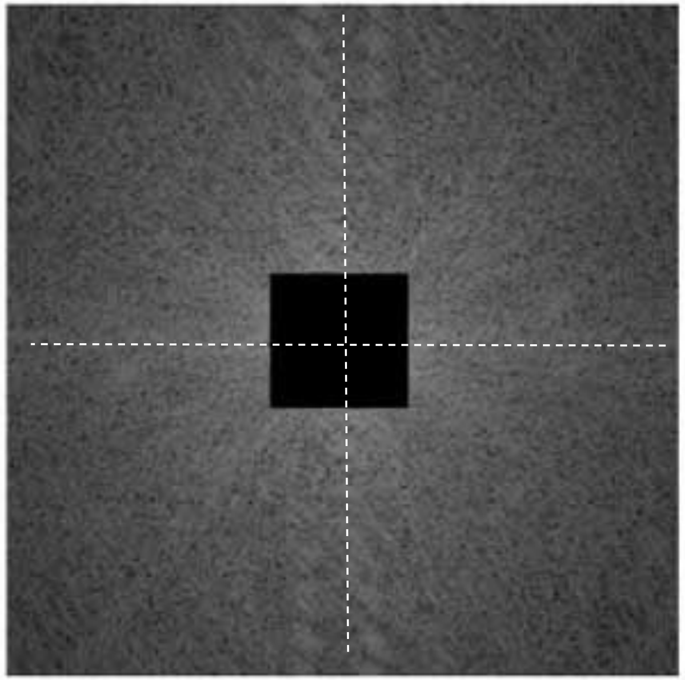
How different parts of transform space contribute to image space



The center of Transform space contains **low spatial frequency** information

Functional Magnetic Resonance Imaging, Huettel et al. © Sinauer, 2004, 2009

How different parts of transform space contribute to image space



The outer portion of Transform space contains
high spatial frequency information

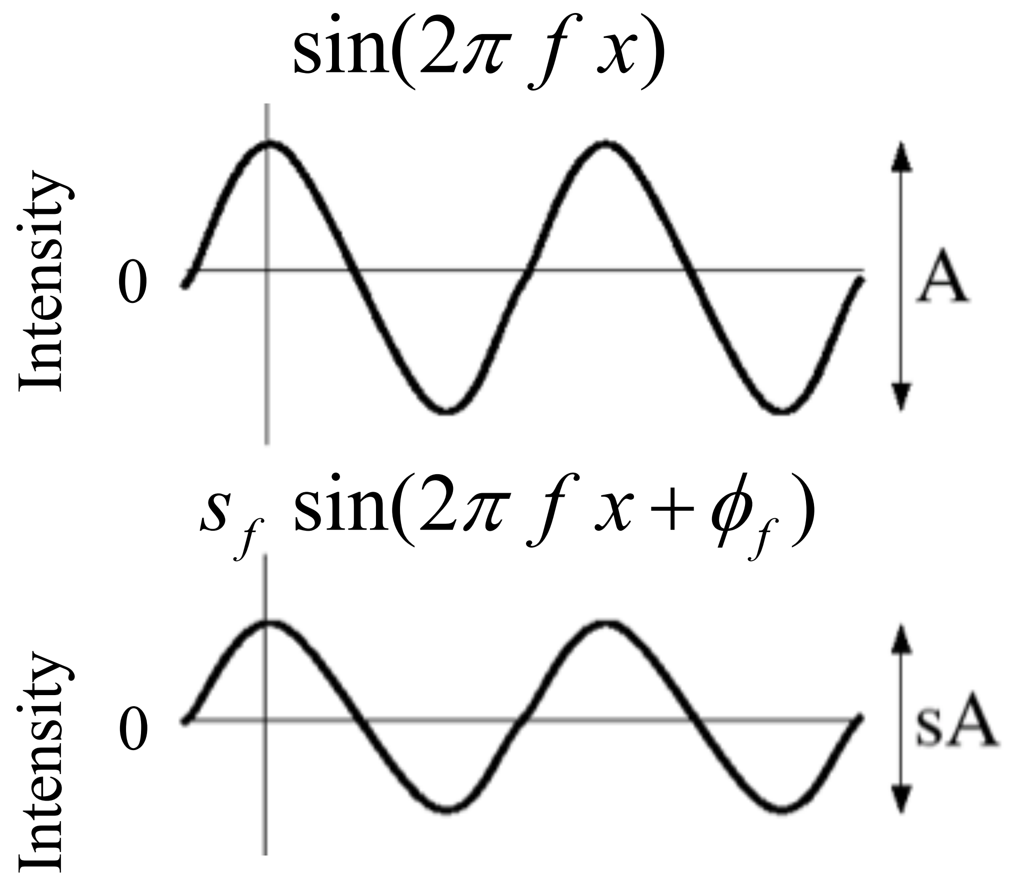
The modulation transfer function (MTF)

- The MTF of the human optics
- Pointspread

Harmonics have a special role in shift-invariant linear systems

- Harmonics are (almost) eigenfunctions. They pass through at the same frequency, but scaled and shifted in phase.
- The true eigenfunctions are complex exponentials

$$Ae^{i\phi} = a\cos(\phi) + ib\sin(\phi))$$

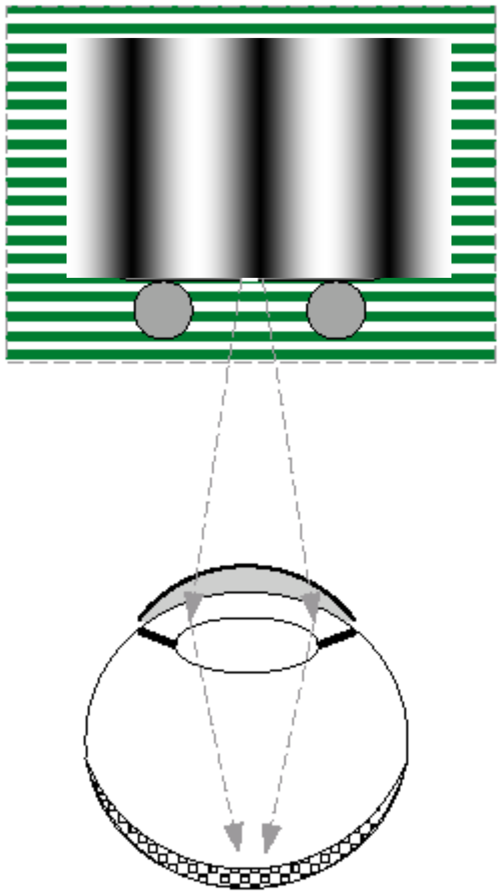


But – light is not negative
What to do?

The Modulation Transfer Function

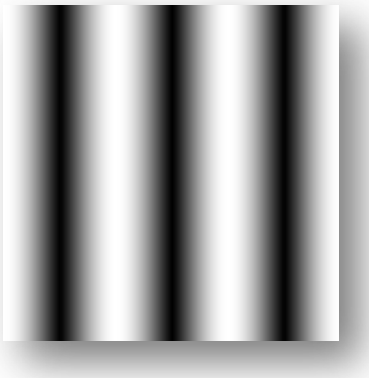
Contrast reduction depends on spatial frequency and wavelength

Luminance (cd/m²)

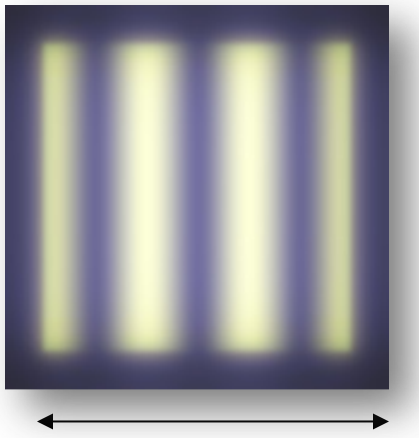


Illuminance (lux)

Harmonic input (radiance)

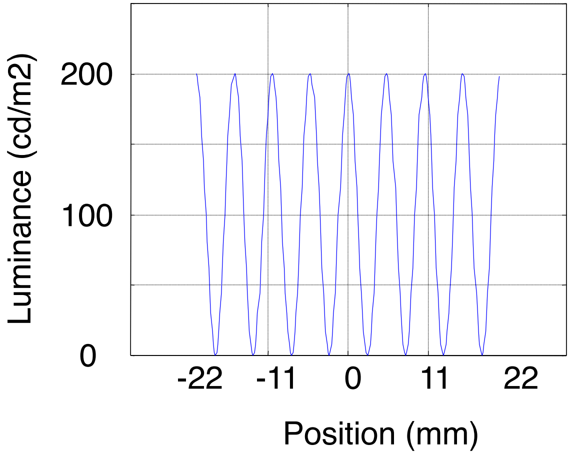


Retinal image (irradiance)

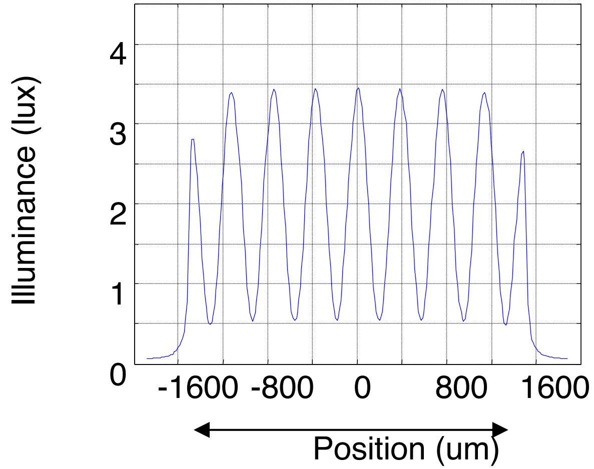


600 um

Harmonic input



100% contrast



75% contrast

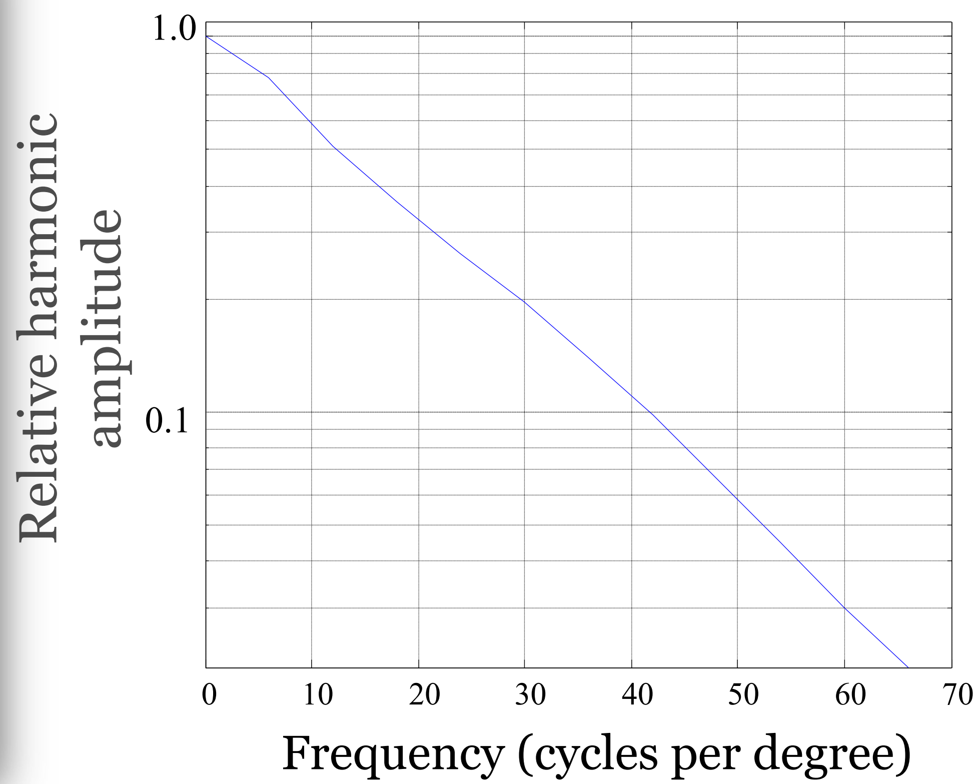
Modulation Transfer Function (MTF)

The reduction in the frequency modulation is a very useful way to characterize the quality of an optical system

The Westheimer function and the Fourier Transform (FFT)

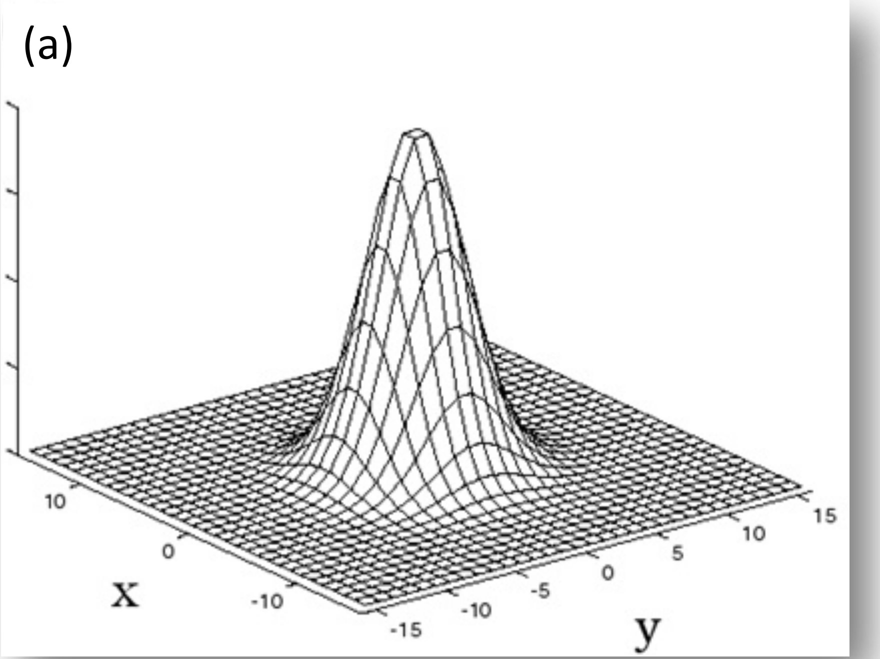
The modulation transfer function (MTF) describes how the amplitude of each harmonic is scaled by the optics. If we know the linespread function, we can calculate the MTF using the Fast Fourier Transform. In this code snippet, we use the function westheimerLSF to estimate the human MTF. That function takes its input as a spatial variable in sec of arc

```
xSec = -300:300;     % 600 sec, total = 10 arc min
westheimerMTF = abs(fft(westheimerLSF(xSec)));

% One cycle spans 10 min of arc, so freq=1 is 6 c/deg
freq = (0:11)*6;
vcNewGraphWin;
semilogy(freq,westheimerMTF(1:12)); grid on;
xlabel('Freq (cpd)'); ylabel('Relative contrast');
set(gca,'ylim',[0 1.1])
title('Westheimer MTF');
```

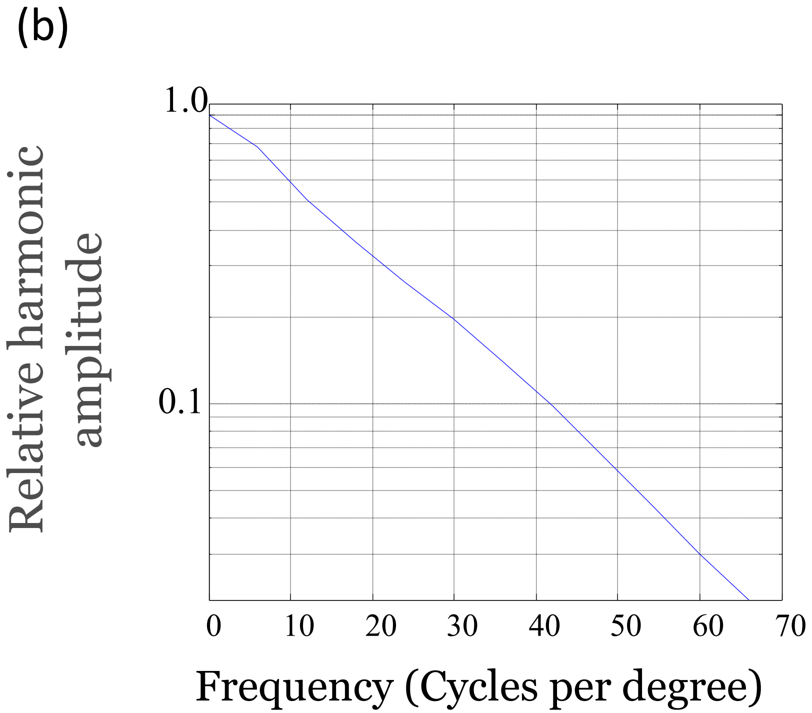


Two summaries of optical quality

Point spread or line spread

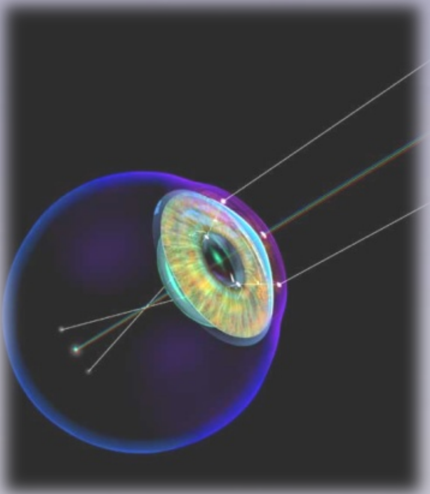


Modulation transfer function (MTF)



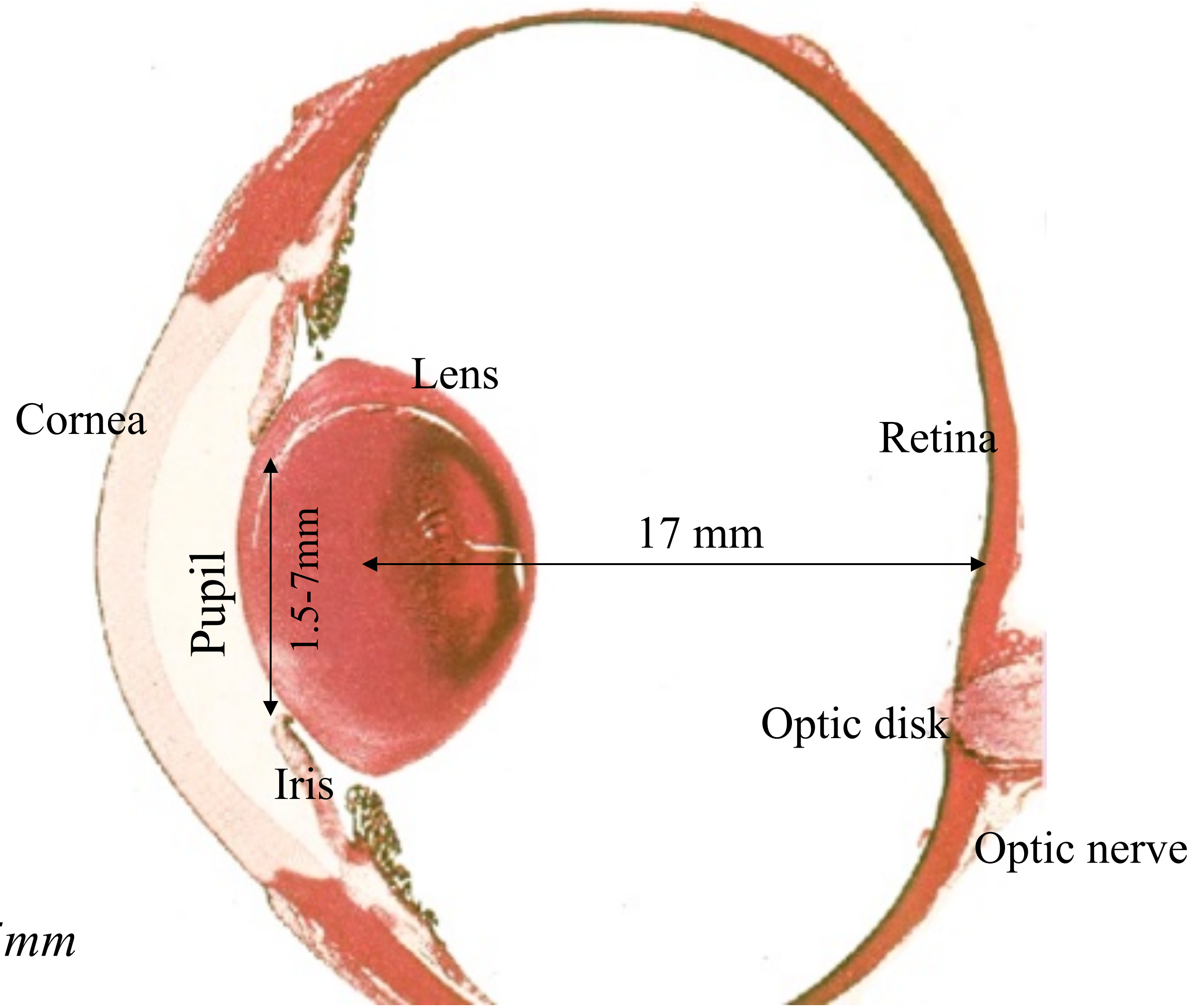
Human image formation

- Human eye and retina: anatomy
- Pupil size and lens accommodation
- Human Linespread and MTF
- Astigmatism



s_humanLSF
s_humanOptics

Human Eye in Cross-Section



F-number ~ 2.4-11
Retinal thickness ~ 0.5mm

Lens accommodation

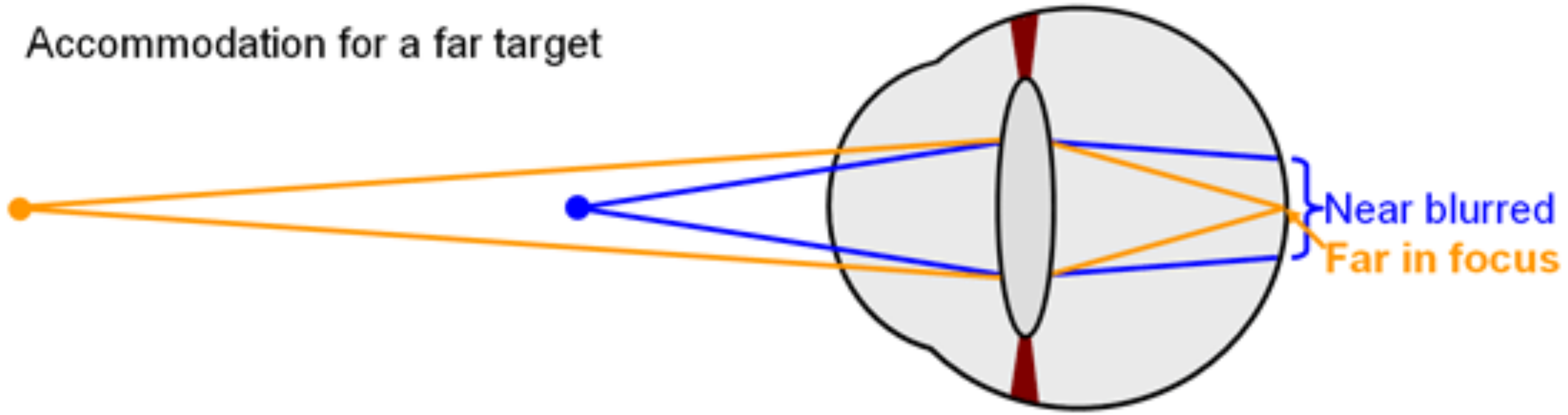


Image from: http://www.sapdesignguild.org/editions/edition9/print_vision_physiology.asp

Lens accommodation

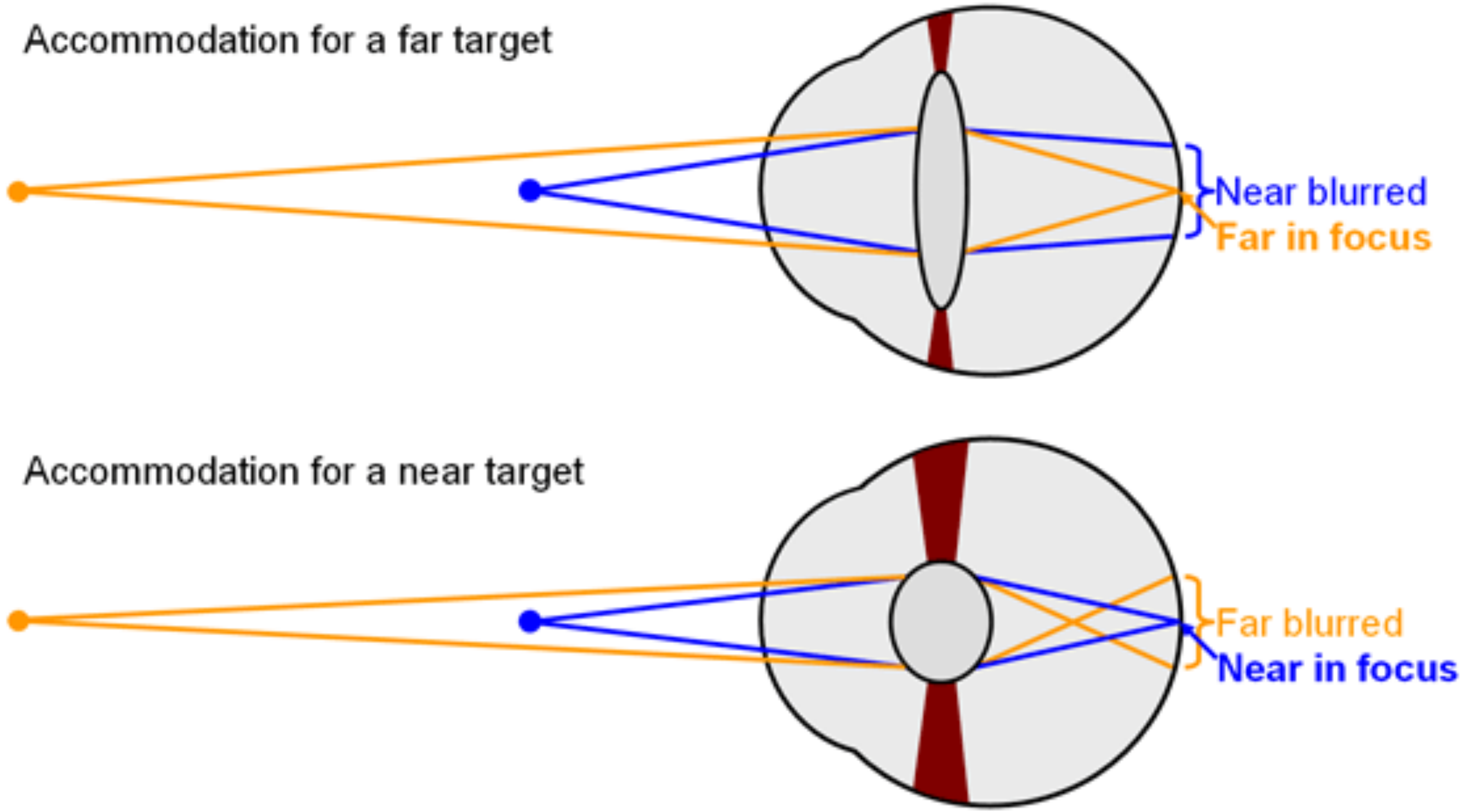
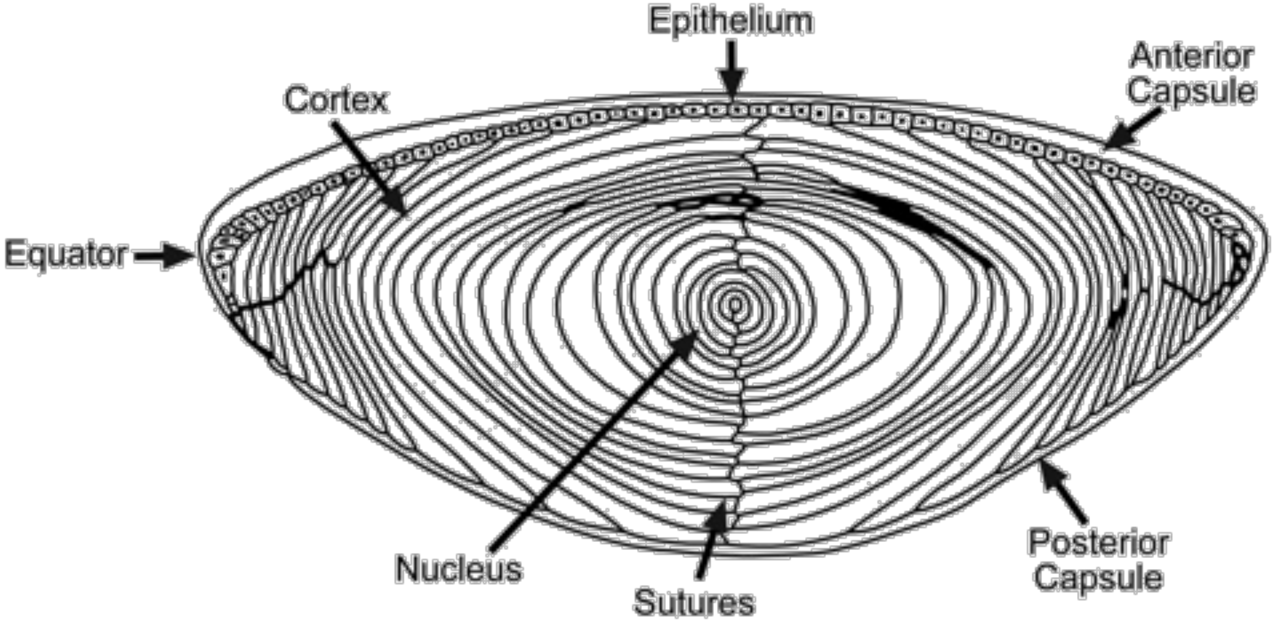


Image from: http://www.sapdesignguild.org/editions/edition9/print_vision_physiology.asp

Engineering requirements for the lens

- Flexible – that's how accommodation works
- Transparent (no blood vessels)
- Last about 100 years – optimistic
- **Solution**: Lens is like an onion – each sheet contains Fibers that interlock with little ball-socket connectors



http://www.optics.rochester.edu/workgroups/cml/opt307/spr06/joe/index.htm

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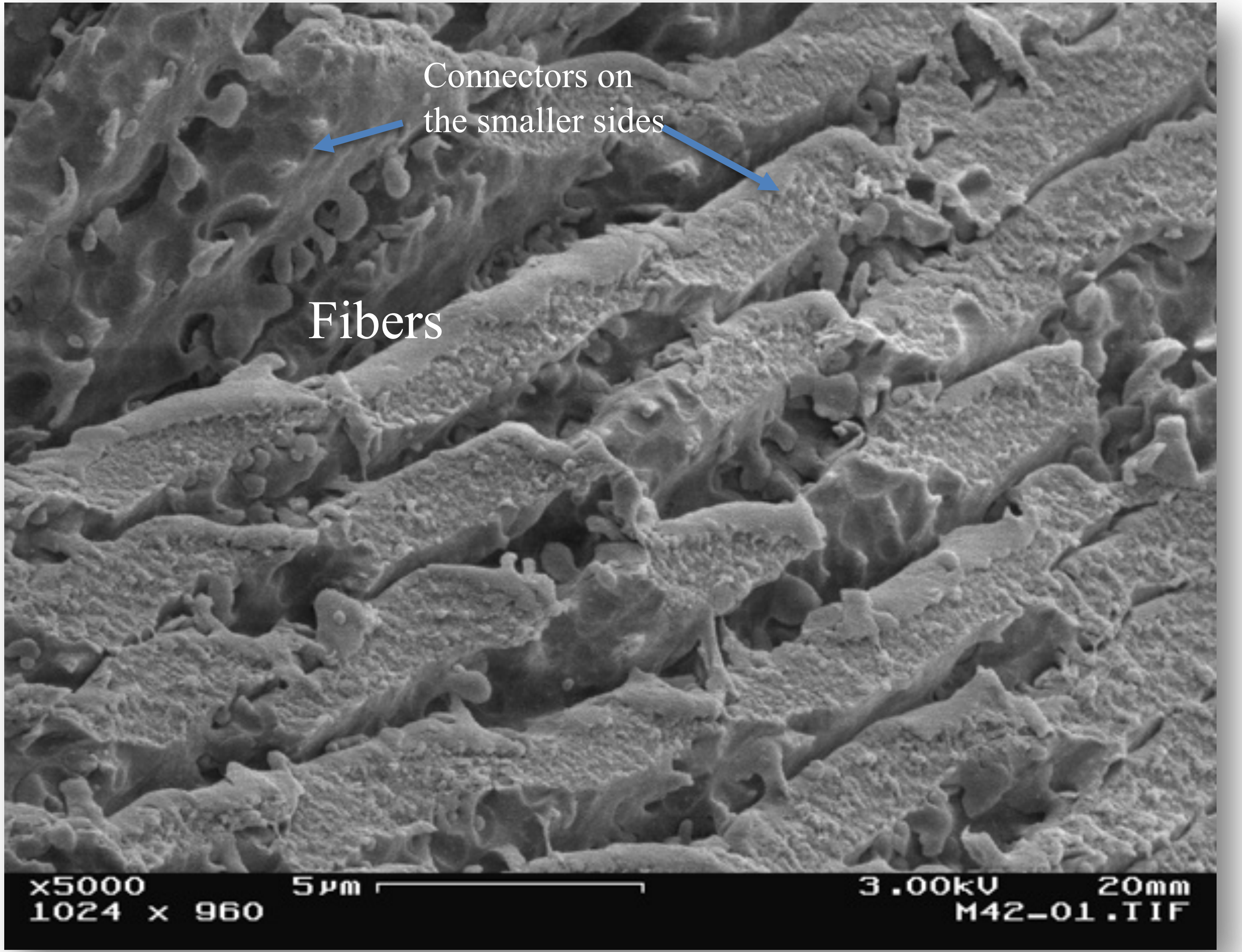


Figure 1.1 - Lens Fiber cut away view. Note interconnecting ball and socket structures on short edges. Also note relative absence of ball and sockets on planar side of superficial fibers allowing planar movement.

http://www.optics.rochester.edu/workgroups/cml/opt307/spr06/joe/index.htm

Engineering requirements for the lens

- Transparent – no blood vessels, capillaries, lipid membranes (which scatter light)
- Flexible – that's how accommodation works
- Last about 100 years – optimistic
- Pores in the fibers allow the nutrient laden **vitreous humor** to bath and support the fibers in the absence of a vascular network.

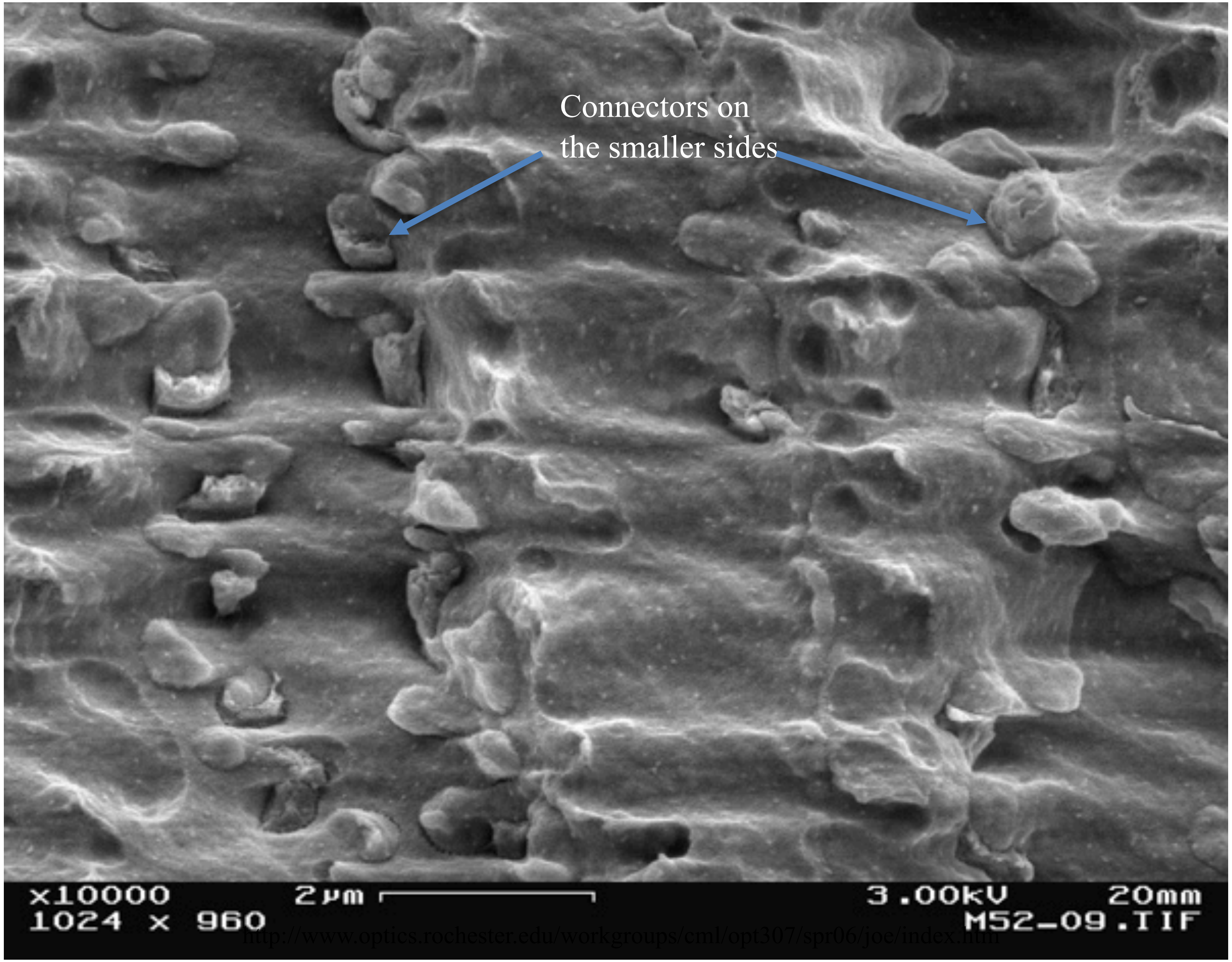
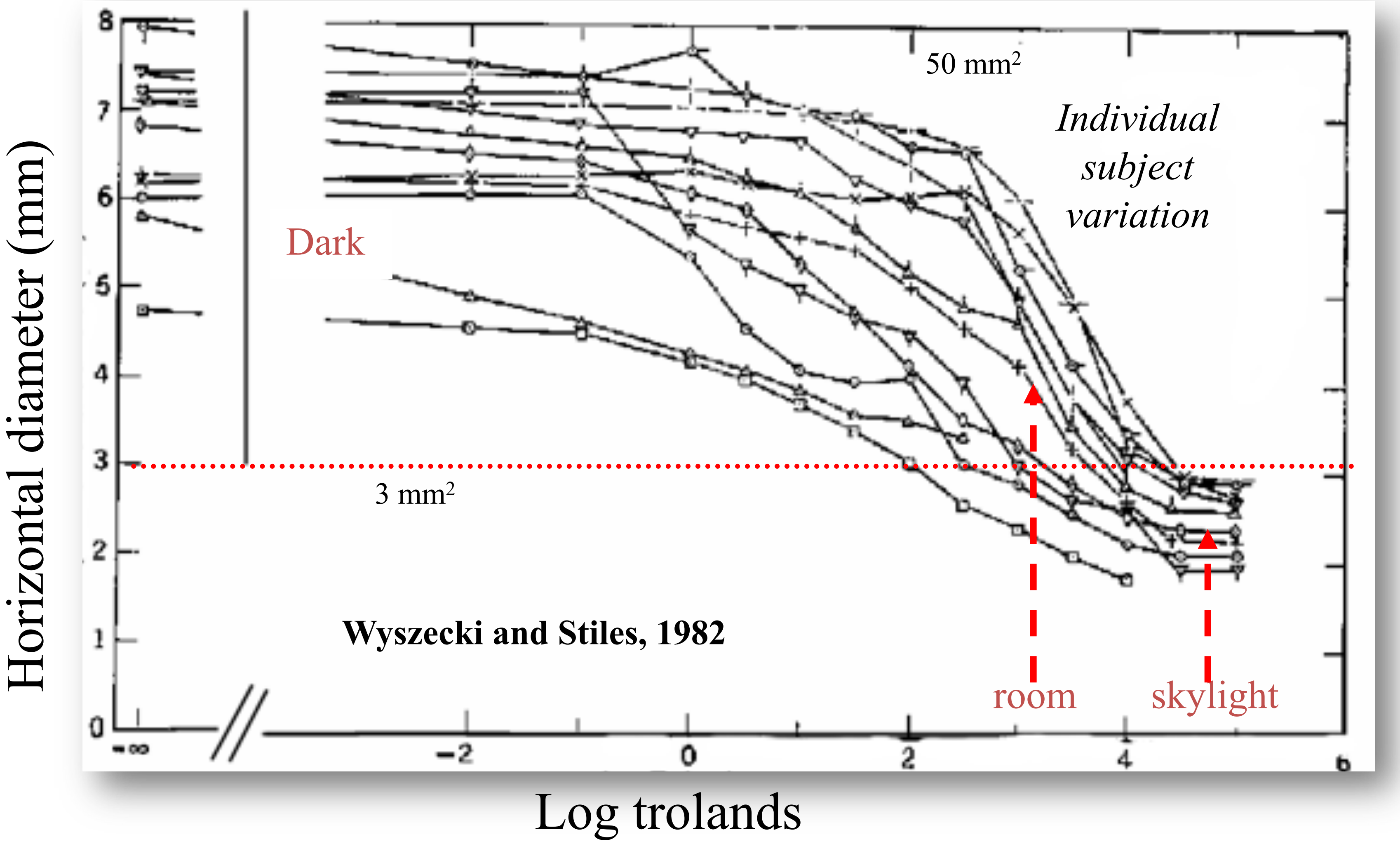
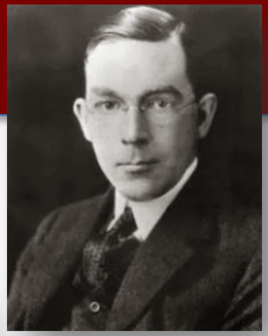


Figure 1.3 - Lens Fiber Ball and Socket close up. Note the ball and socket joint interlocking at superficial cortical fiber edges. Planar surface of fibers interlock with next layer of offset fibers (removed).

Pupil diameters vary with light level

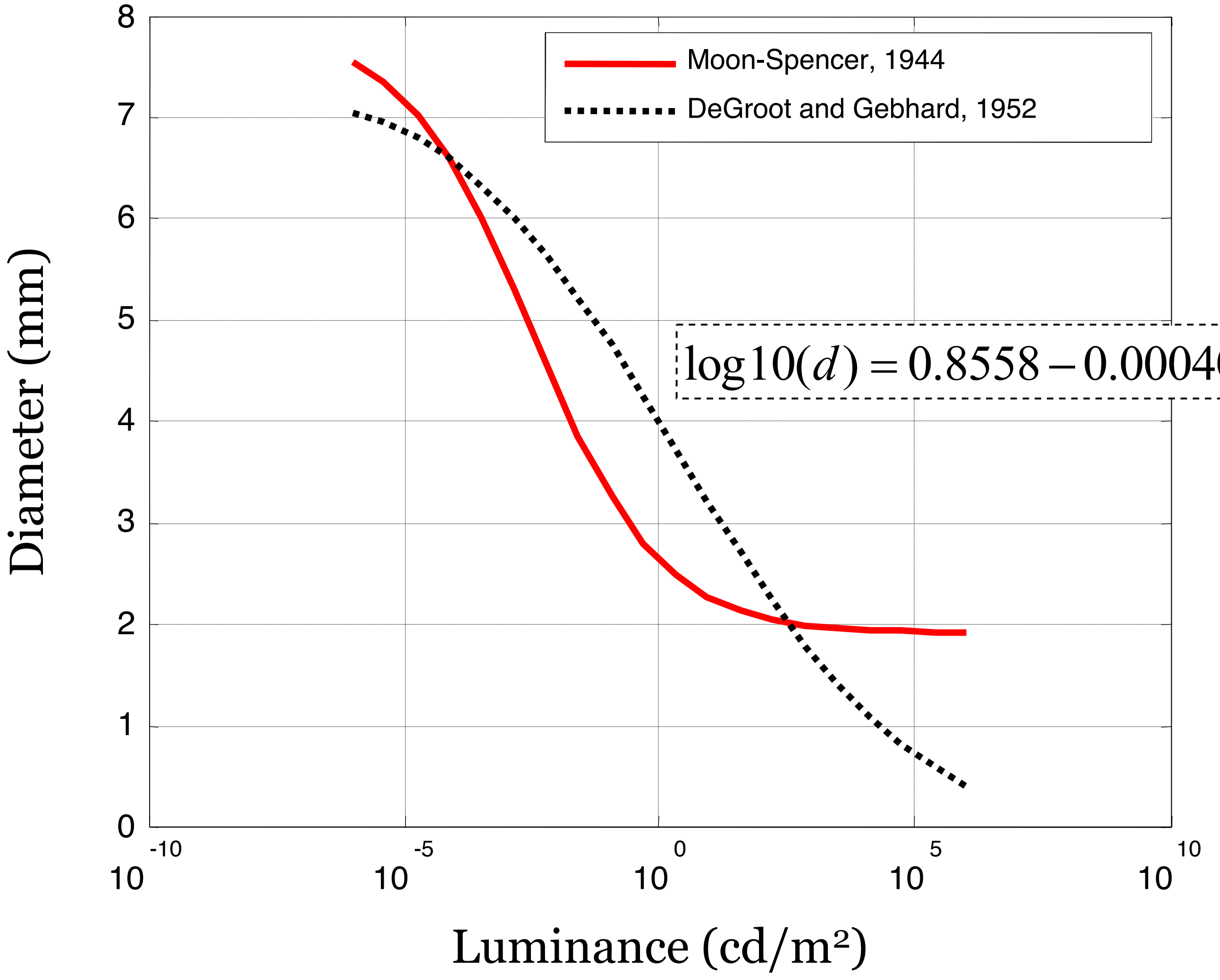
Log Trolands are related to candela/m^2 but accounting for the pupil size





Leonard Troland

Pupil Size Changes, Influencing Retinal Illuminance and Acuity



$$d = 4.9 - 3\tanh(0.4(\log(L)+1))$$

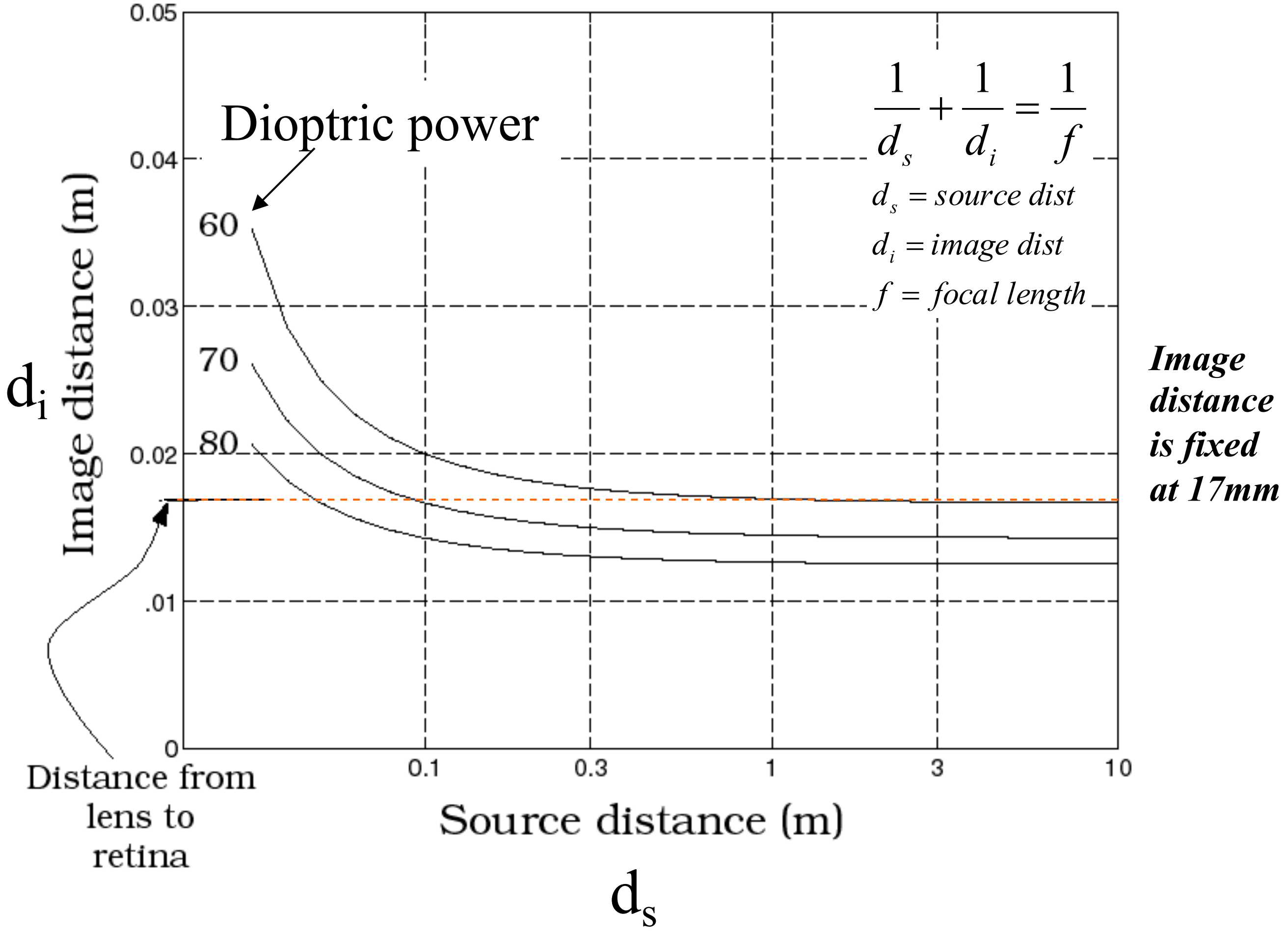
$$\log 10(d) = 0.8558 - 0.000401(\log(L)+8.6)^3$$

Human accommodation

When the source is distant, muscles in the eye transform the lens shape to a lower power

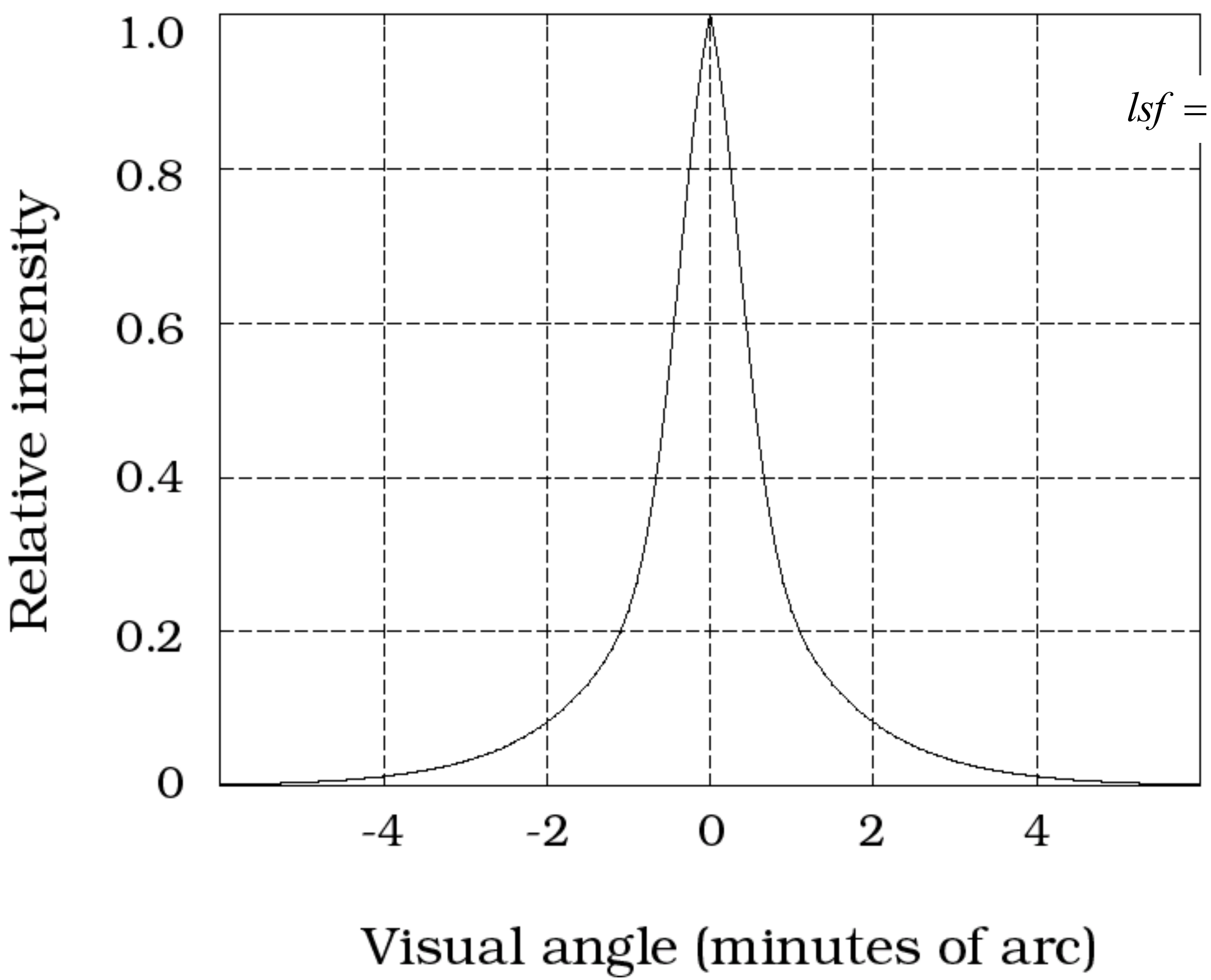
When the source is close, the muscles make the lens higher power

This process is called accommodation



Inferred human line spread (Westheimer)

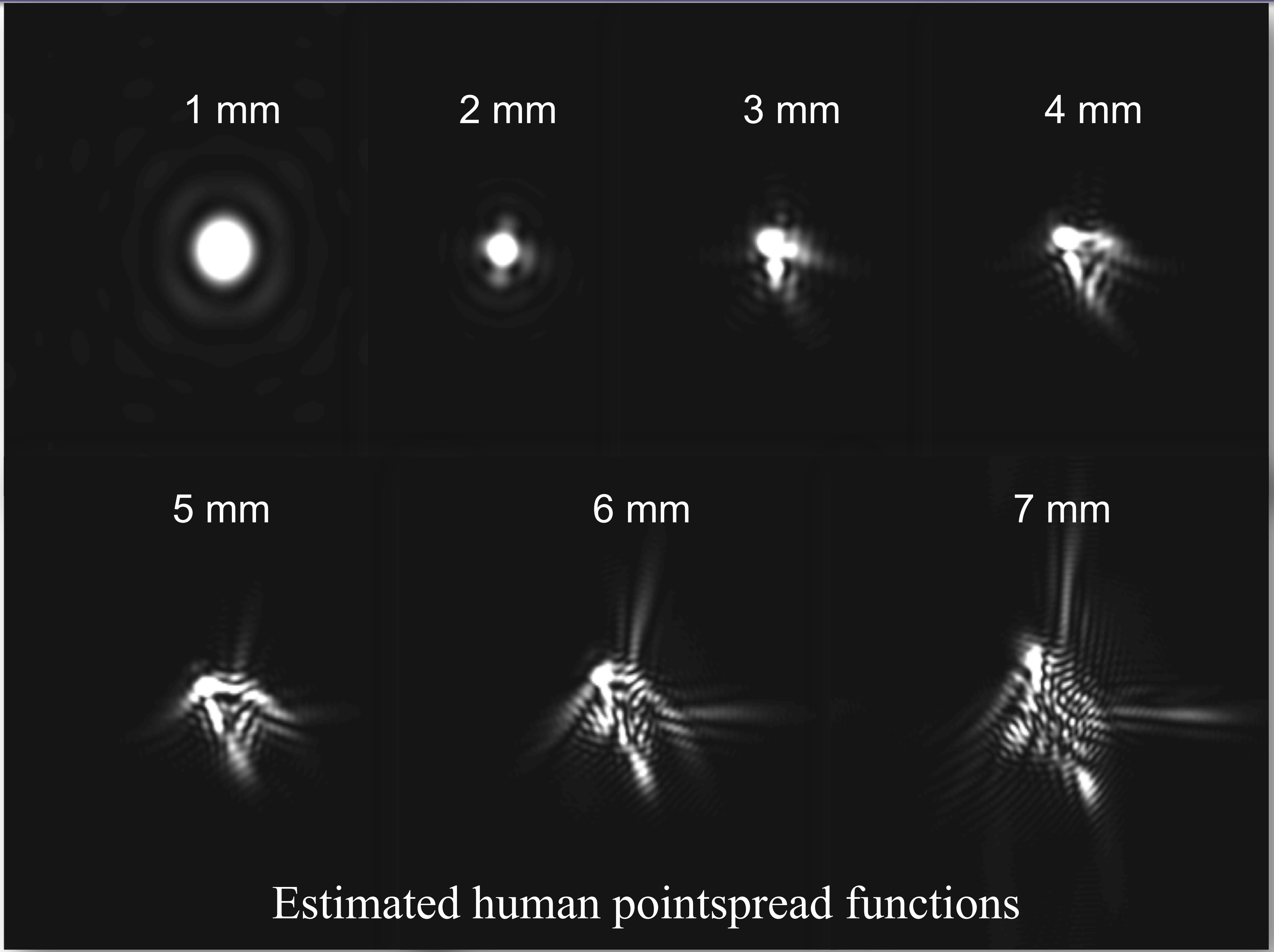
ISET: westheimerLSF



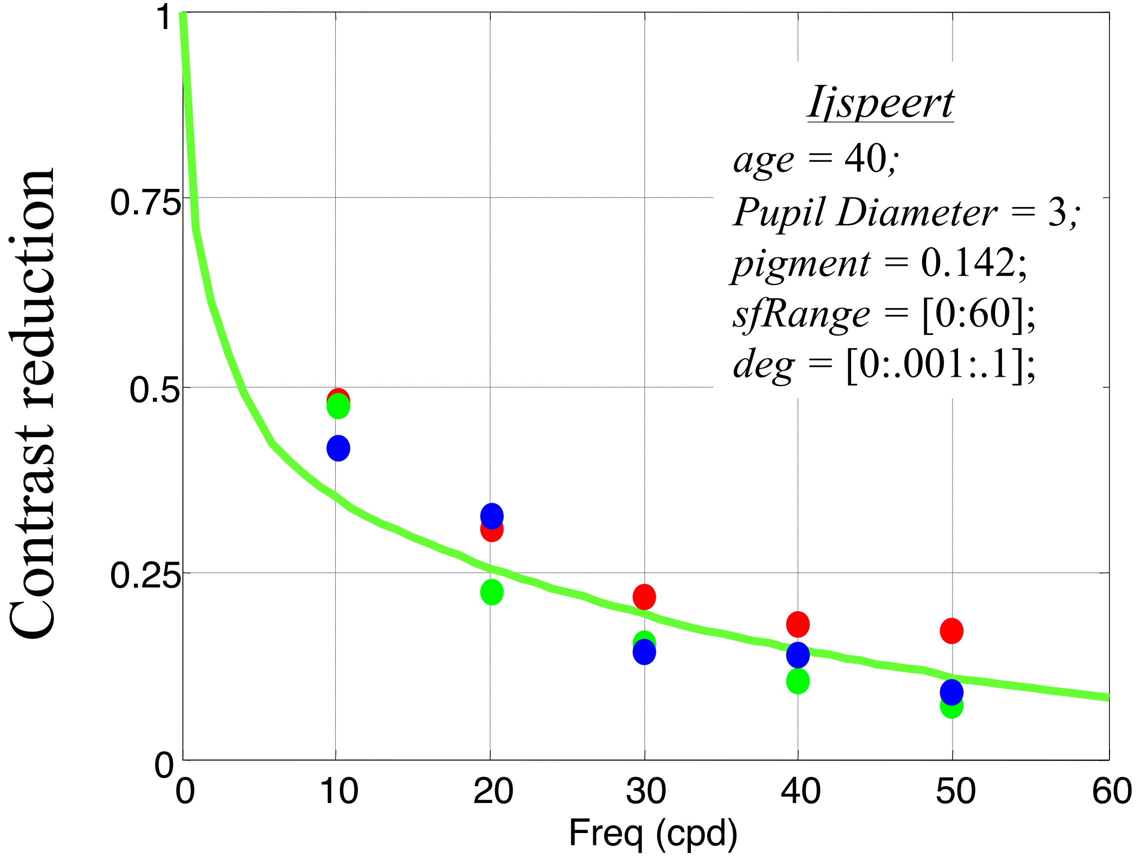
$$lsf = 0.47\exp(-3.3x^2) + 0.53\exp(-0.93|x|)$$

ISET: westheimerLSF

Visual acuity is worse as pupils dilate (Faber, 2001, CFAO)



Theoretical modulation transfer function compared with data

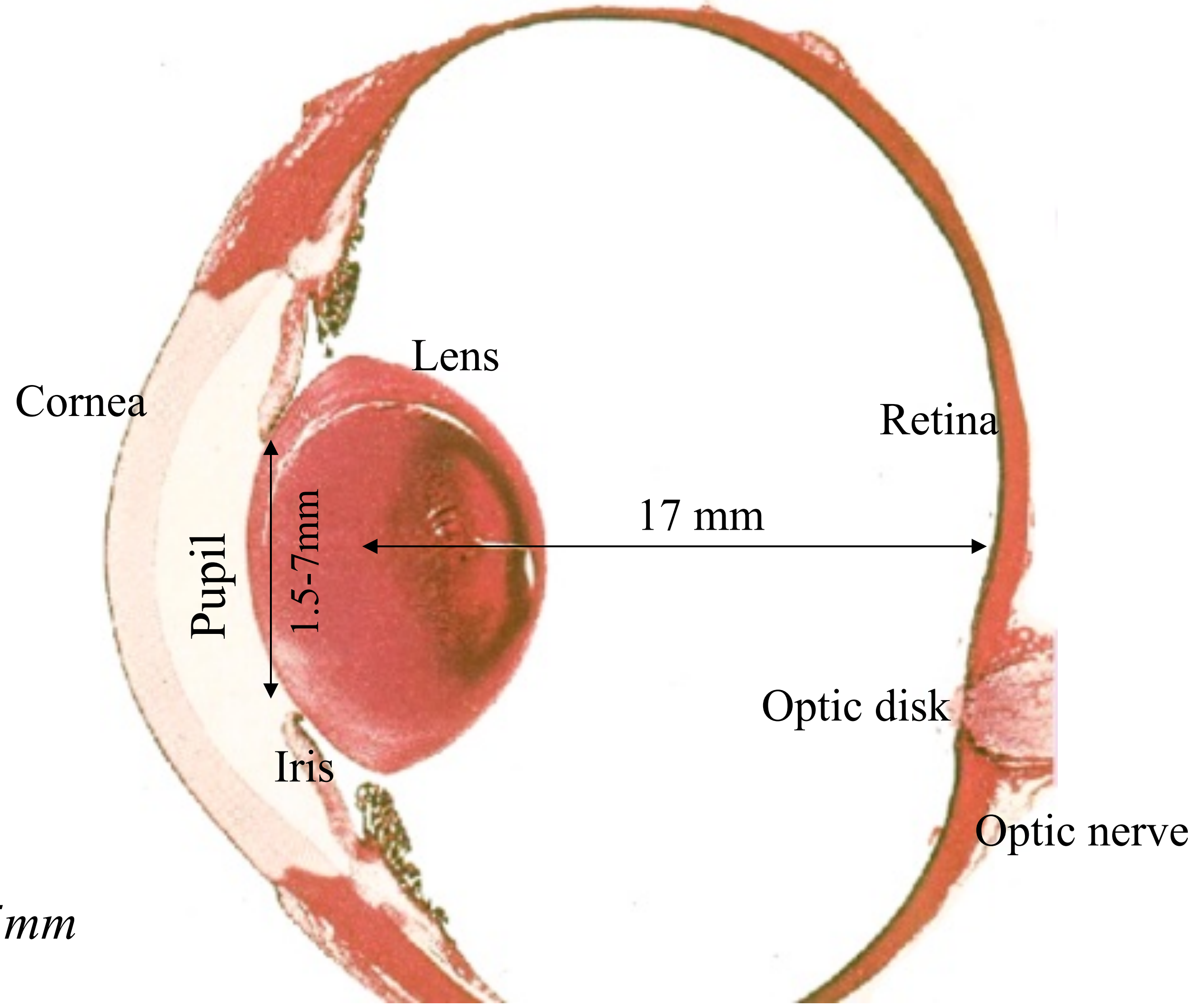


Longitudinal chromatic aberration

- Focus and wavelength
- Example images
- Dioptric power by wavelength
- MTF and linespread by wavelength

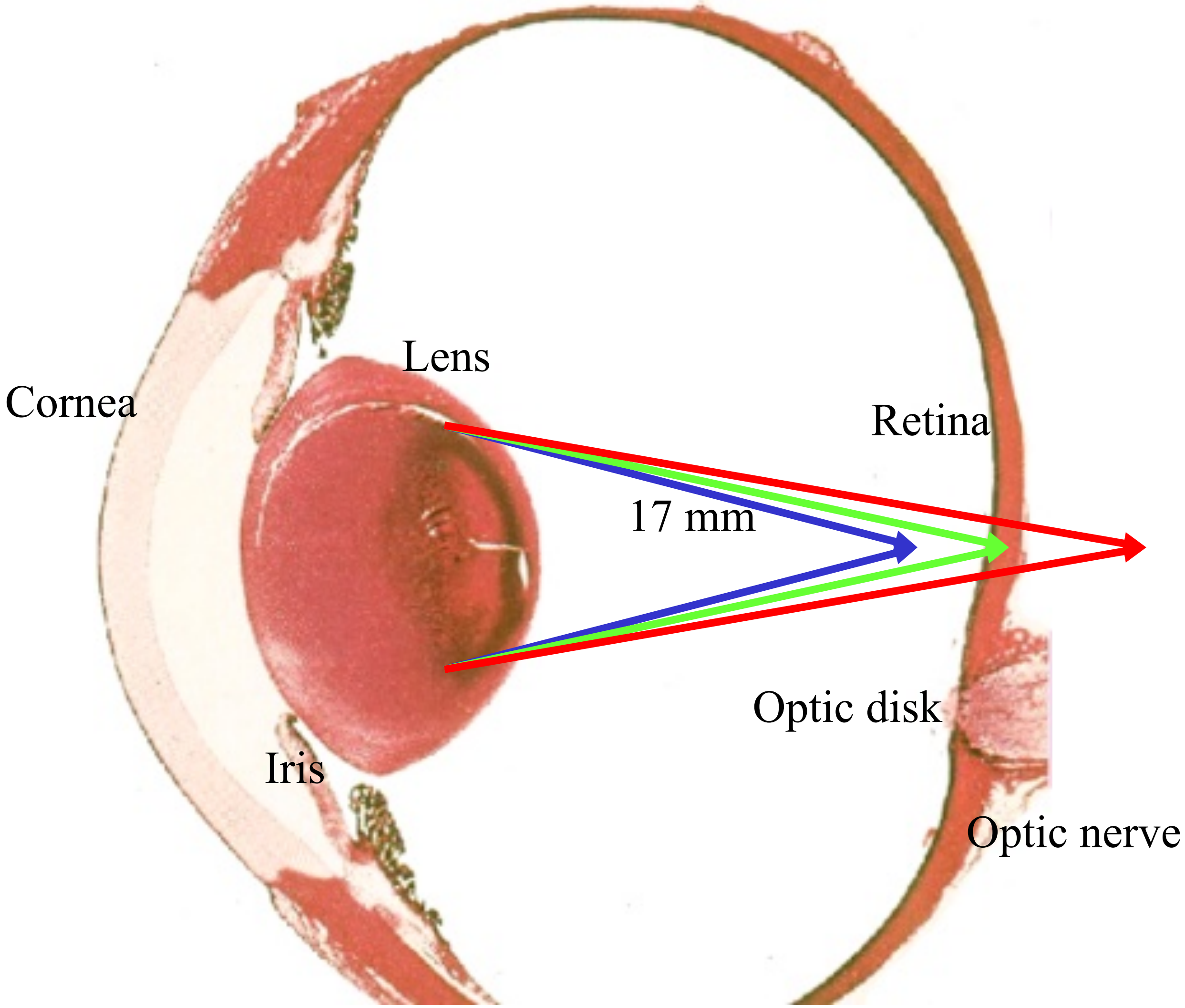
s_HumanLSF
s_HumanOptics

Human eye in cross-section

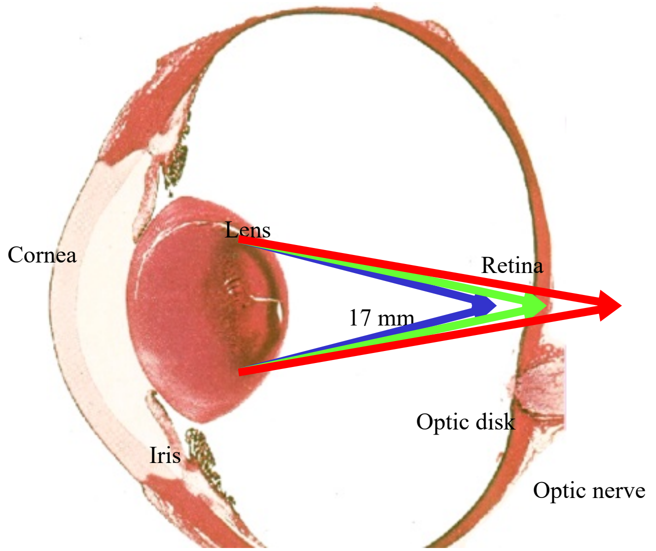


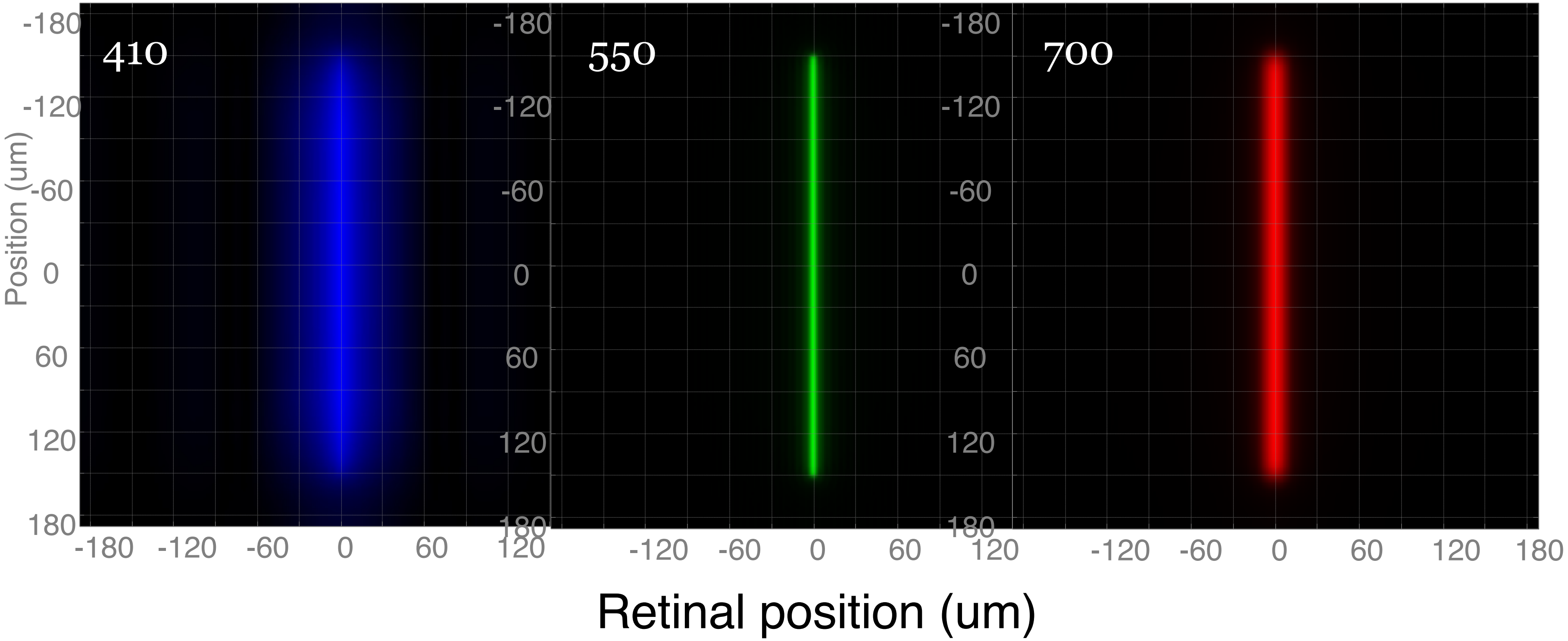
F-number ~ 2.4-11
Retinal thickness ~ 0.5mm

Chromatic aberration is a difference in optical focus across wavelength



At the retina, the line spread differs by wavelength

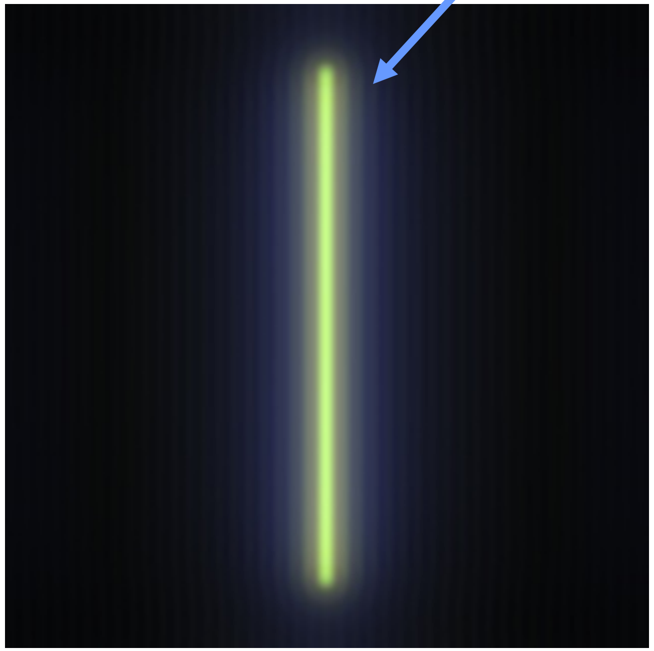


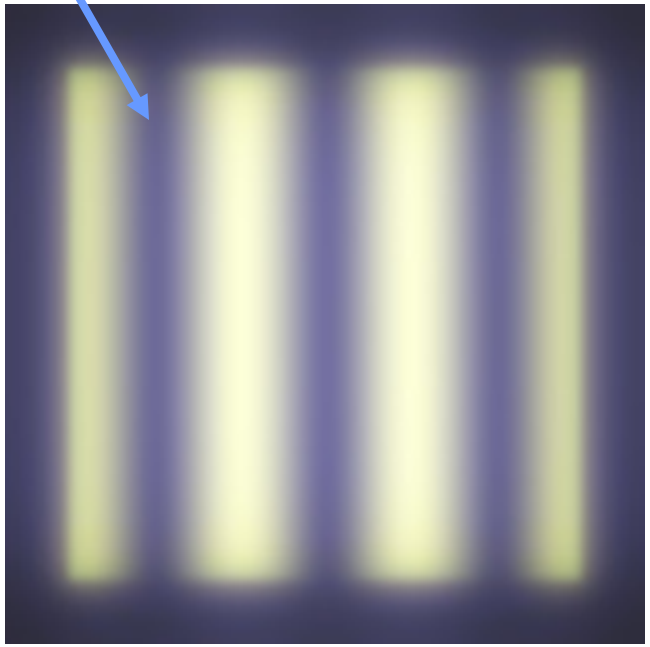


ISET: s_HumanLSF

Example: Wavelength-dependent spread

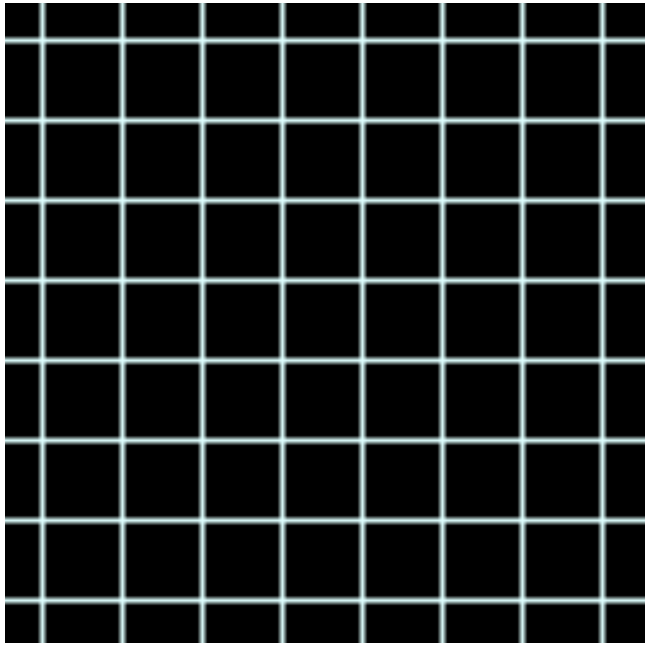
Short-wavelength light spreads more



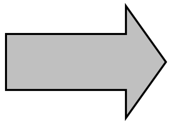


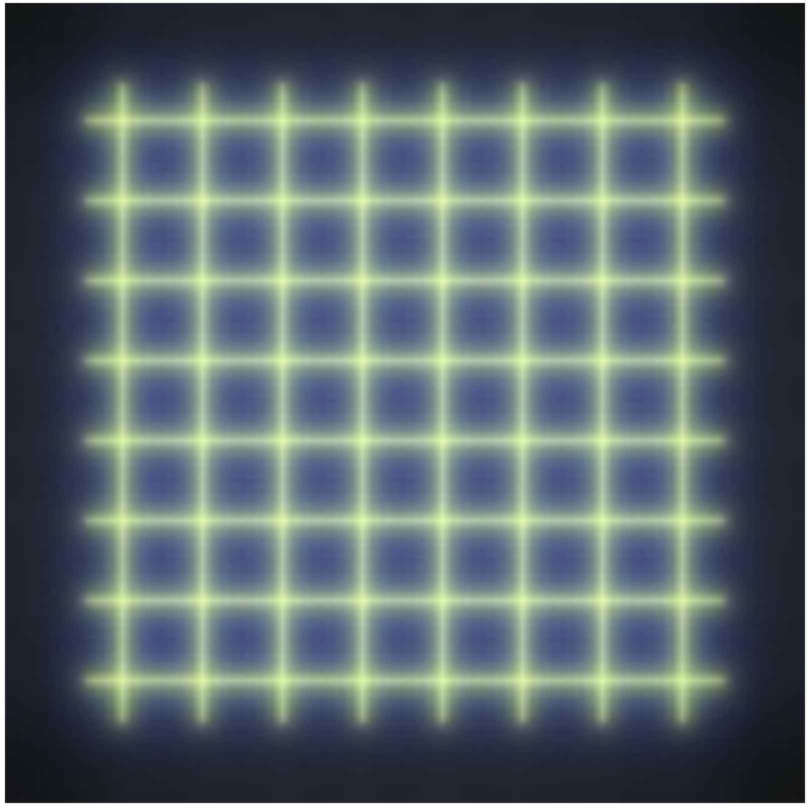
Example: Wavelength-dependent spread

Broadband radiance produces chromatic irradiance

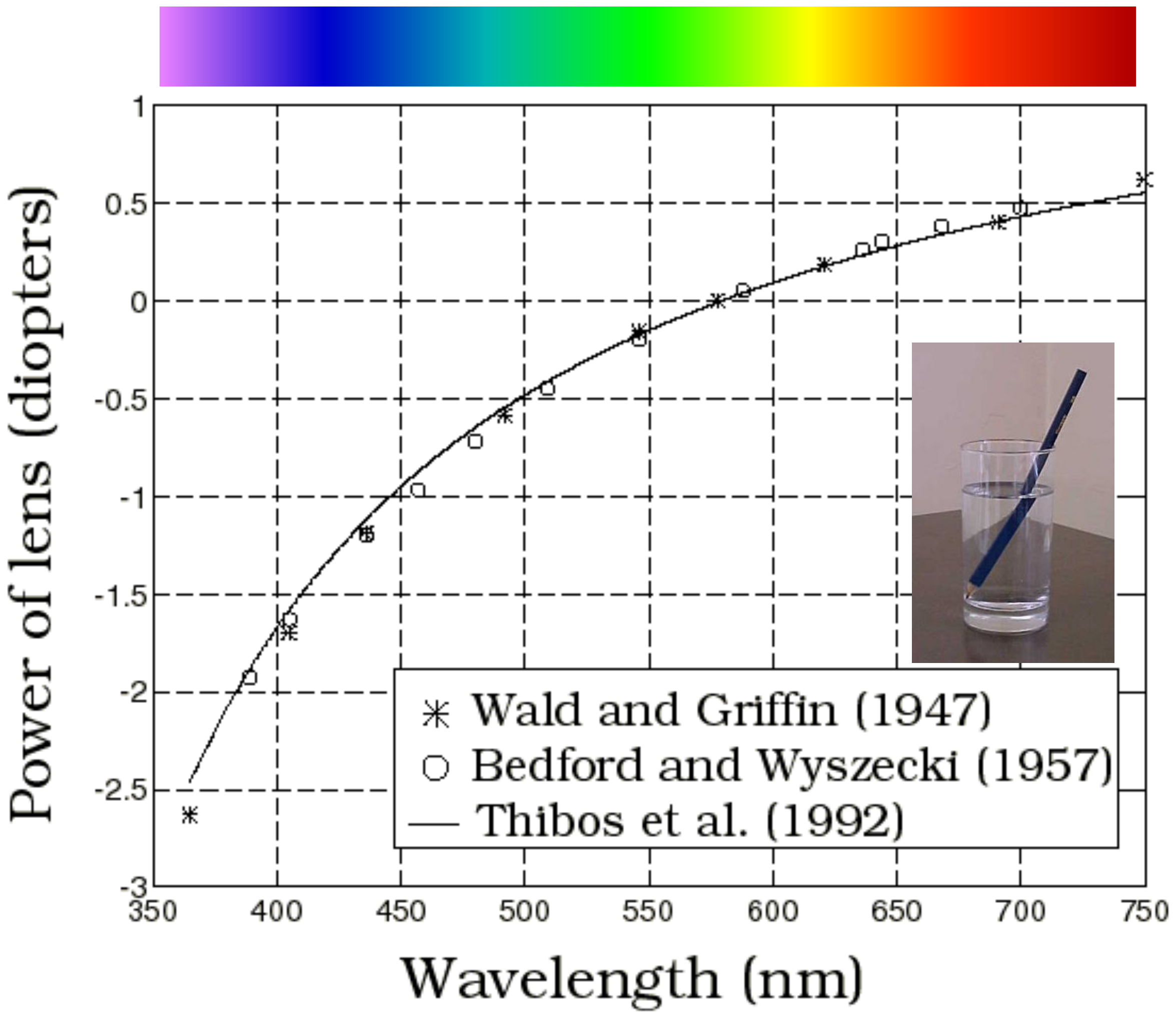


Human optics





Chromatic aberration is measured by the variation in optical power across wavelength; very similar across people



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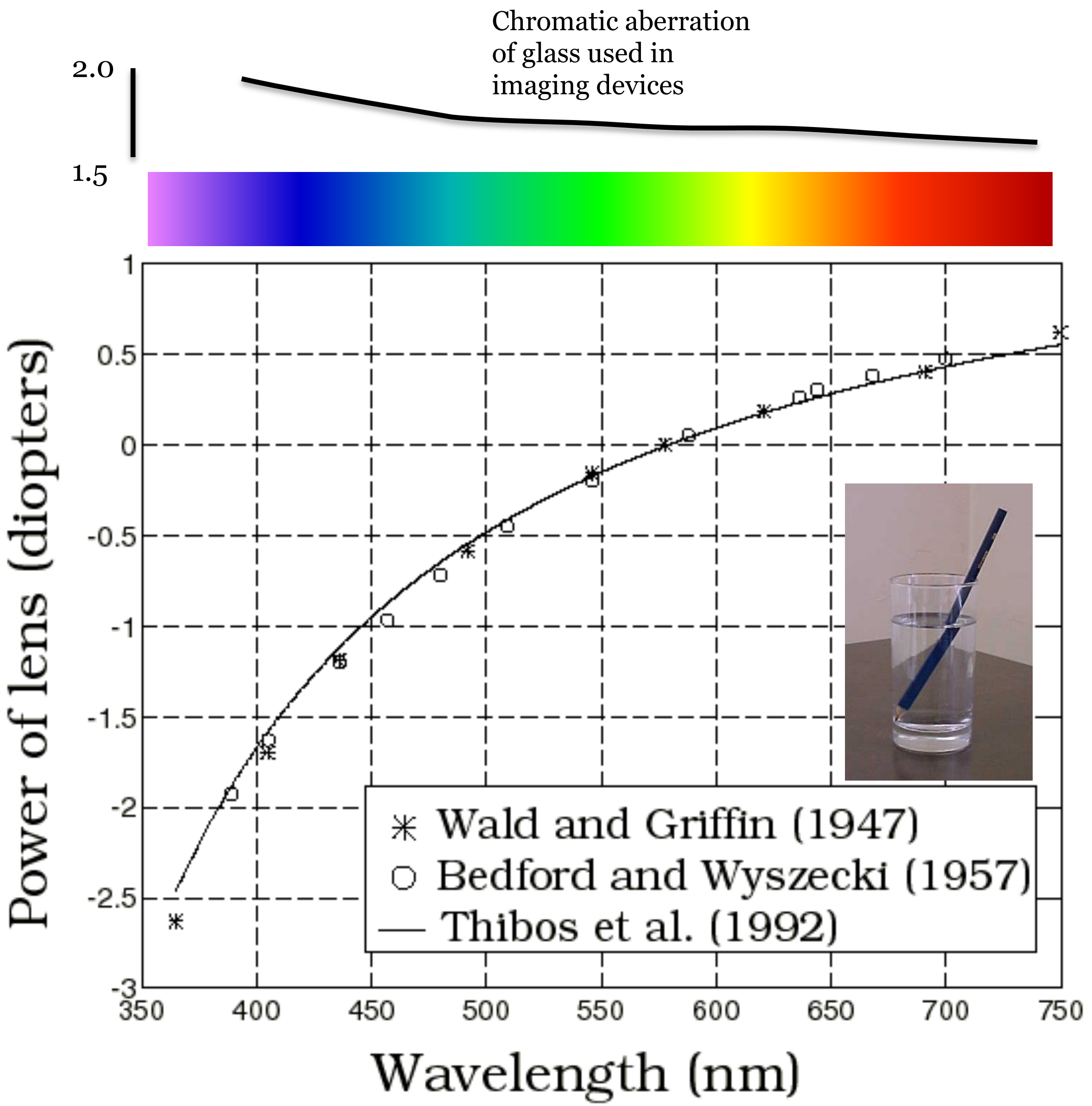
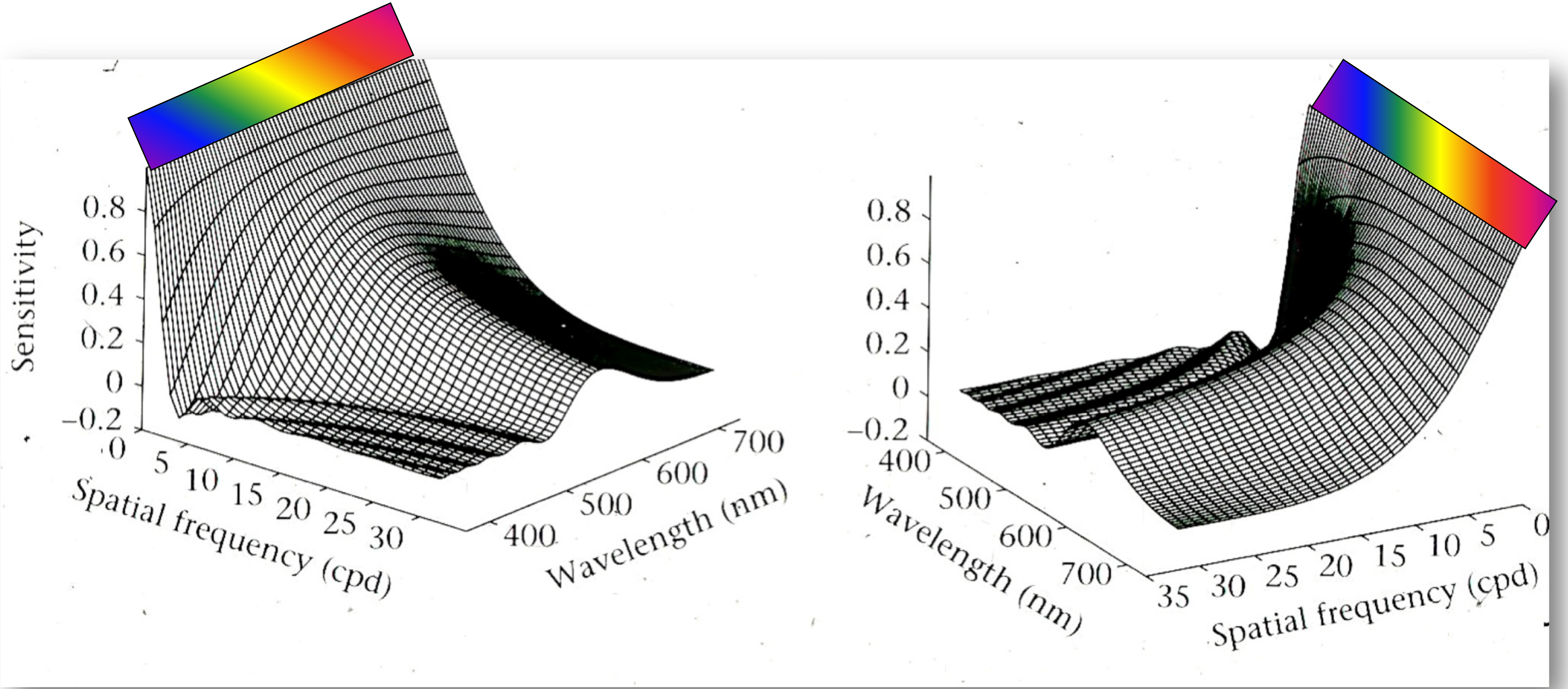


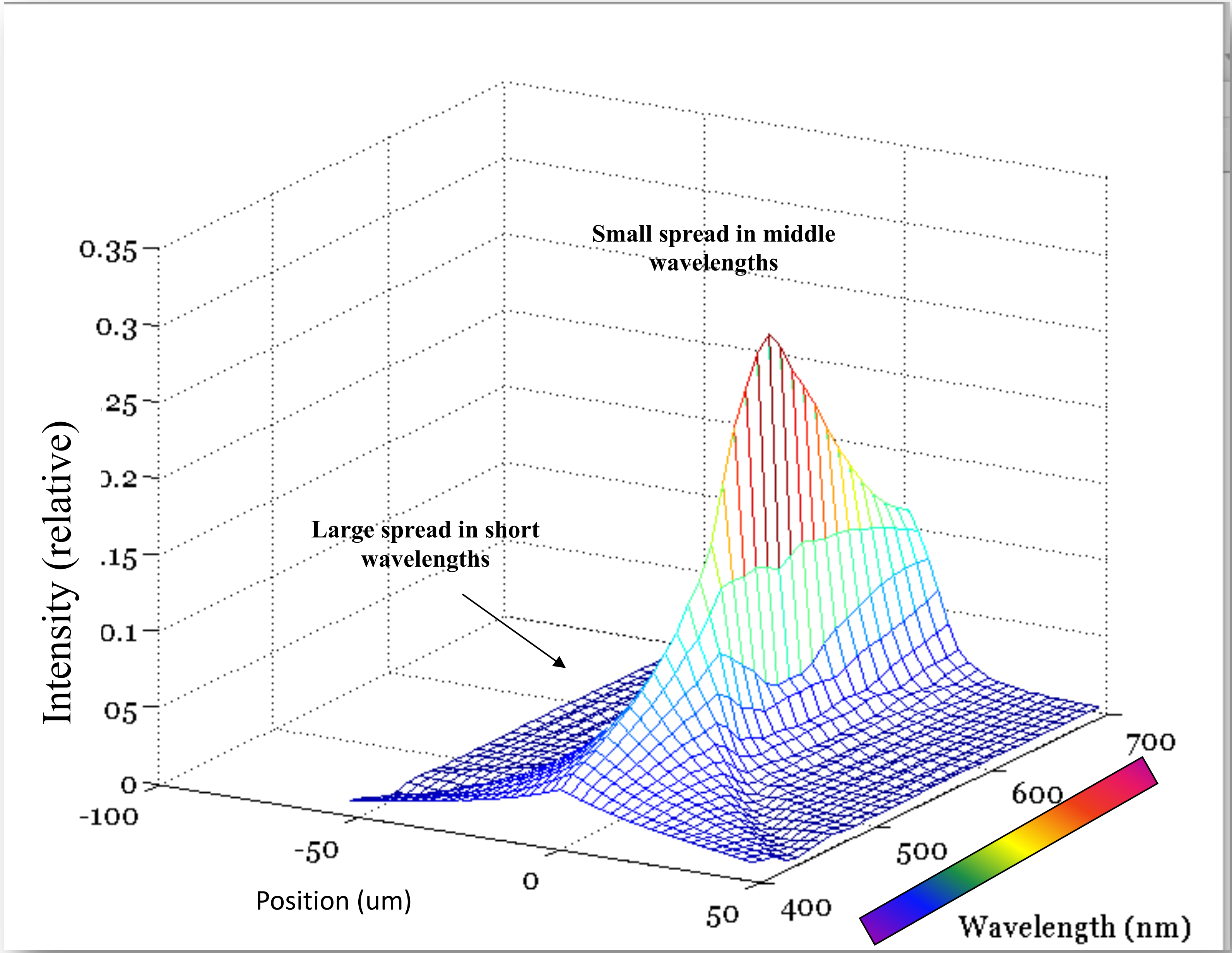
Chromatic MTF of the human optics

(Marimont and Wandell, J. Opt. Soc Am. A., 1994)

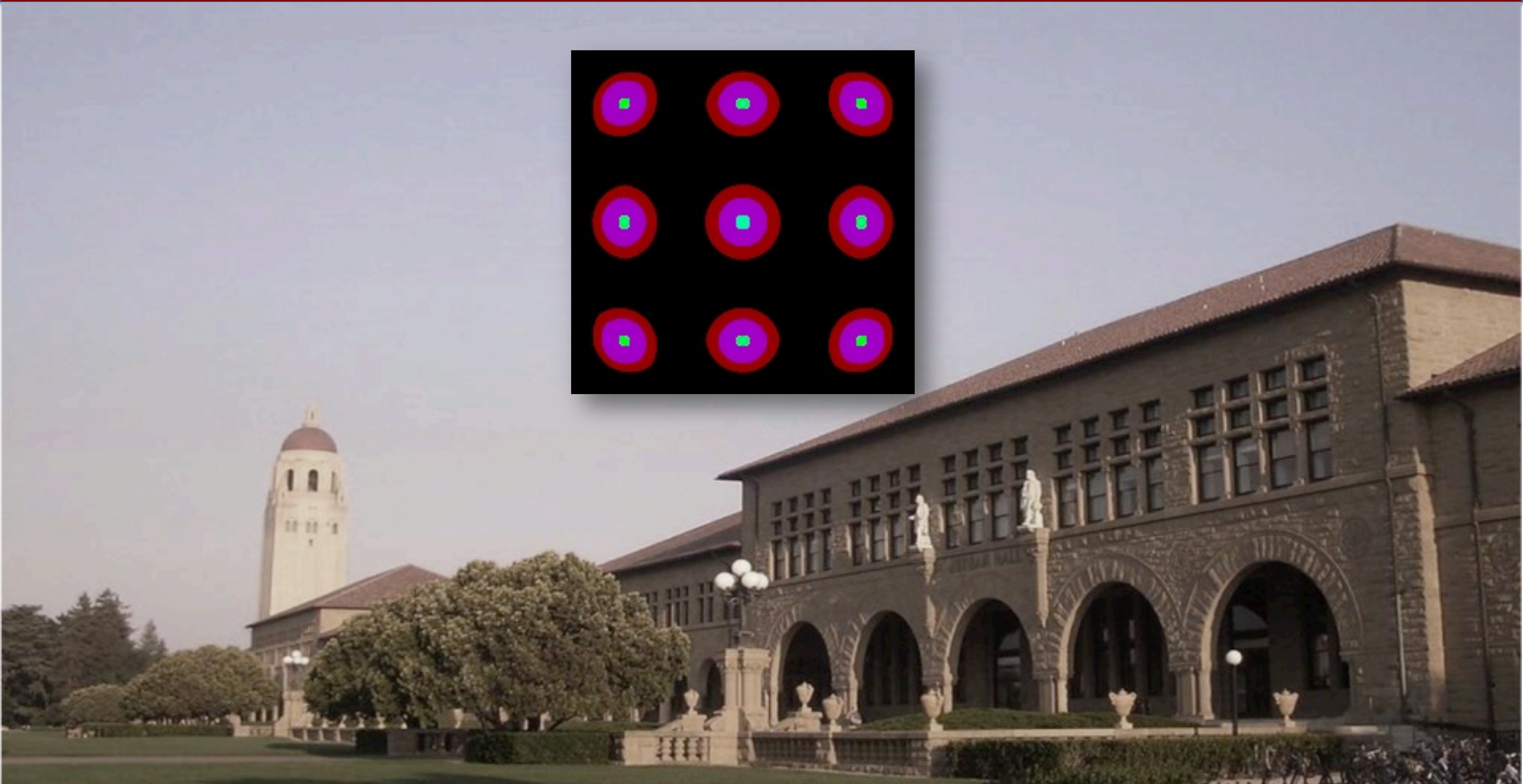


Chromatic aberration summarized by the line spread function

ISET: humanLSF



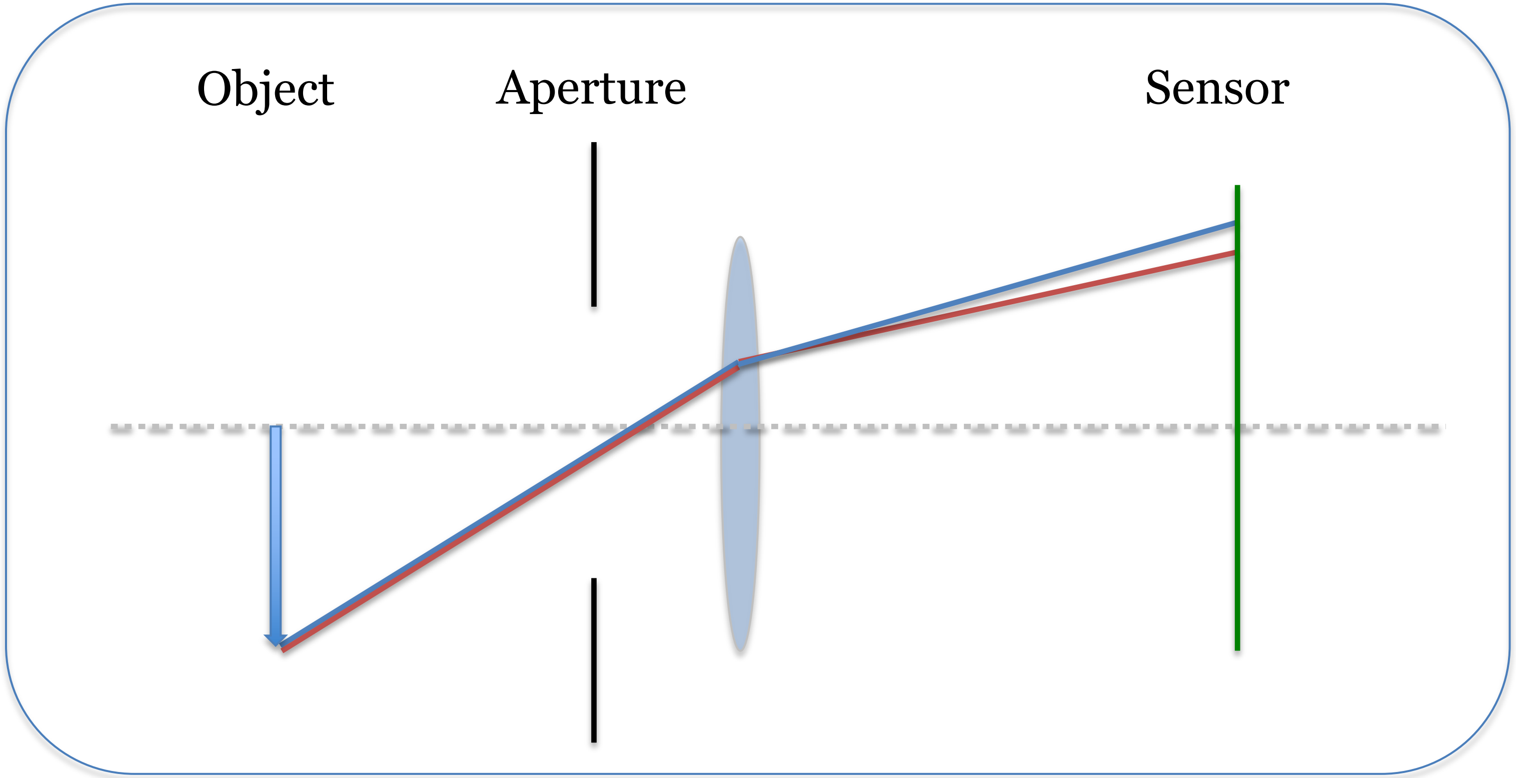
Transverse chromatic aberration



Transverse chromatic aberration

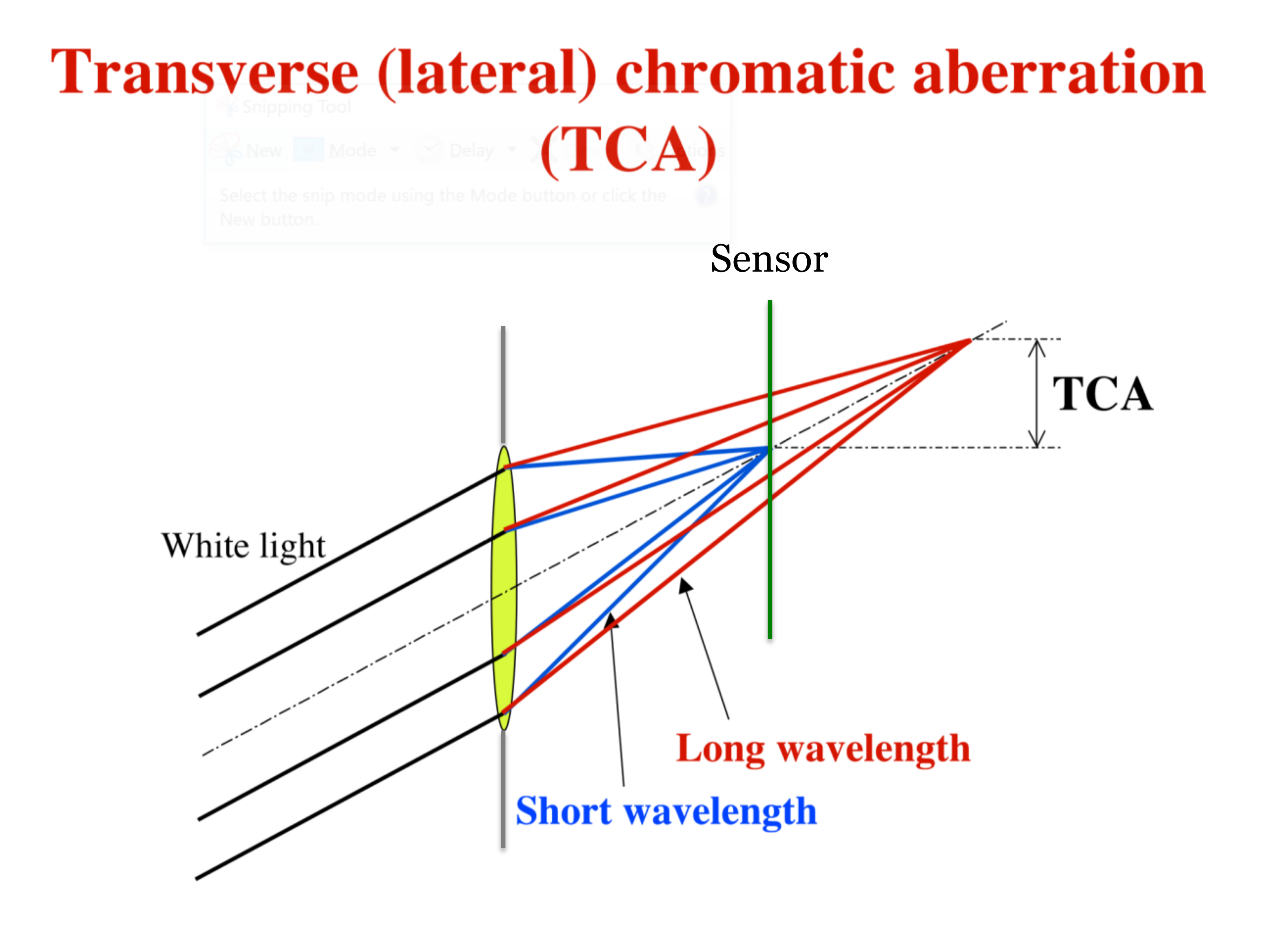
- Two rays starting at the same position on the object, passing through the center of the aperture (chief rays) are refracted by different amounts

- When the rays from this same object point arrive at the sensor, they are displaced laterally (transverse)



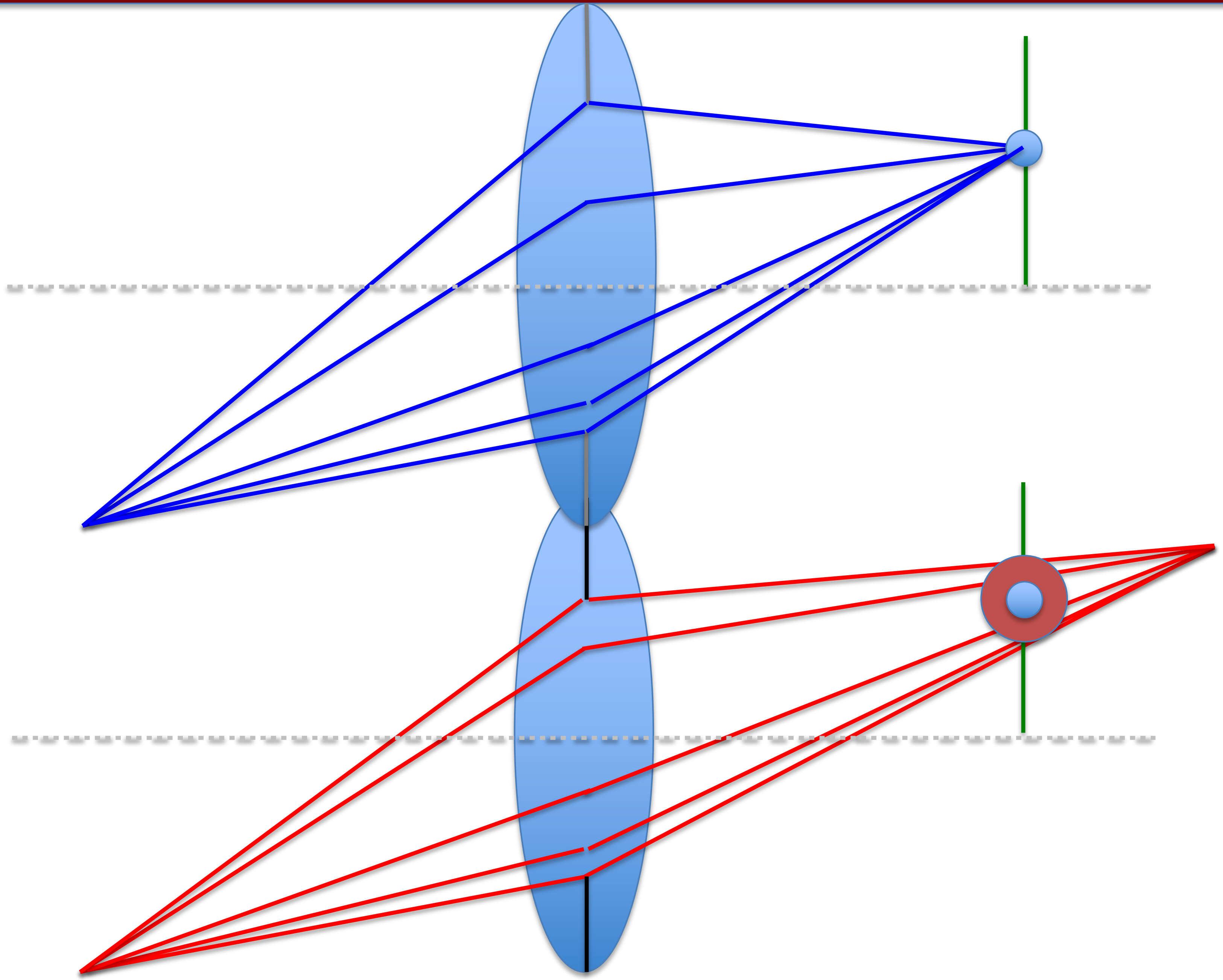
Transverse chromatic aberration

- Another way to specify the transverse component is to measure what happens when we trace parallel rays of red and blue light
- They will be brought into focus at different distances; longitudinal chromatic aberration
- In this case, when the aperture is in the middle of the lens, the red and blue rays are centered at the same position



Transverse chromatic aberration

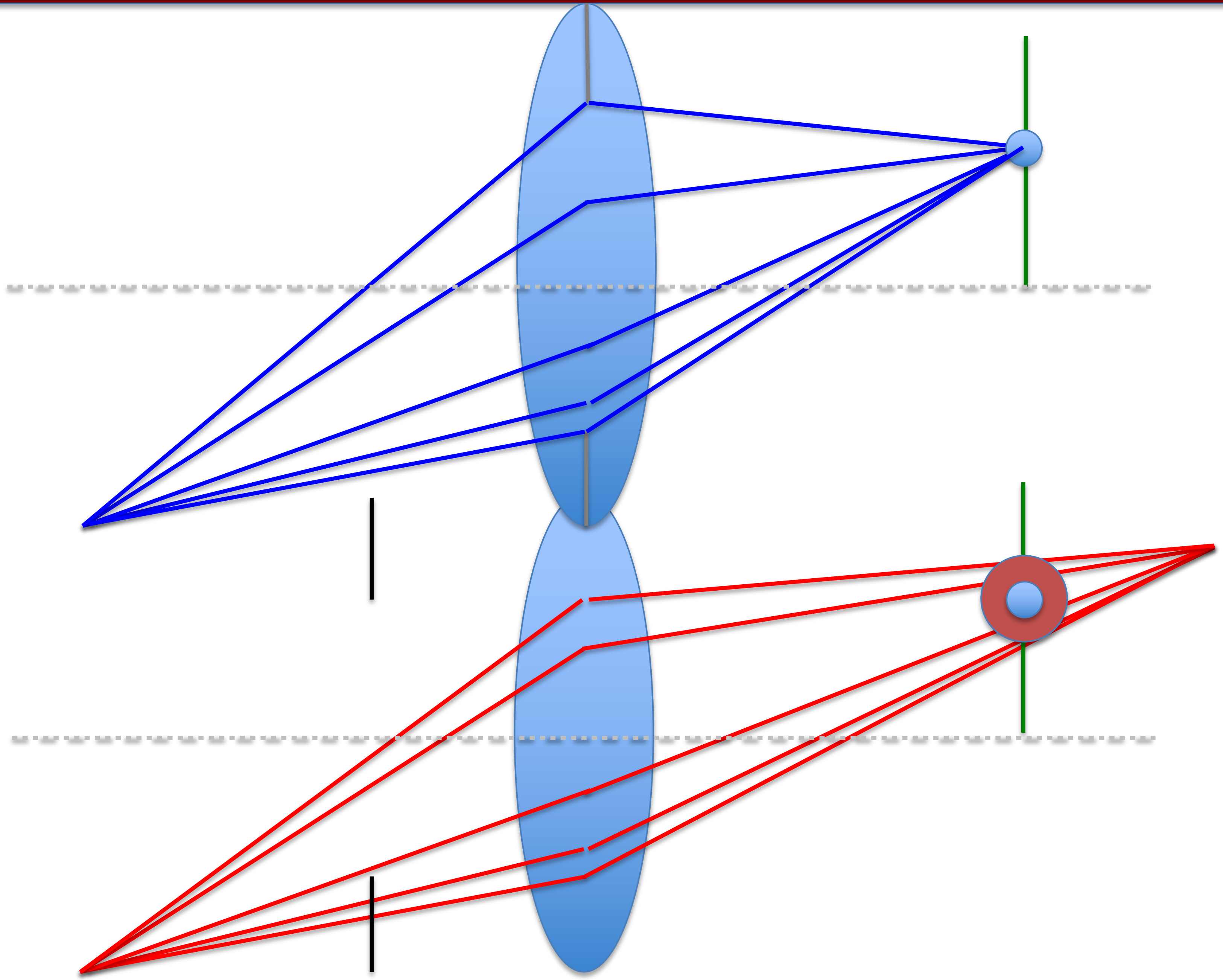
- Transverse chromatic aberration sounds like it is similar to longitudinal (axial) chromatic aberration
- They are both unwanted effects of wavelength, but they are not caused by the same mechanism
- This image shows axial chromatic aberration



Longitudinal and transverse chromatic aberration both arise from differences in the index of refraction of the lens material

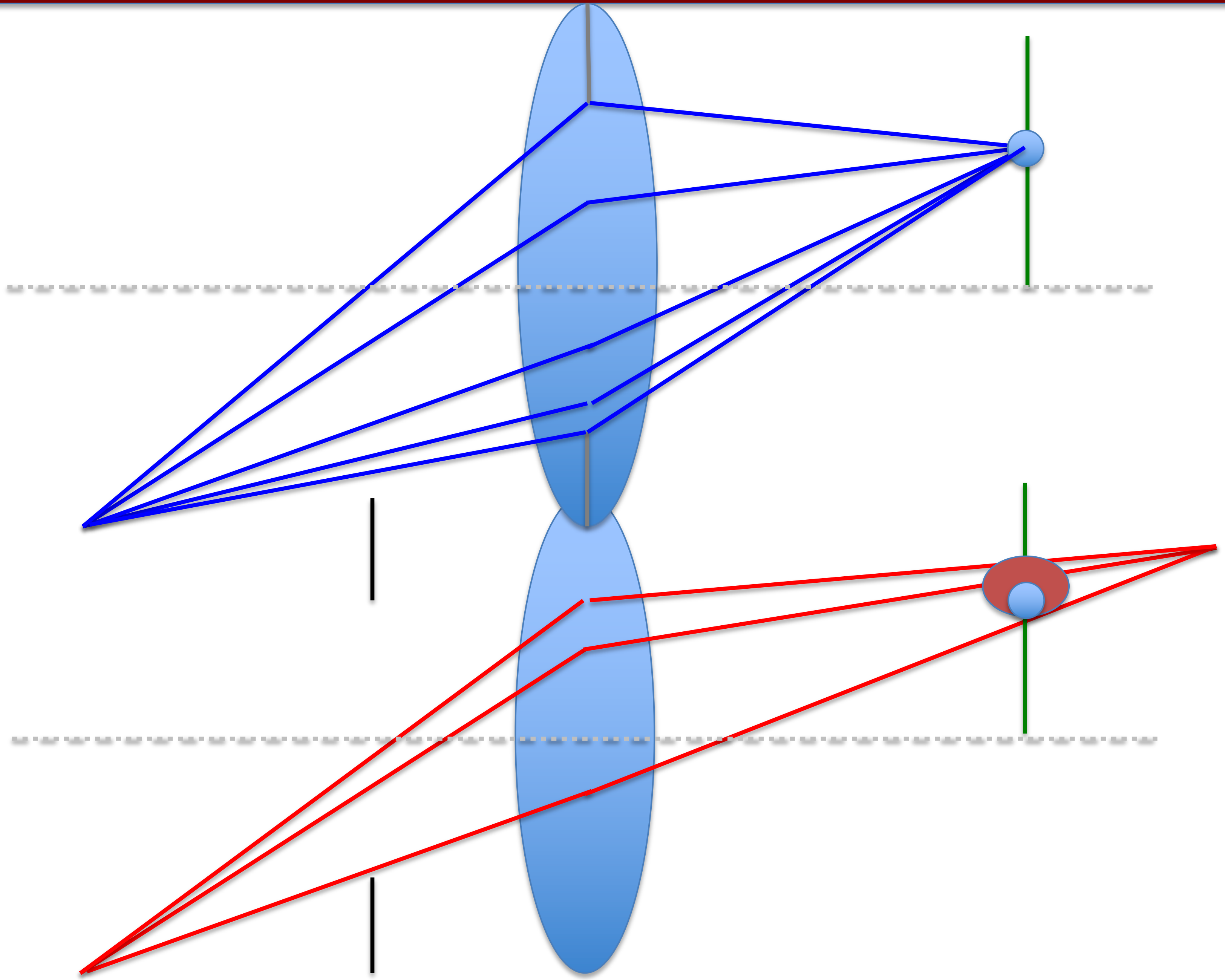
Transverse chromatic aberration

- Suppose we shift the aperture position from the middle of the lens to in front of the lens (red rays)



Transverse chromatic aberration

- Suppose we shift the aperture position from the middle of the lens to in front of the lens (red rays)
- Shifting the aperture blocks some of the red rays
- Removing these rays changes shifts the image center up

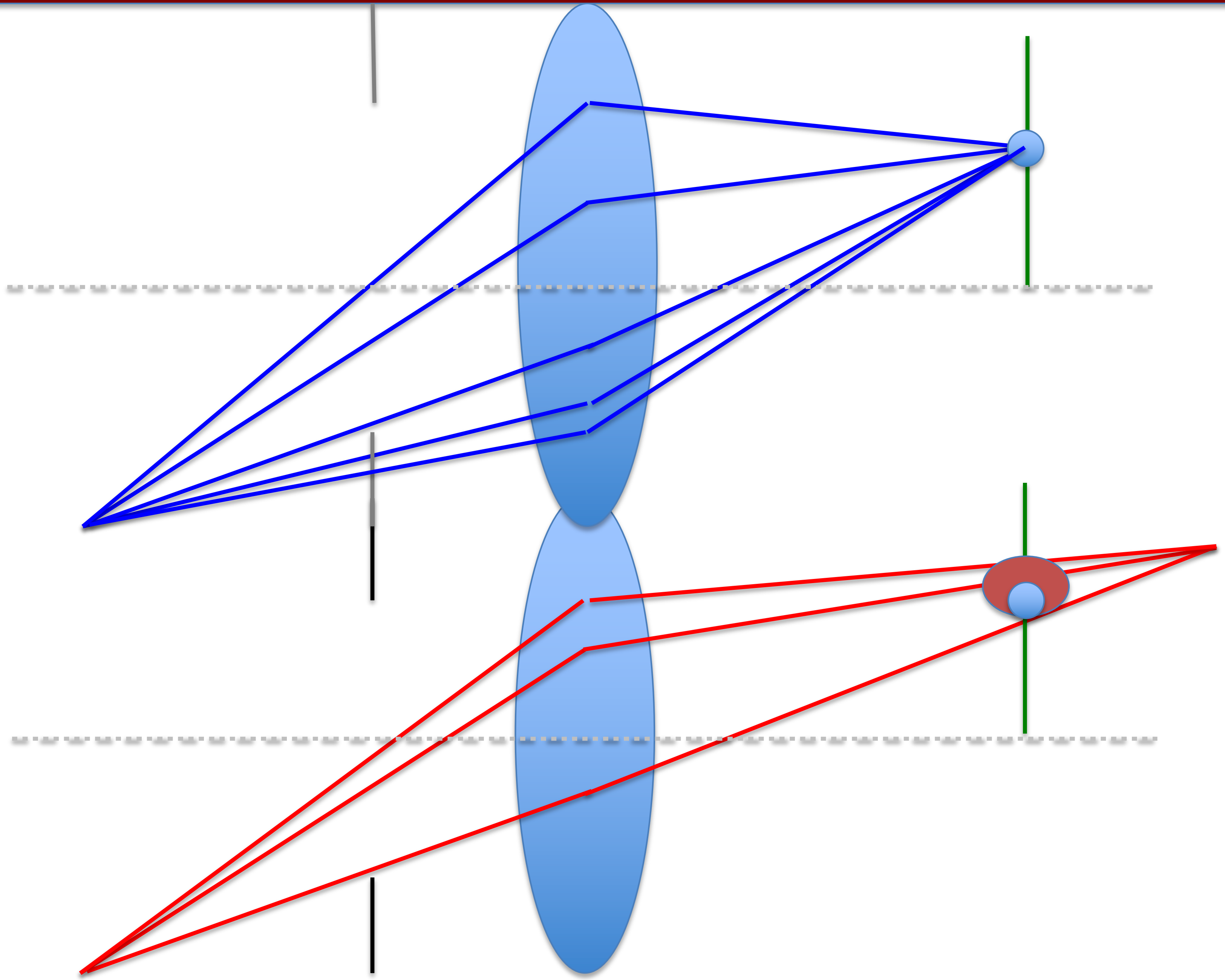


Transverse chromatic aberration

- Suppose we shift the aperture position from the middle of the lens to in front of the lens (blue rays)

- The blue rays were converged to a small point of focus

- Removing some of them doesn't change the mean position of the blue spot



Transverse chromatic aberration

- Suppose we shift the aperture position from the middle of the lens to in front of the lens (blue rays)

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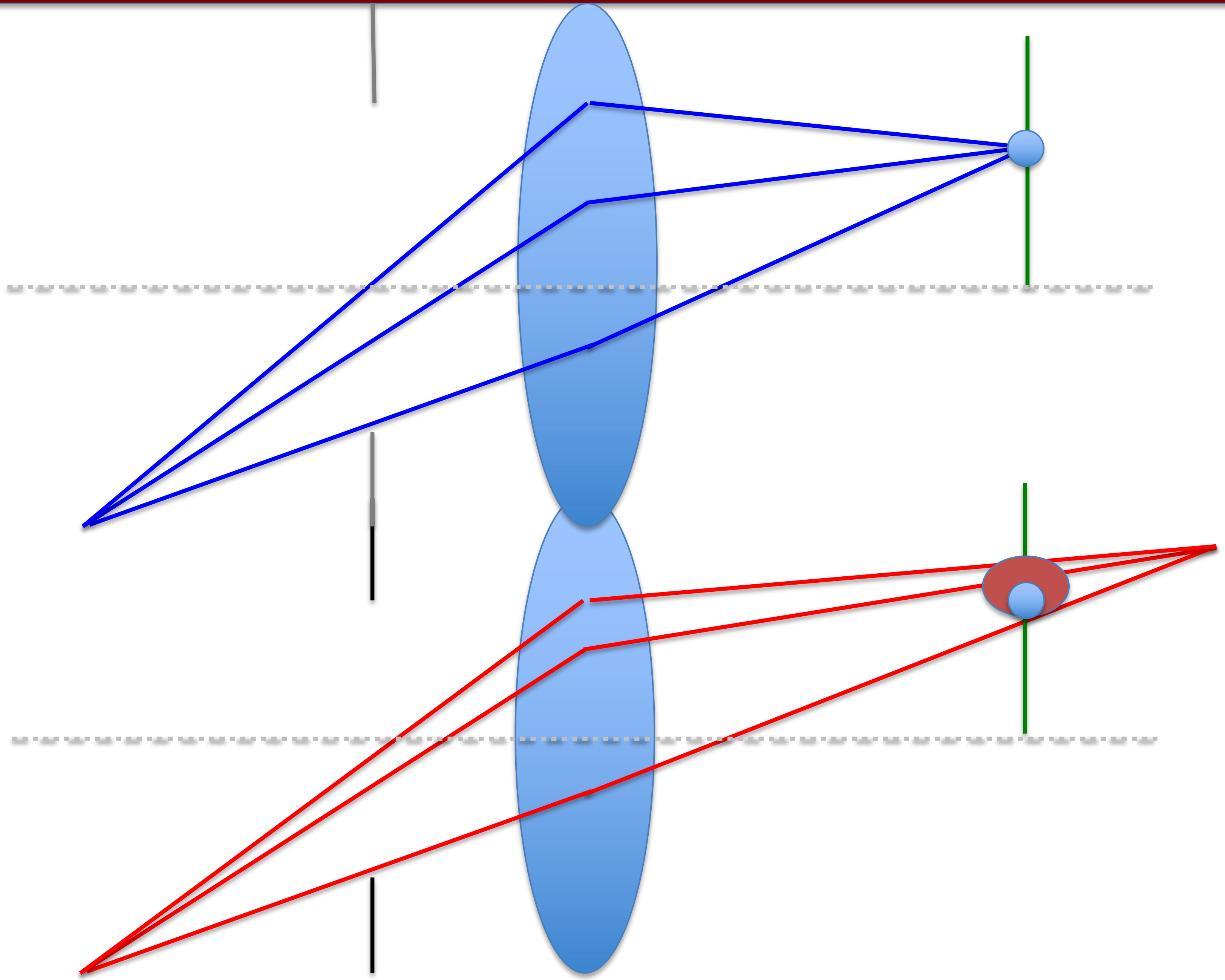
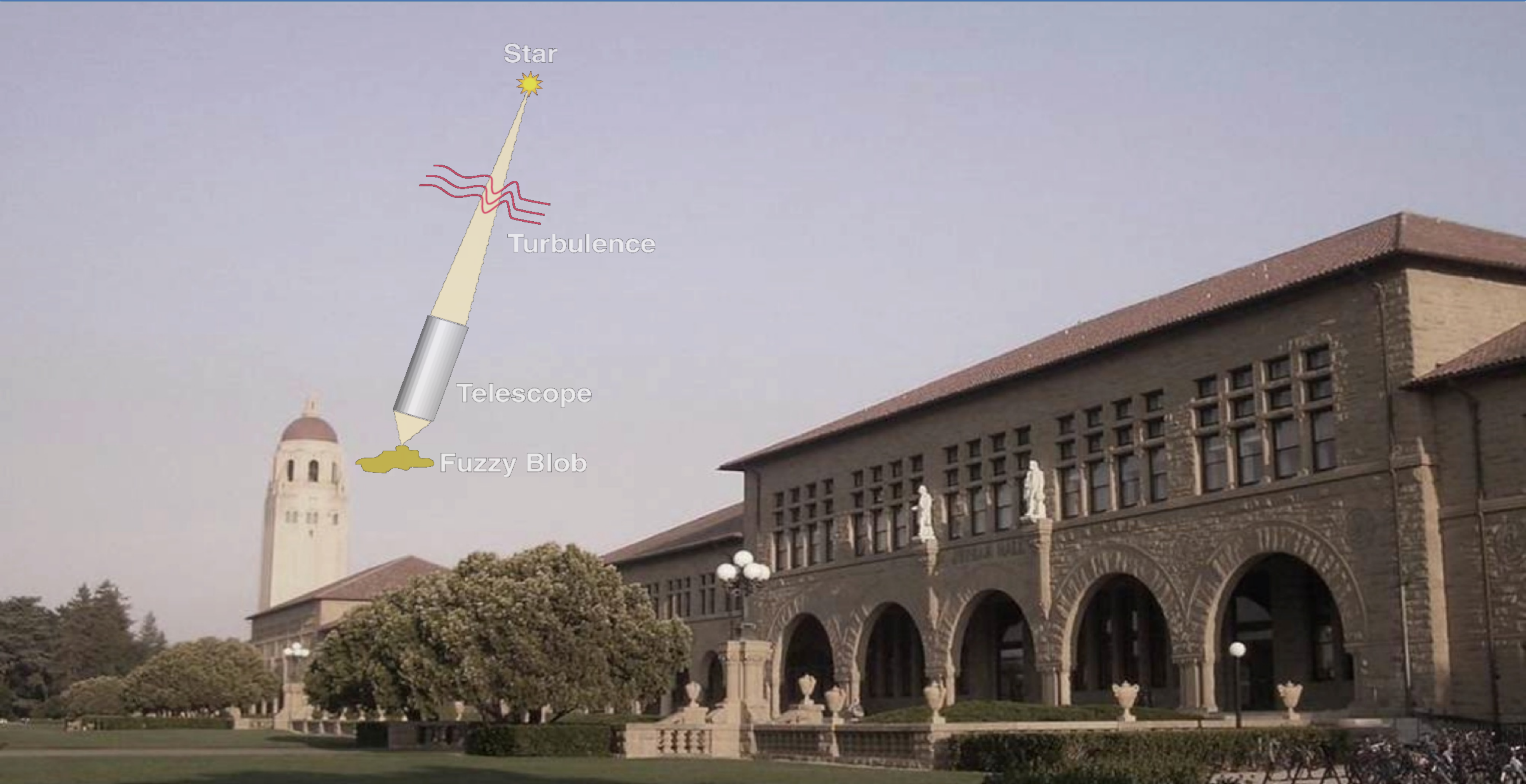
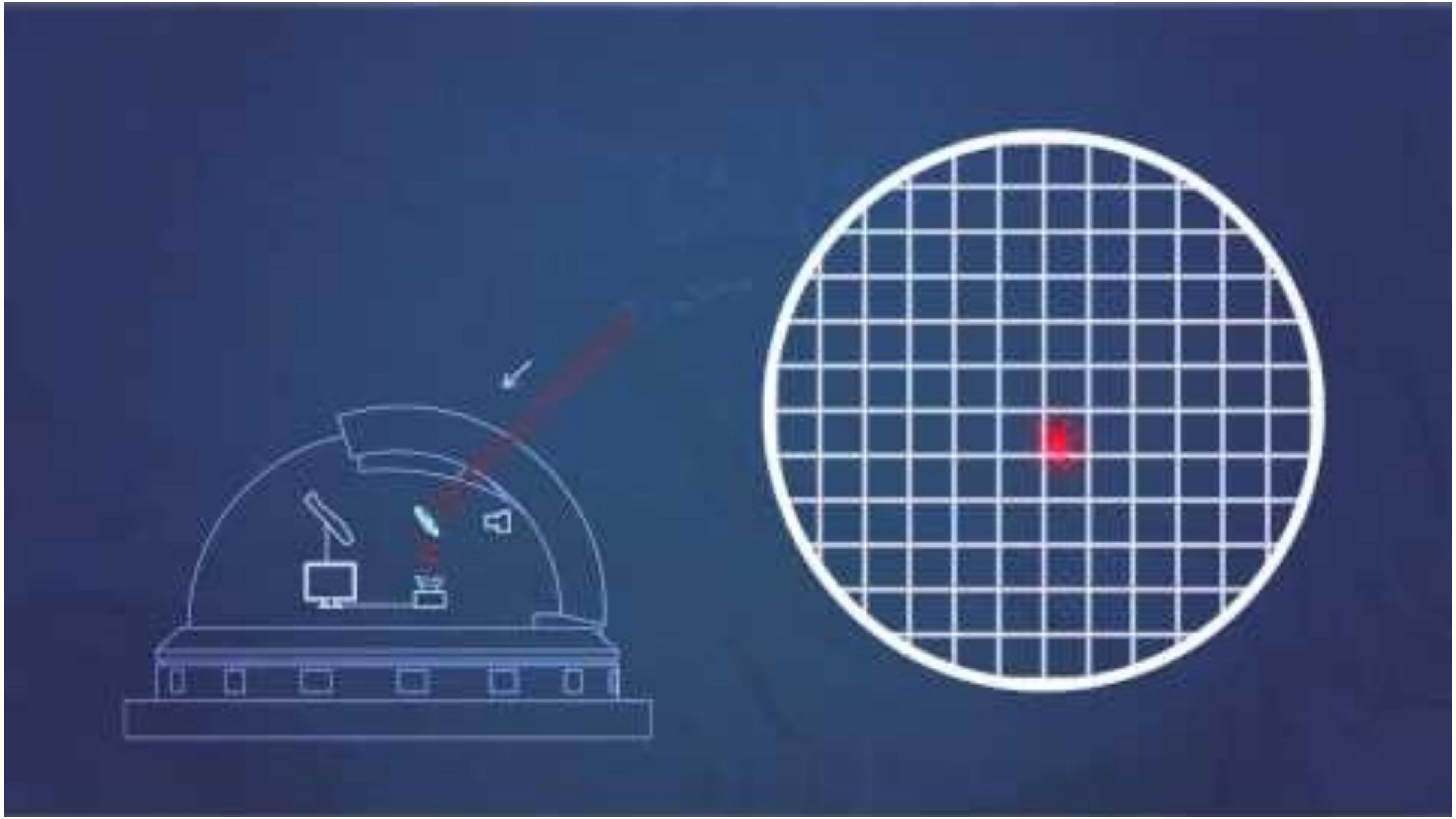


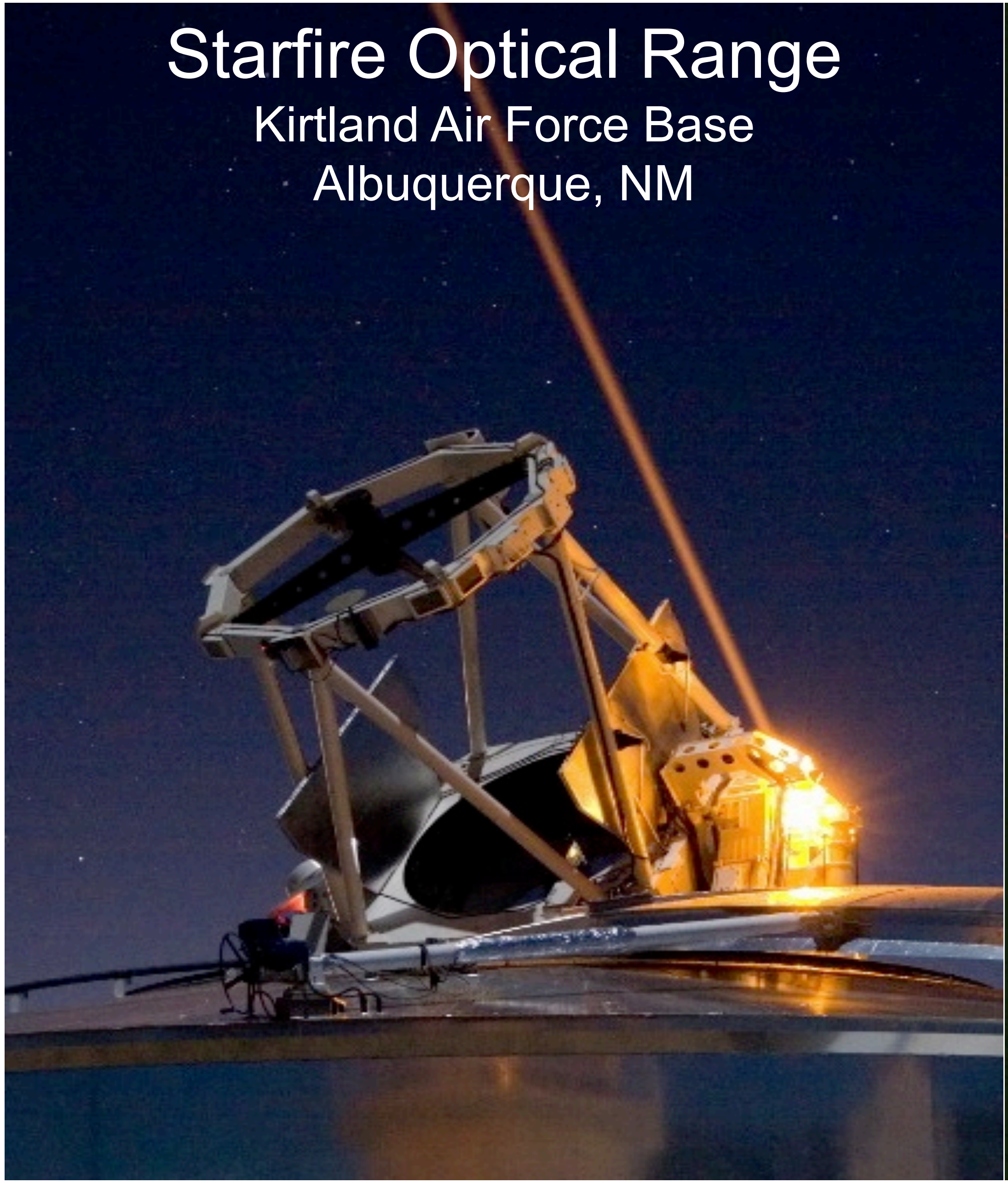
Adaptive optics

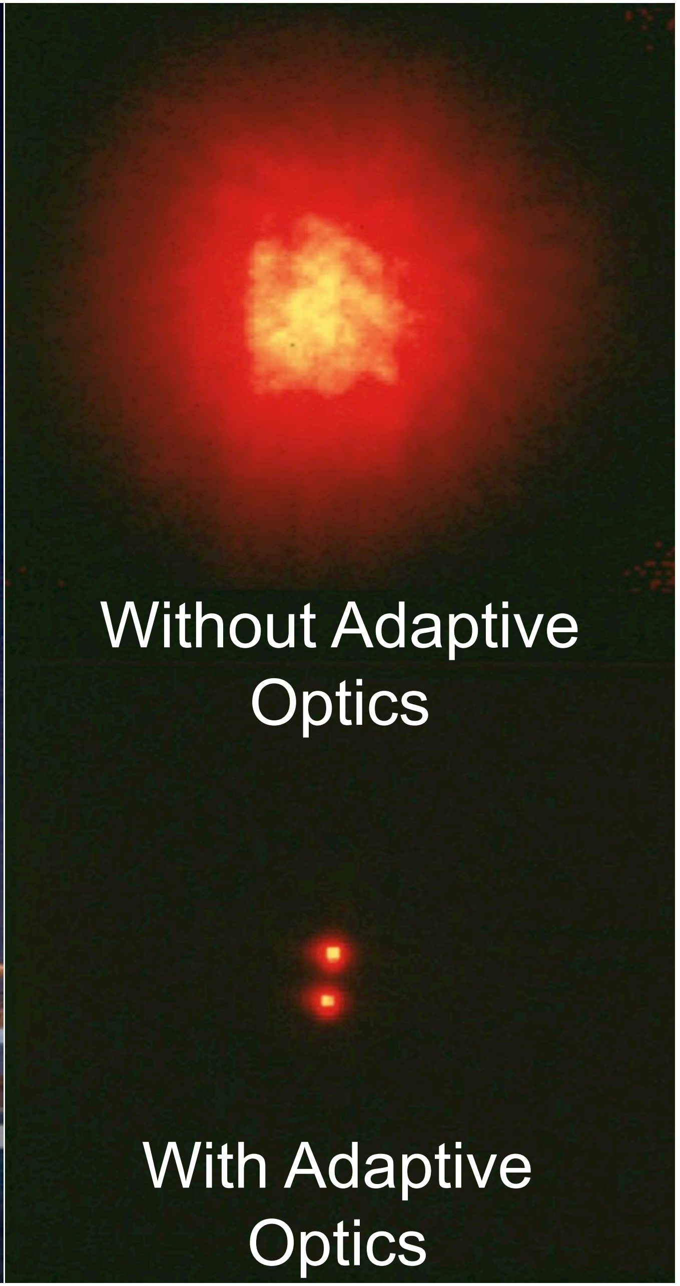


Movie of astronomical use of adaptive optics

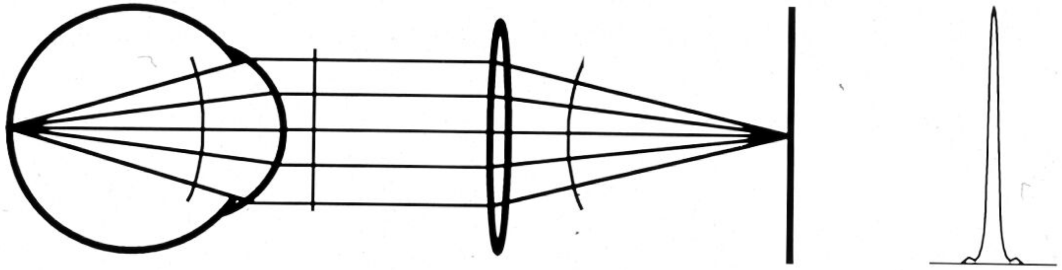


- What an astronomical system looks like
- The laser creates a point source and the AO system measures the turbulence from the system
- The correction is a very big effect

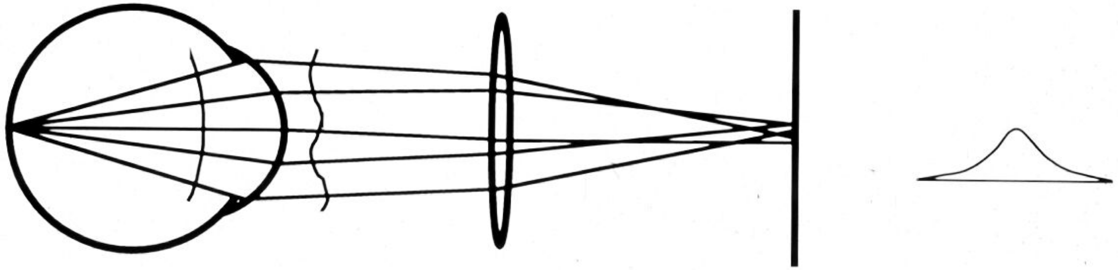




Adaptive optics: Reaching the diffraction limit

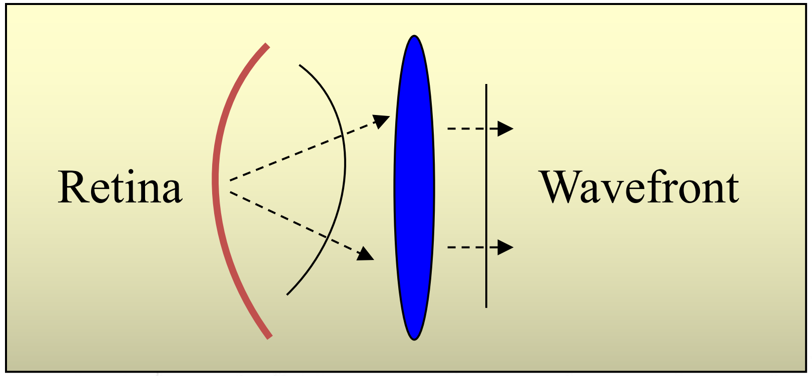


Perfect eye

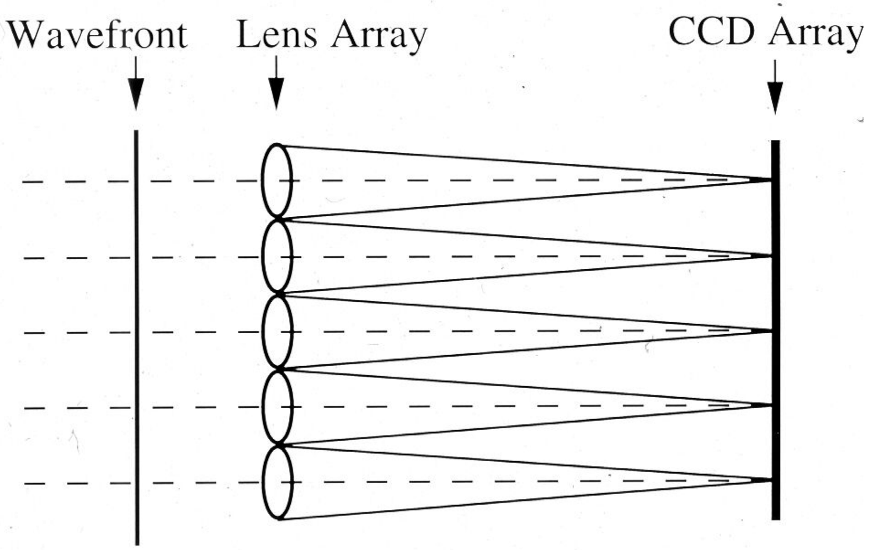


Aberrated eye

Hartmann-Shack wavefront sensor measures the image imperfections

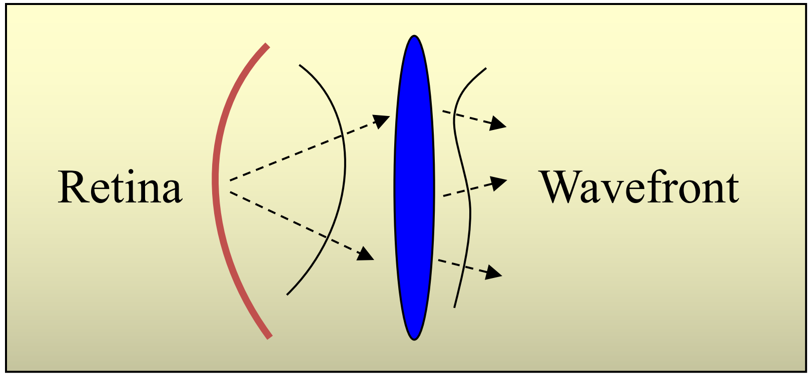


Put a lens here and converge the collimated wavefront to a point

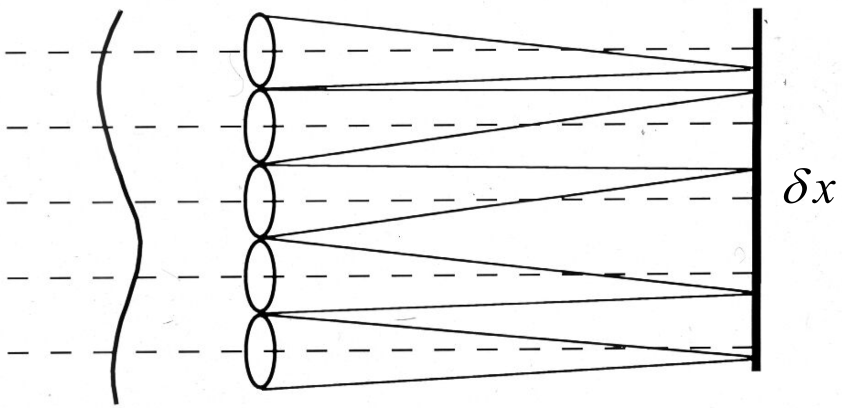


Perfect eye

Hartmann-Shack Wavefront Sensor Measures the image imperfections



Your lens will not converge the wavefront well



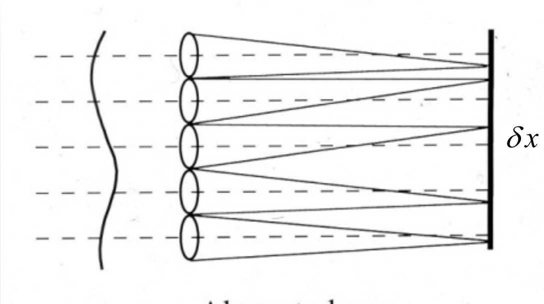
Aberrated eye

Wavefront error

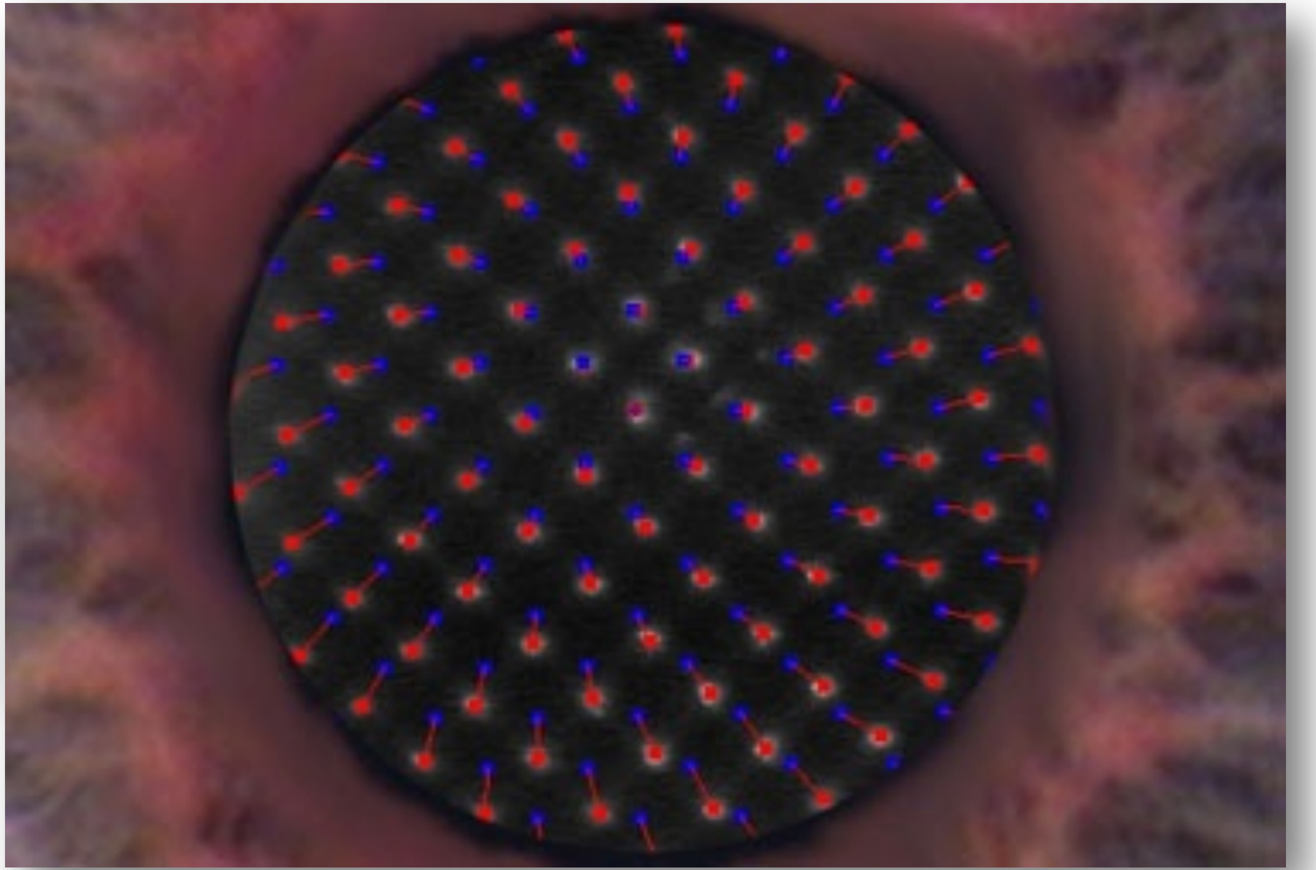
Displacement images at the CCD

Eye with wavefront scanning data superimposed. Data appears as red and blue dots over the eye's iris. Wavefront perturbations cause the red and blue dots to separate.

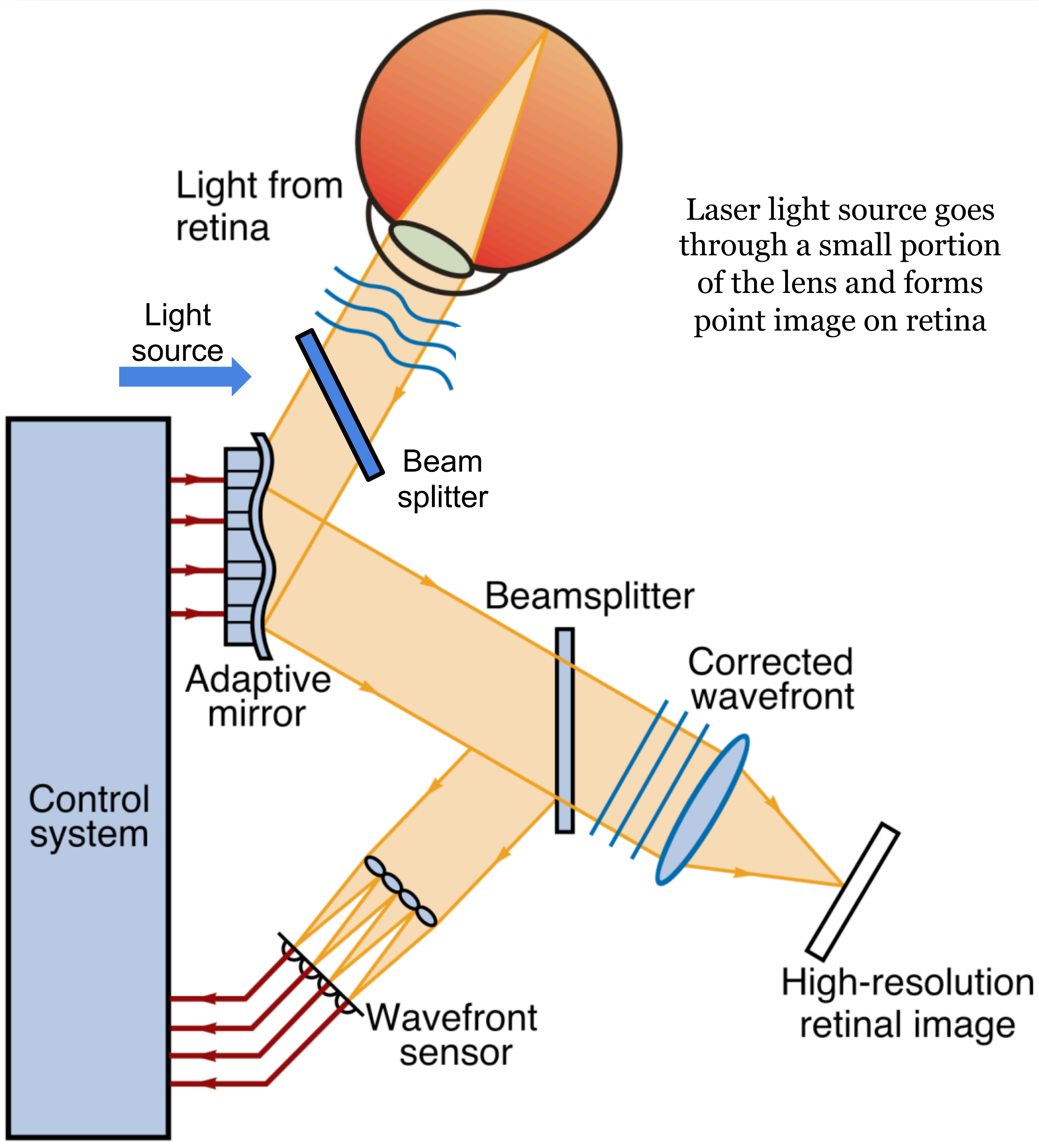
(Credit: J. Schwiegerling)



Aberrated eye

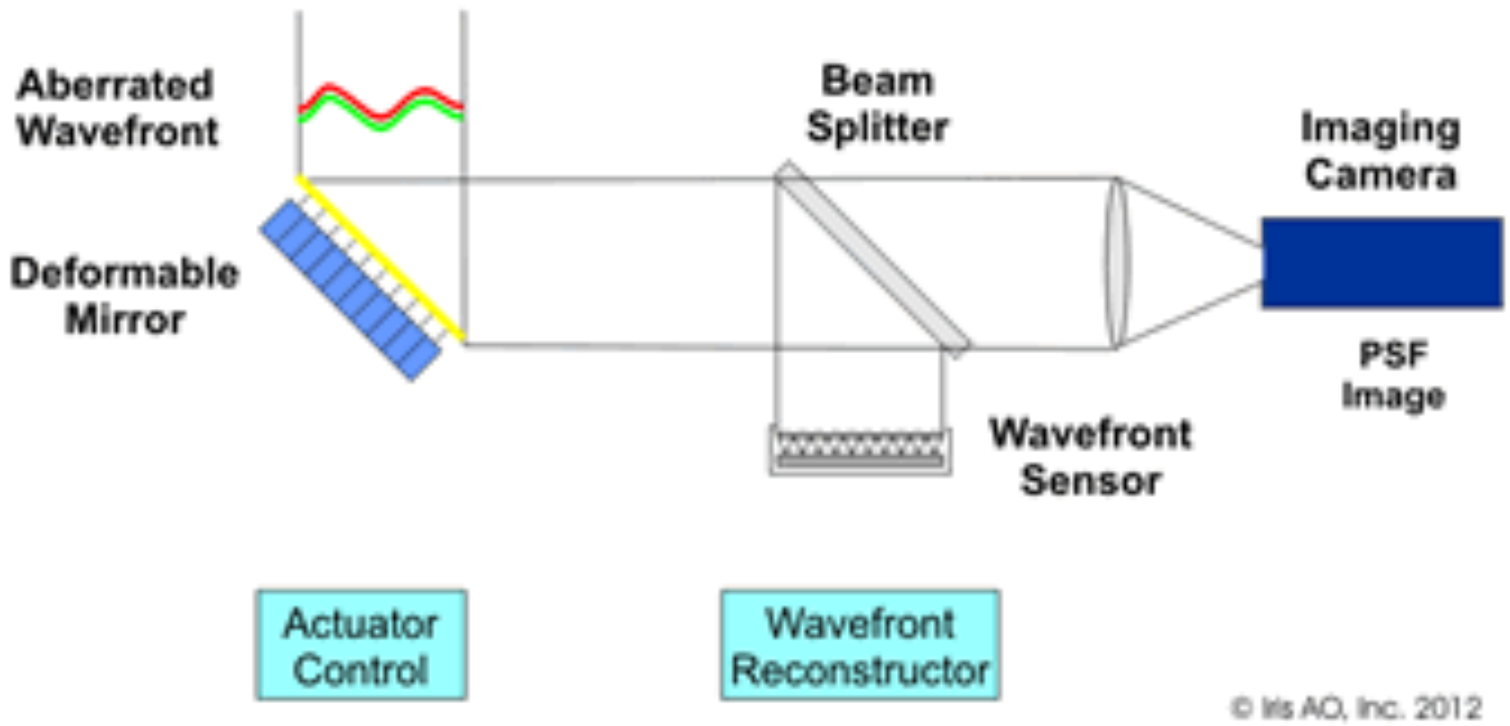


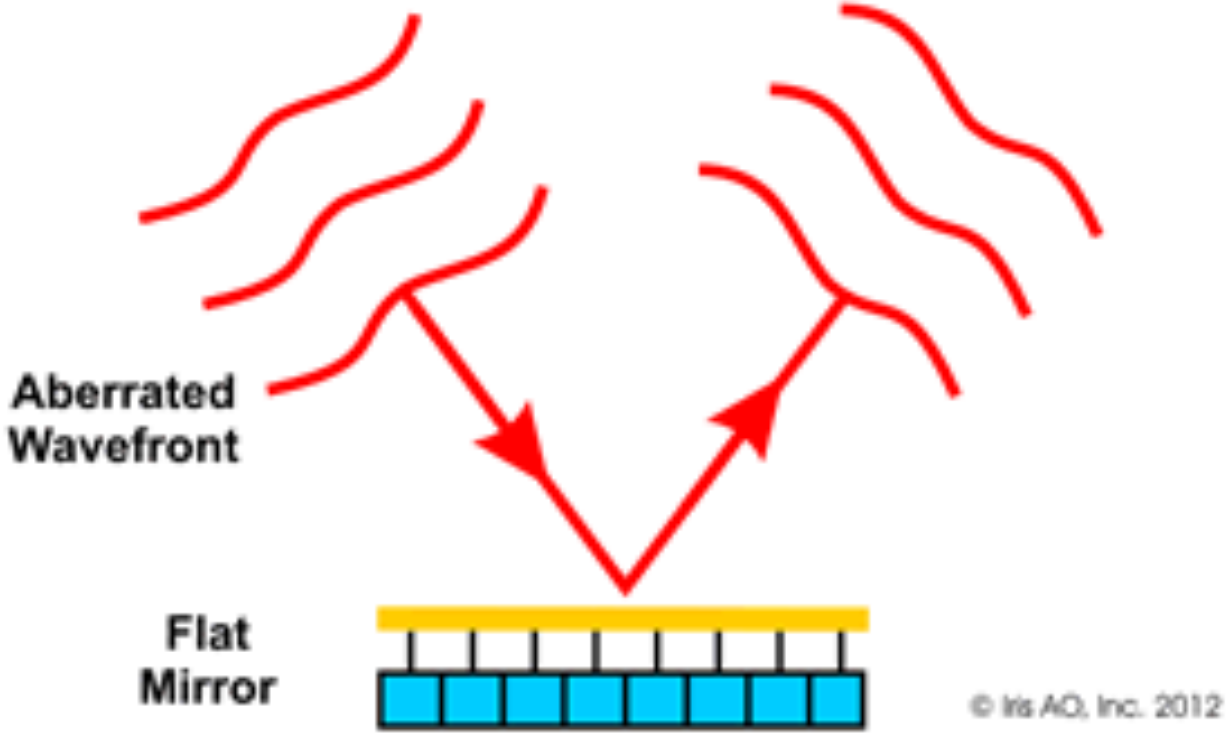
http://www.opticsreport.com/content/article.php?article_id=1005



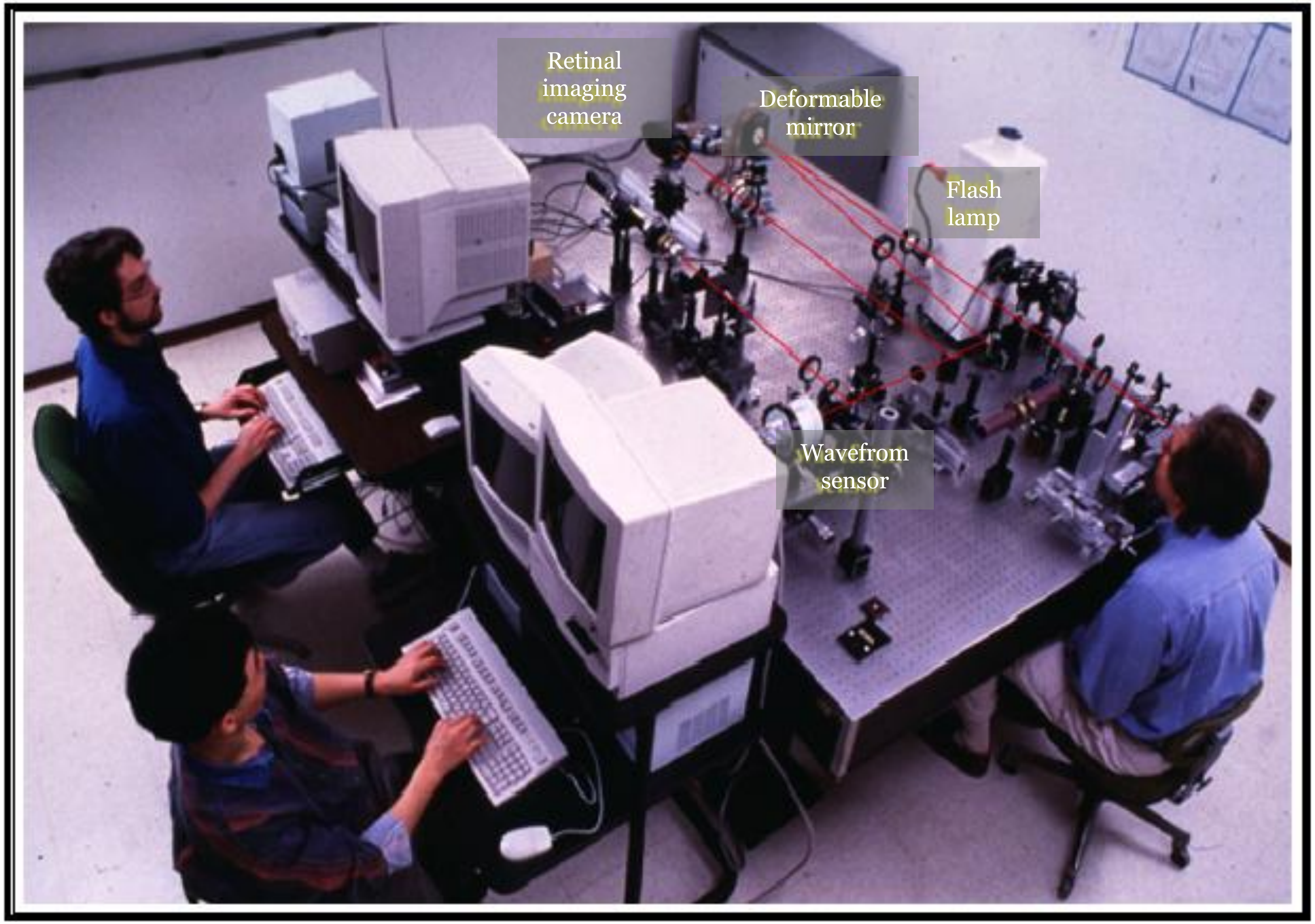
Adaptive optics corrects retinal wavefront aberrations

J. Carroll, D. Gray, A. Roorda, D. R. Williams (2005)


Aberrated Wavefront
Deformable Mirror
Beam Splitter
Imaging Camera
PSF Image
Wavefront Sensor
Actuator Control
Wavefront Reconstructor
© Iris AO, Inc. 2012


Aberrated
Wavefront
Flat
Mirror
© Iris AO, Inc. 2012

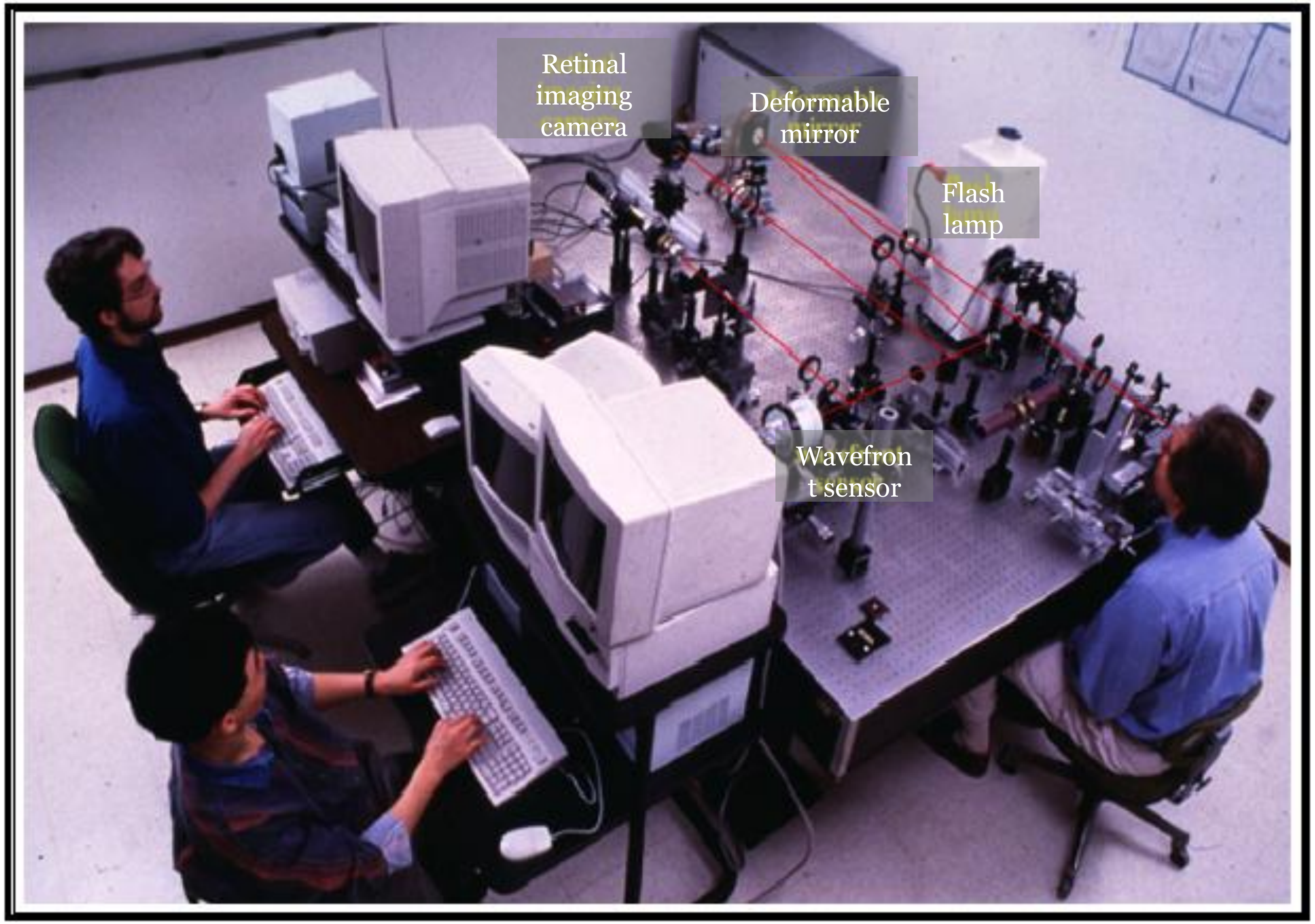
Rochester Adaptive Optics Ophthalmoscope ~ 2000



Rochester Adaptive Optics Ophthalmoscope ~ 2000

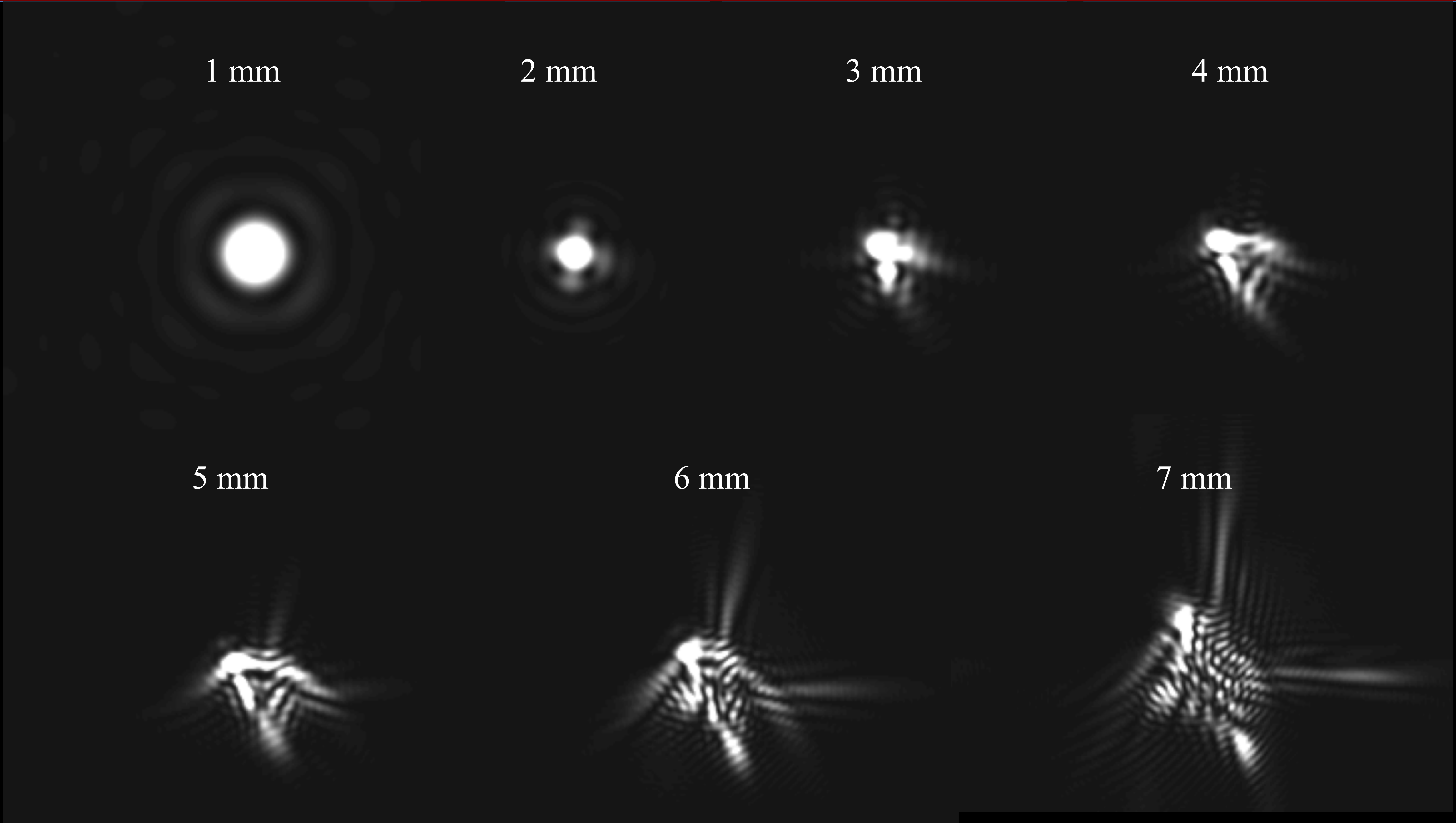
- Following work in astronomy, Dave Williams and his students and postdocs realized that Shack-Hartman wavefront sensors could be used to correct for most aberrations of the in vivo human eye

- They built rigs that allowed them to characterize the optics and then to correct for these aberrations



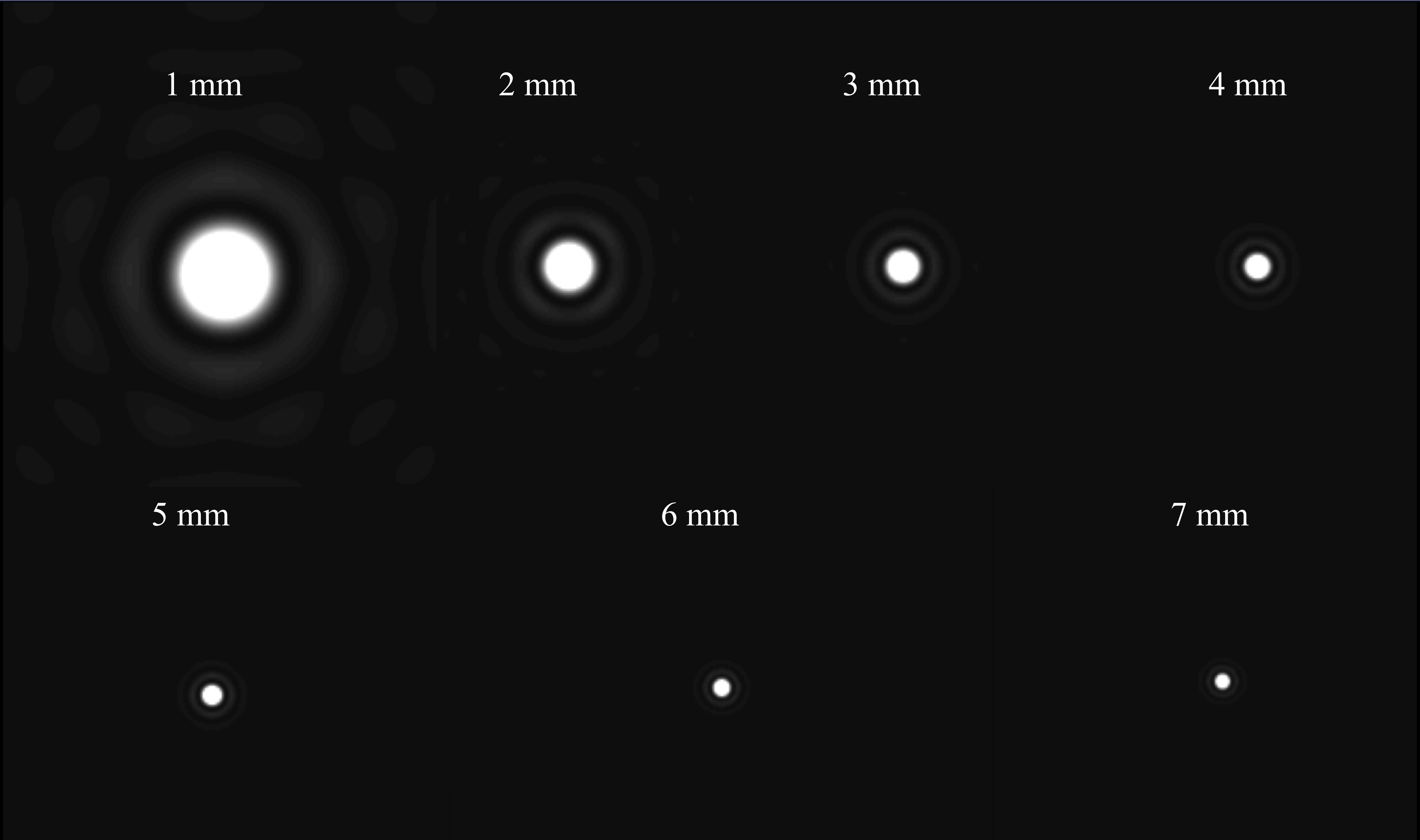
Real Eye Point Spread Functions

Courtesy A. Roorda



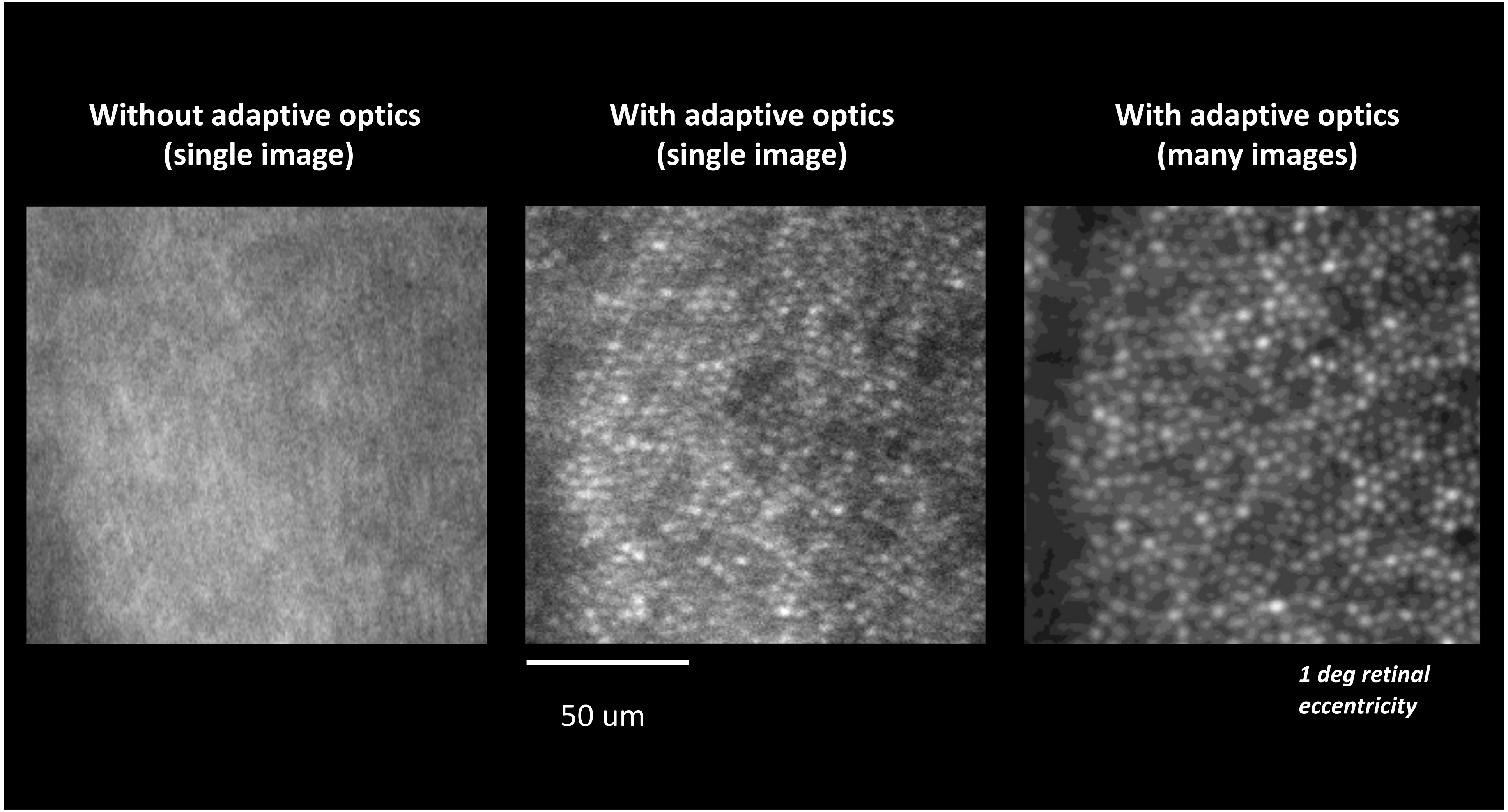
Perfect eye point spread functions

Courtesy A. Roorda



Adaptive optics allows cellular resolution: *in vivo*

Original images from about
2000 from the Williams lab



Application: Seeing The Arrangement of Cone Classes in the Human Eye

(Roorda and Williams et al., 1999)

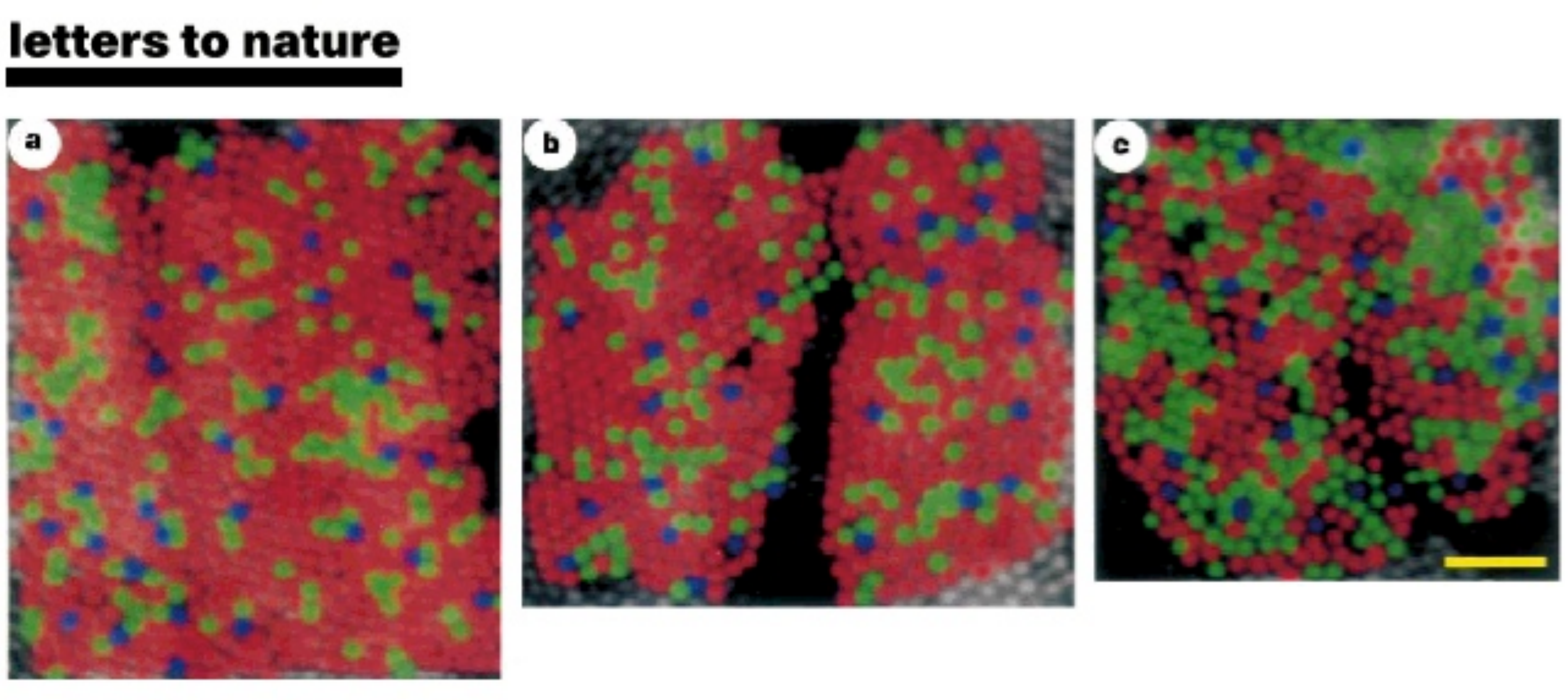
Austin Roorda

Heidi Hofer

- Following work in astronomy, Dave Williams and his students and postdocs realized that Shack-Hartman wavefront sensors could be used to correct for most aberrations of the in vivo human eye

- They built rigs that allowed them to characterize the optics and then to correct for these aberrations

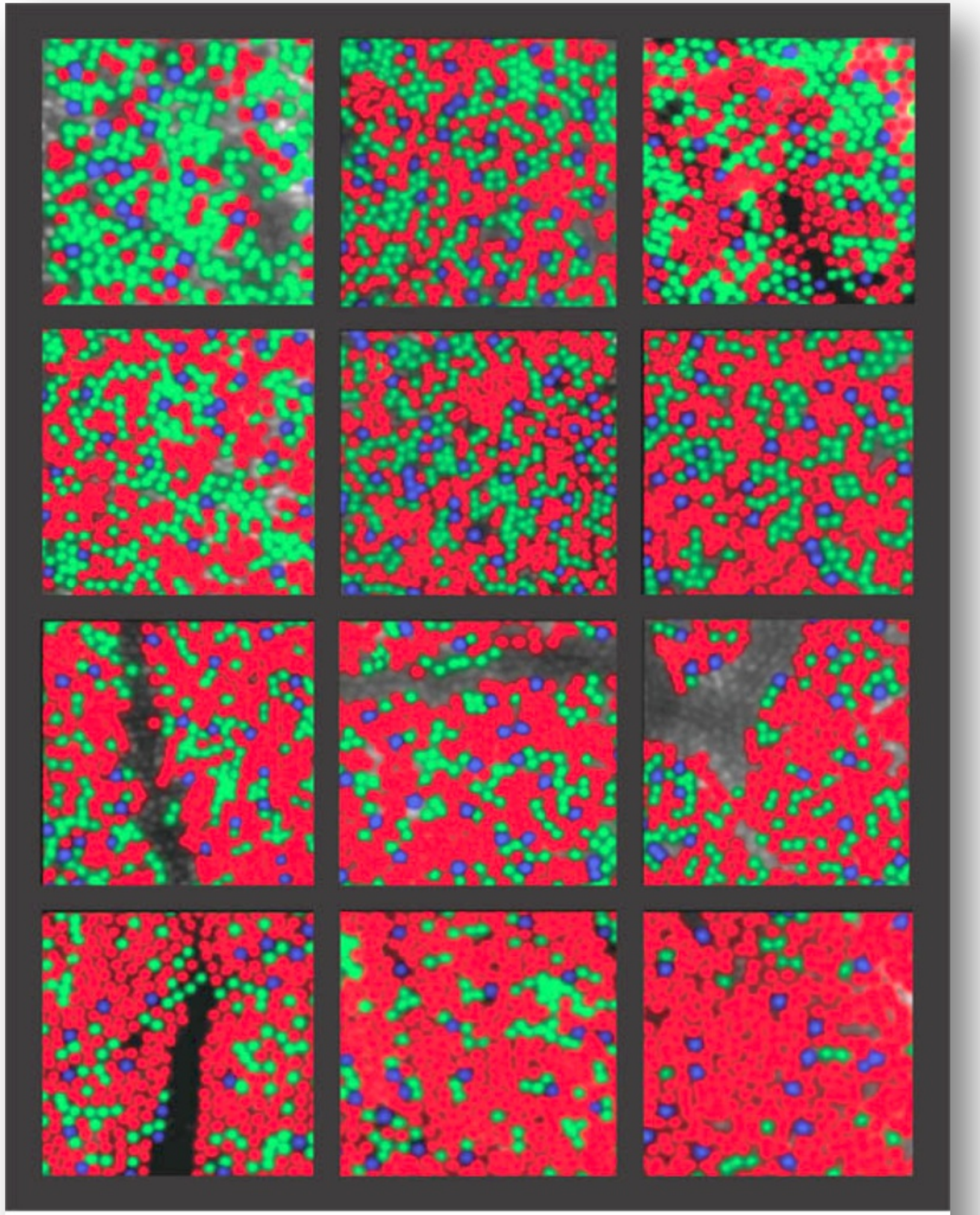
- They developed techniques to estimate which of the three types of cones was present at each location



What if we had the neuroscience data first?

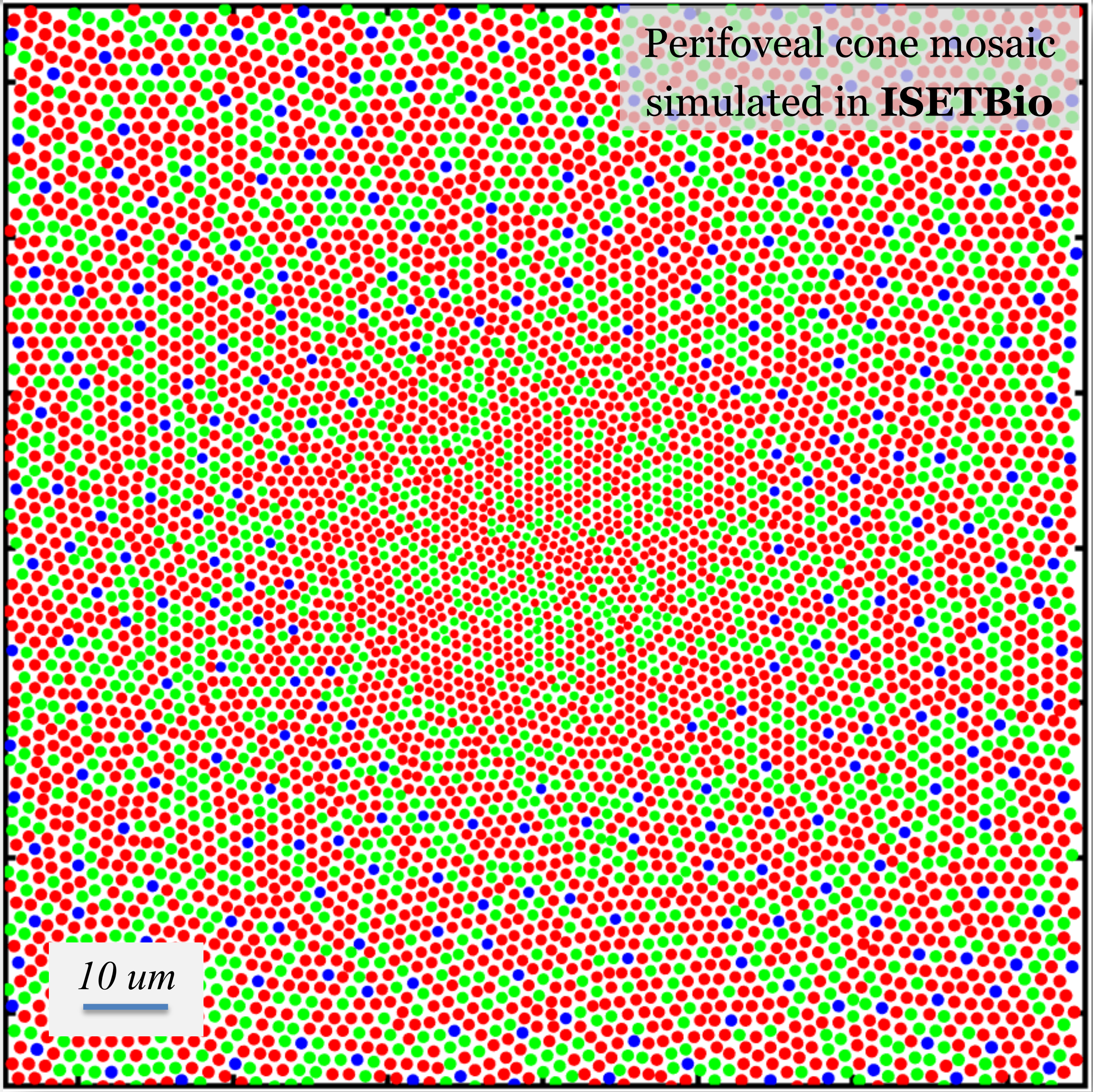
Hofer, H. et al. J. Neurosci. 2005;25:9669-9679

Individual cone mosaics differ greatly in density and ratios of the three cone types

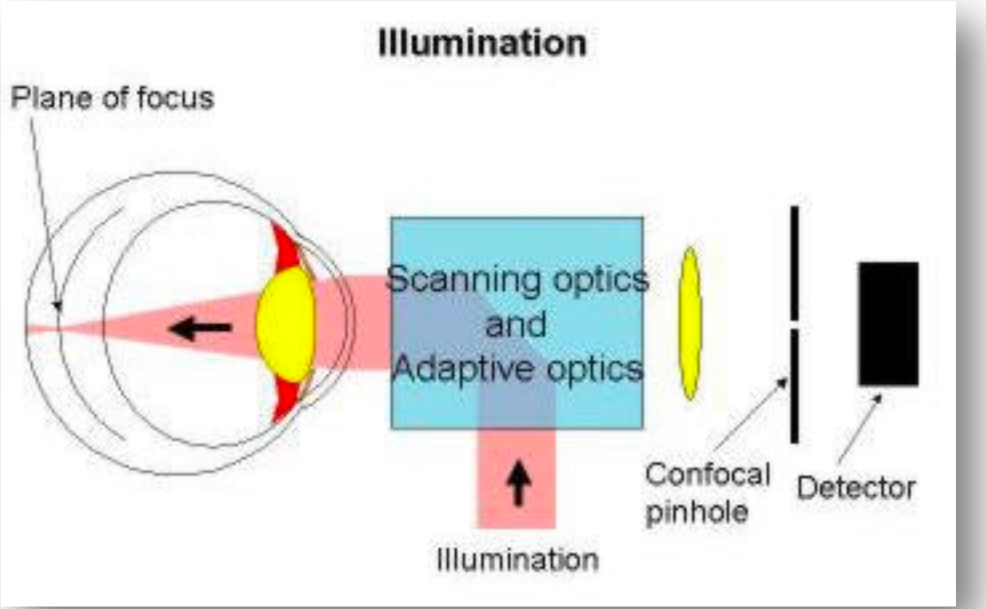


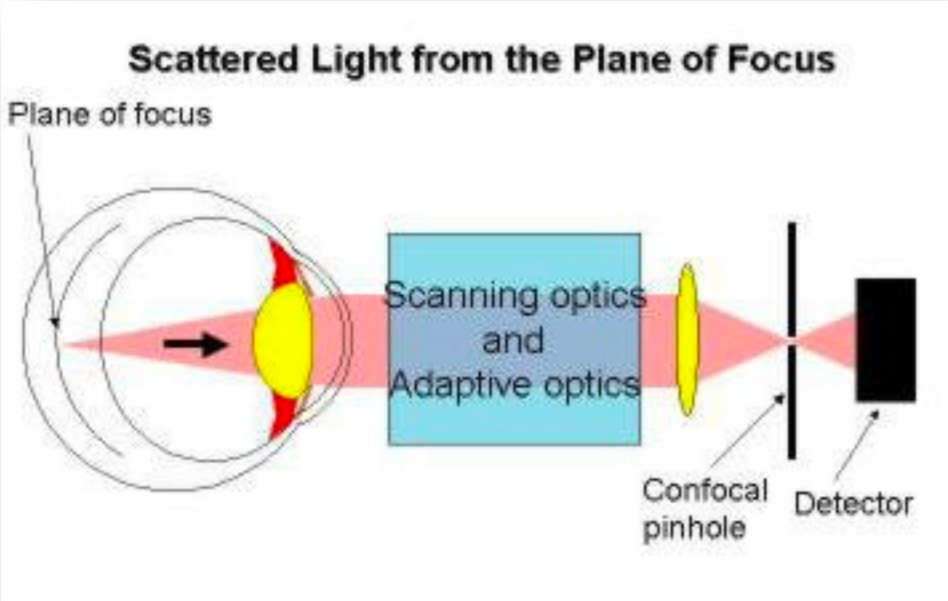
ISETBio: Simulating the cone mosaic

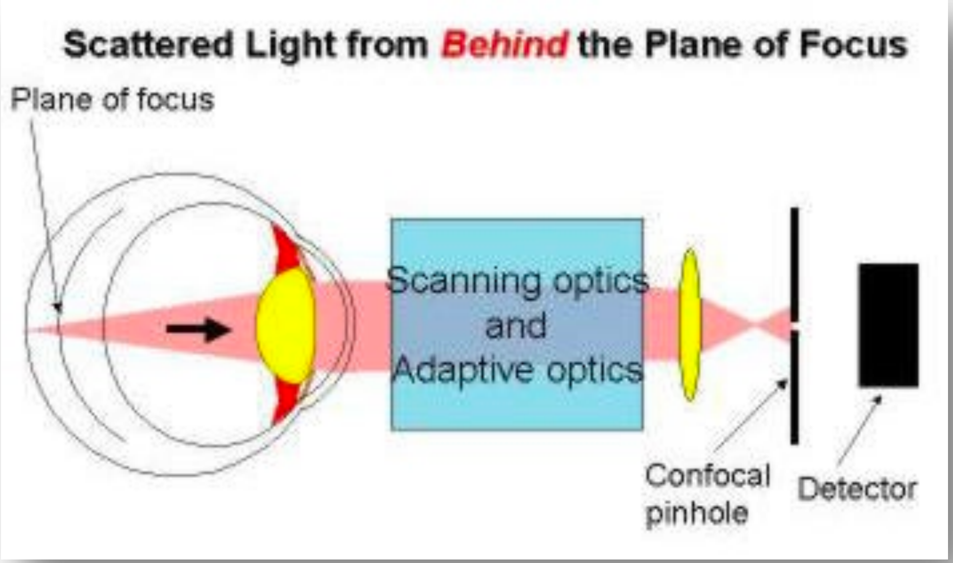
- The cone mosaic is rather complex.
- The image is an ISETBio simulation of the central fovea of the human retina

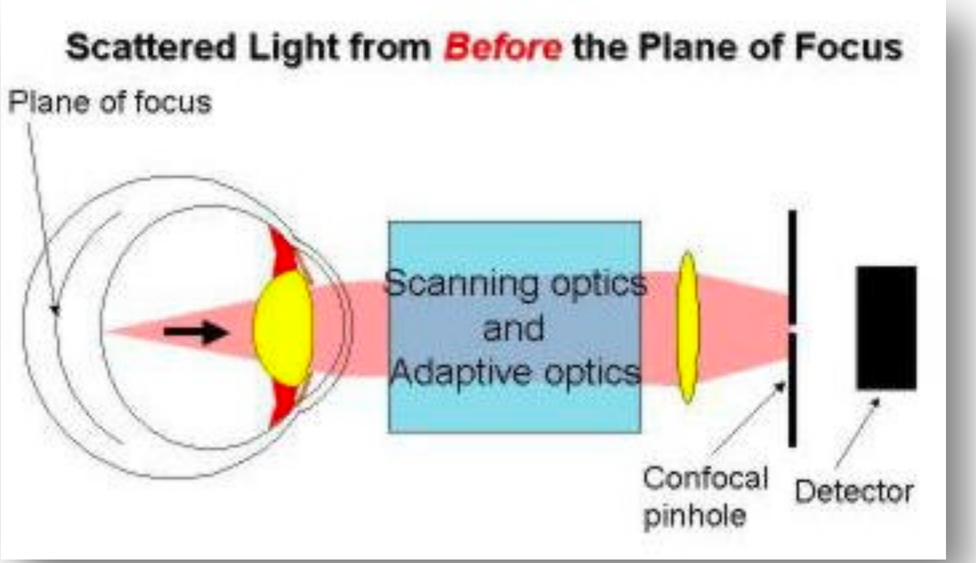


AOSLO – Delivering stimuli



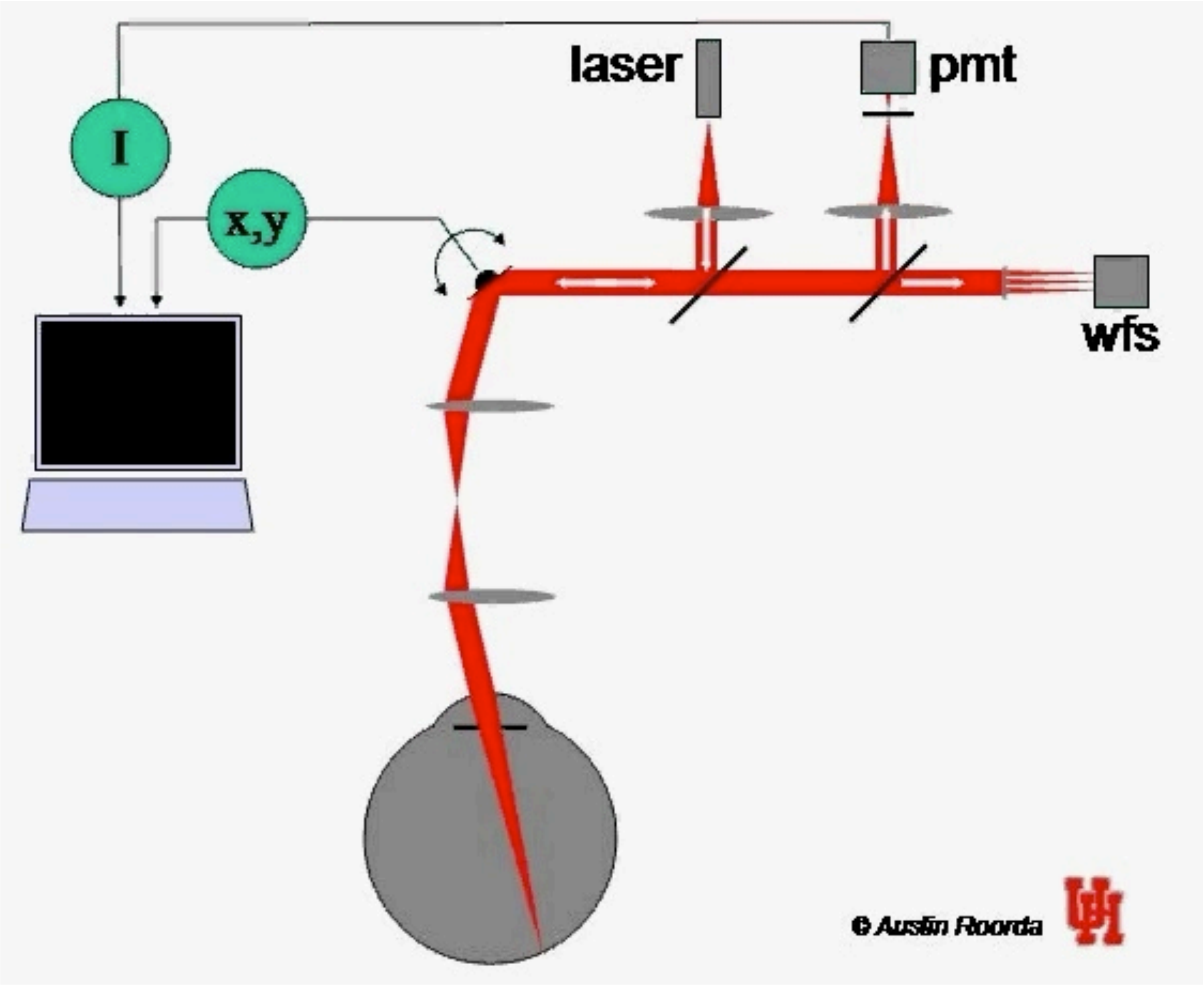






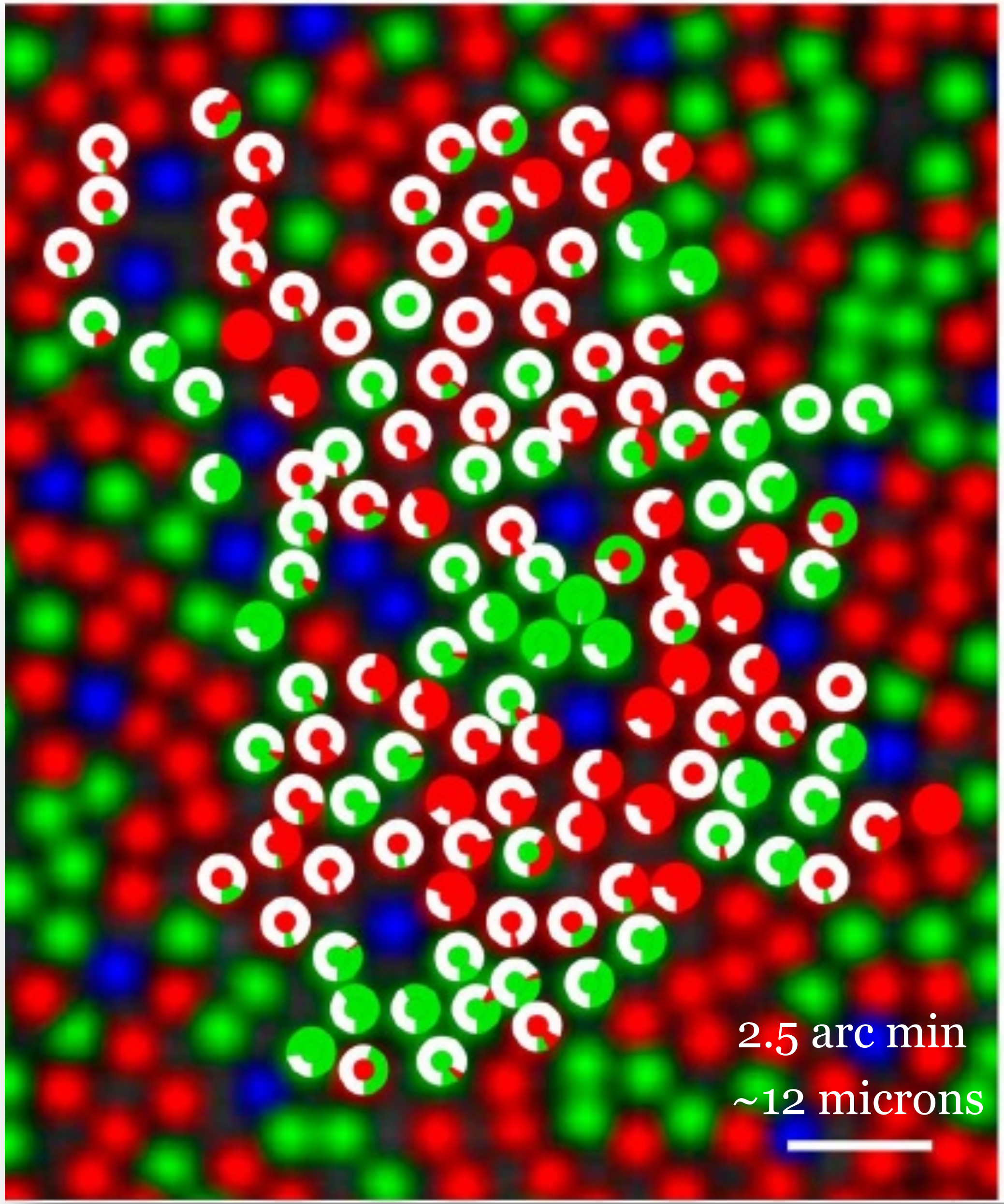
Overview of AO-SLO

It is also possible to draw images on the retina by combining a scanning laser ophthalmoscope (SLO) with the wavefront sensor



Color names when single cones are stimulated on a gray background

- Tracking eye position as well (video version of the system) it is possible to target individual cones for stimulation.
- Subjects name the color appearance of these stimulations, which differ on different stimulations
- A very common response is ‘white’; there are M cones that evoke ‘red’ response and other L cones that reliably evoke a ‘green’ response.
- Some cones evoke red or green responses (the stimulating light is always the same, 543 nm).



A second subject

- Tracking eye position as well (video version of the system) it is possible to target individual cones for stimulation.
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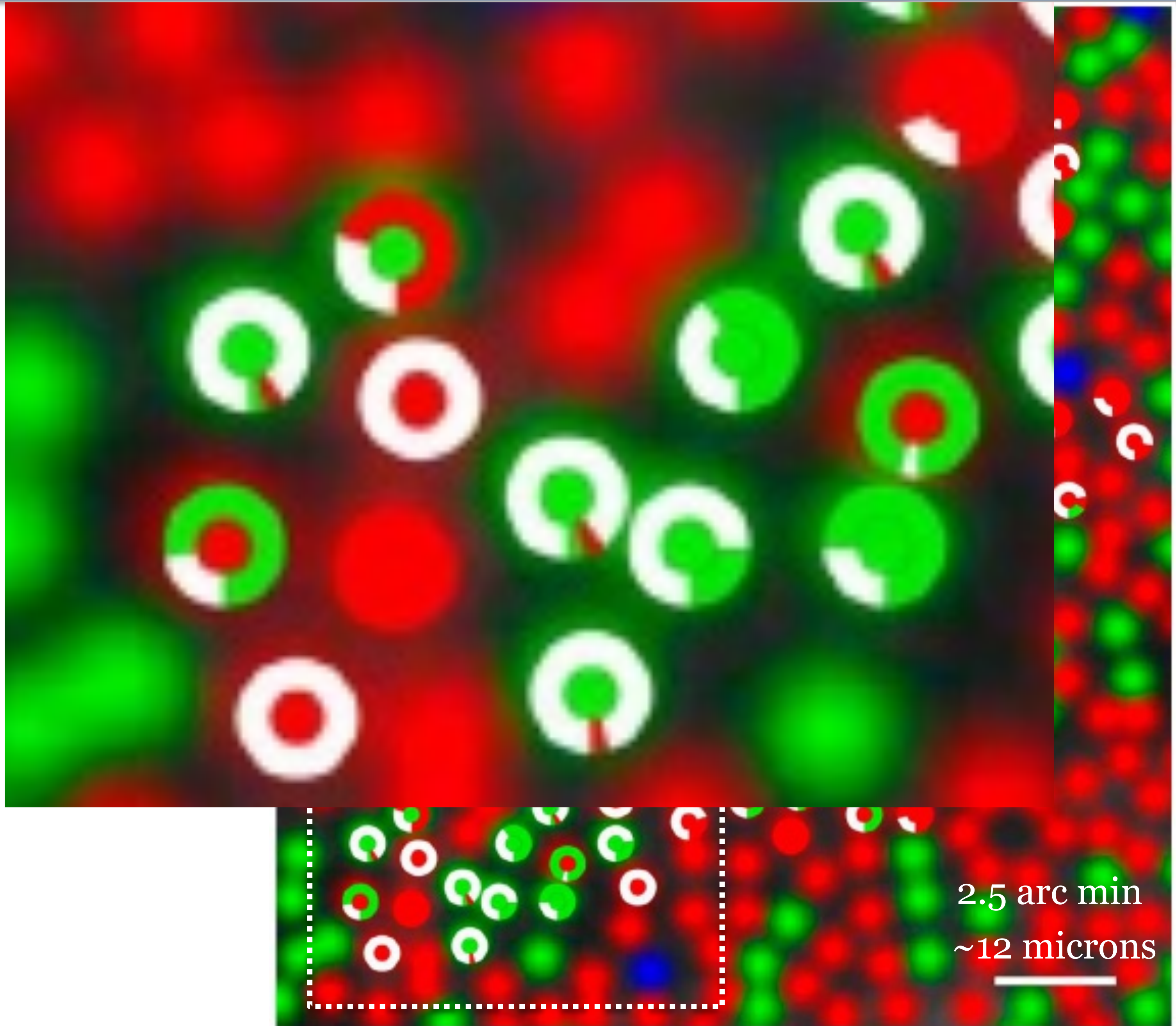
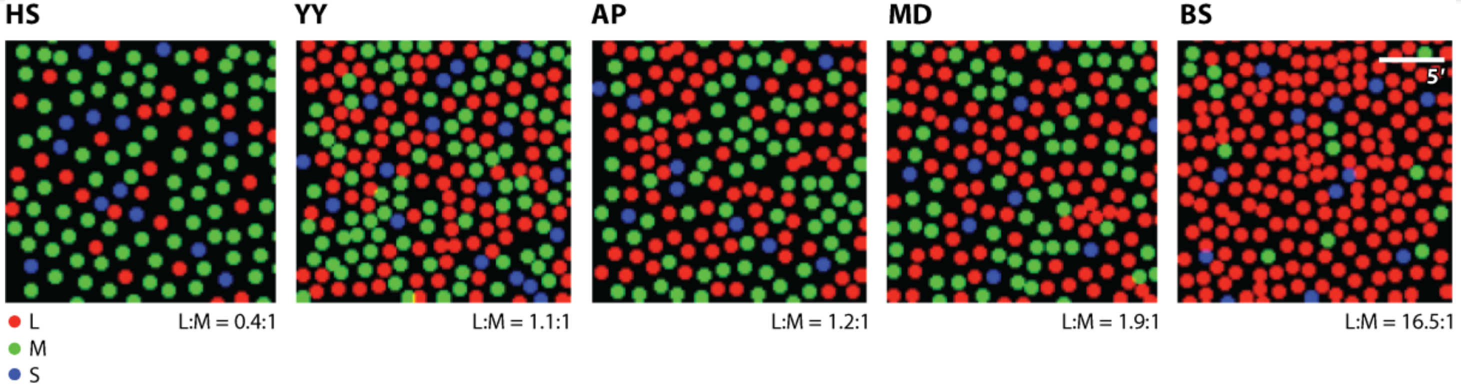
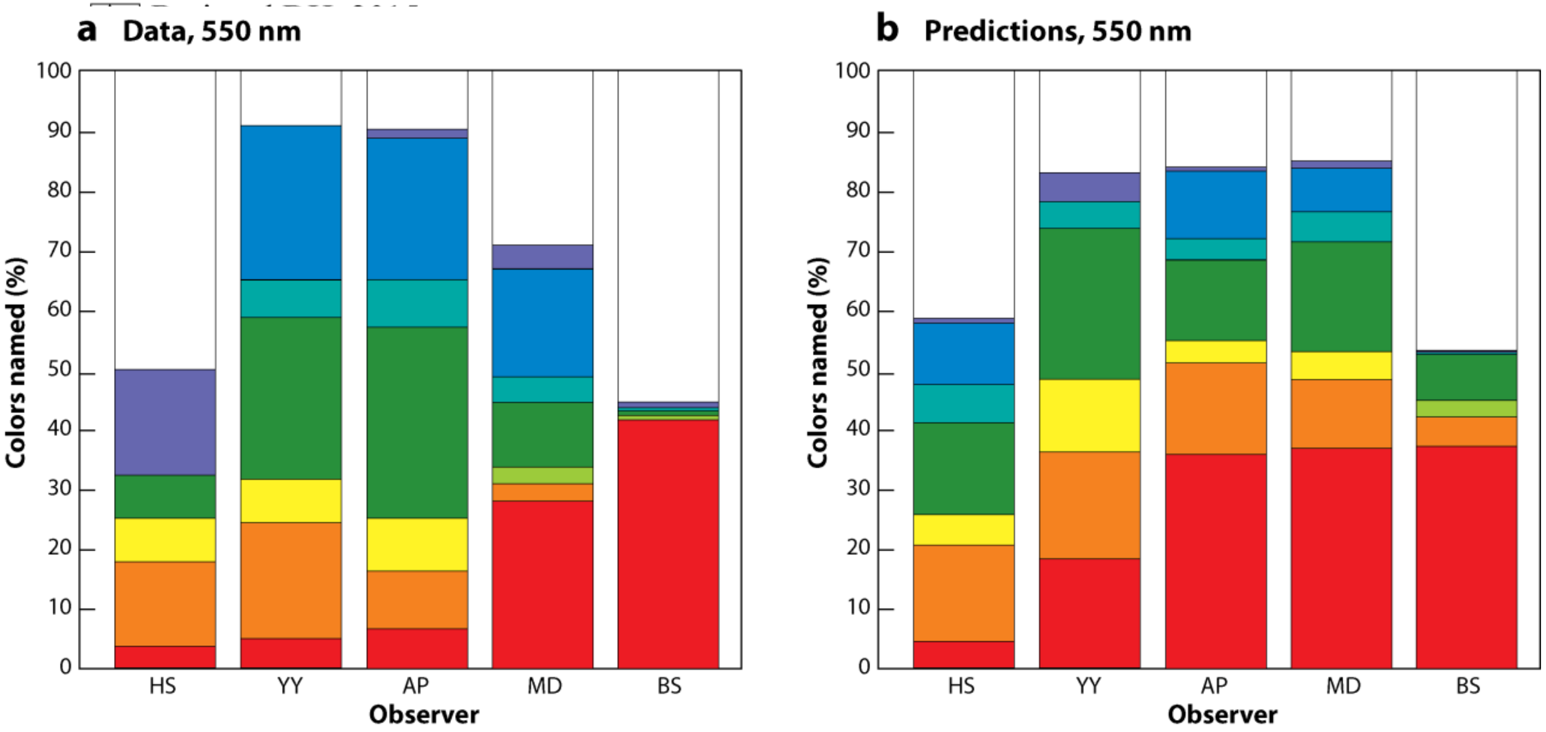


Color names to single cone excitations for 5 observers with very different mosaics

- People differ enormously
- Related experiments by Hofer et al. suggest that people would use more color names in response to single cone excitation

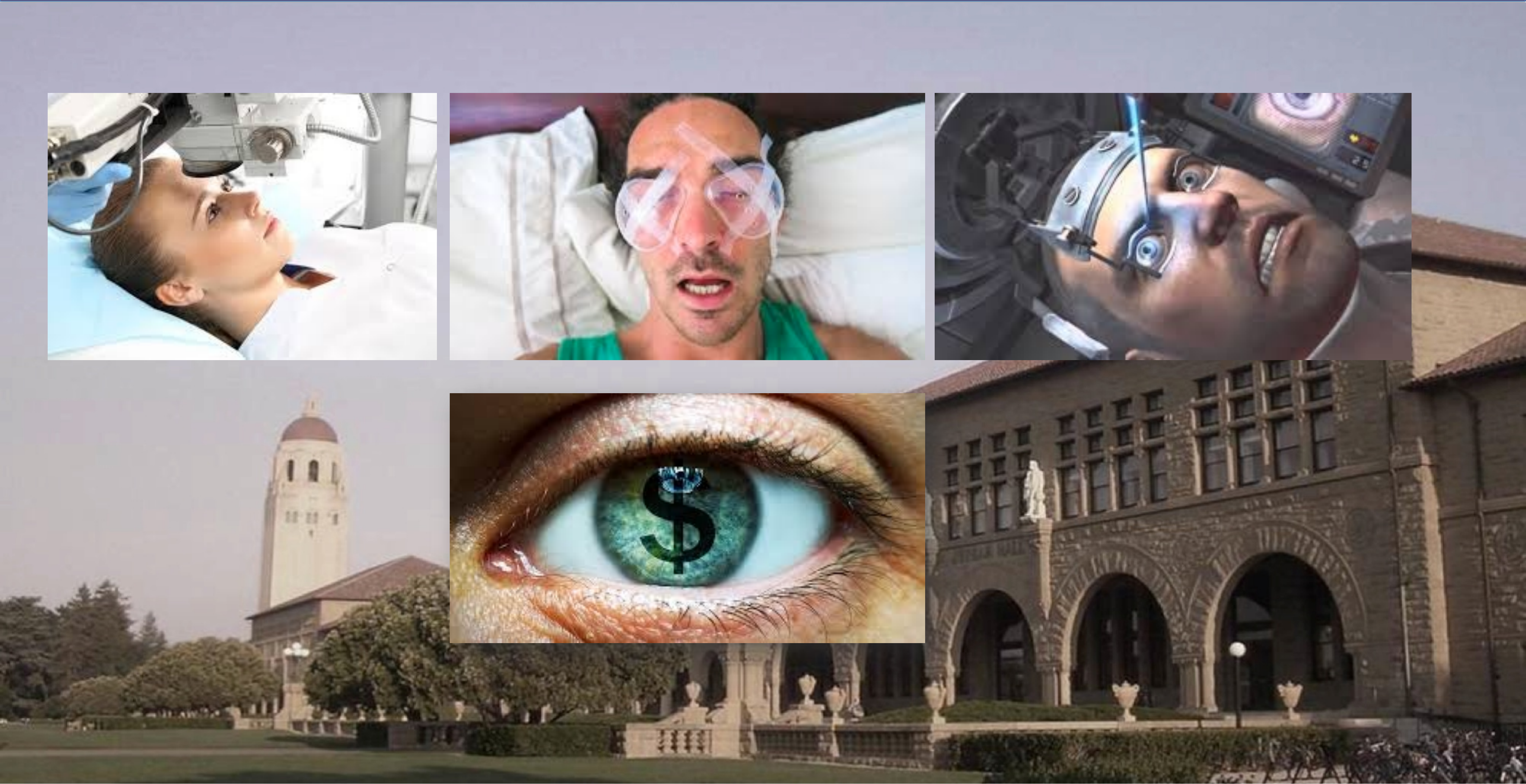




Brainard DH. 2015.
Annu. Rev. Vis. Sci. 1:519–46

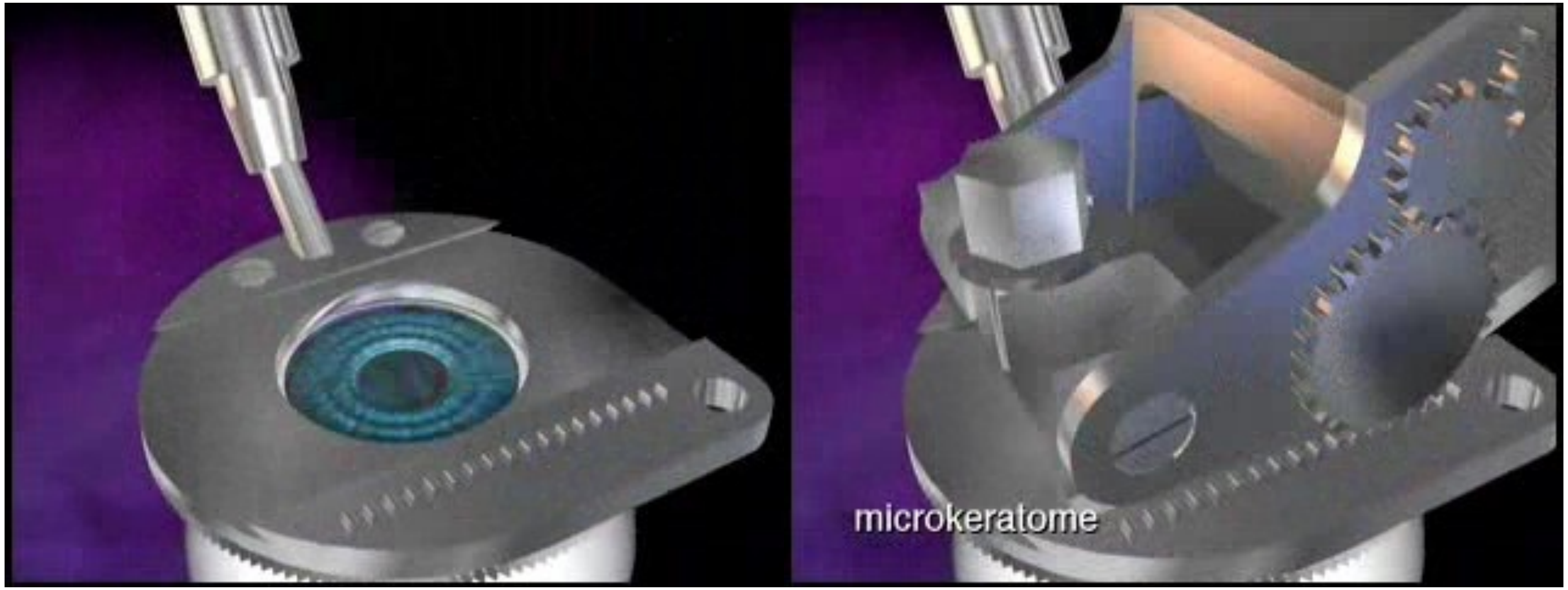
Data from Hofer, Williams and Singer 2005

Laser eye surgery



Laser eye surgery

1. Measure the wavefront aberration
2. Cut a thin flap in the cornea (microkeratome)

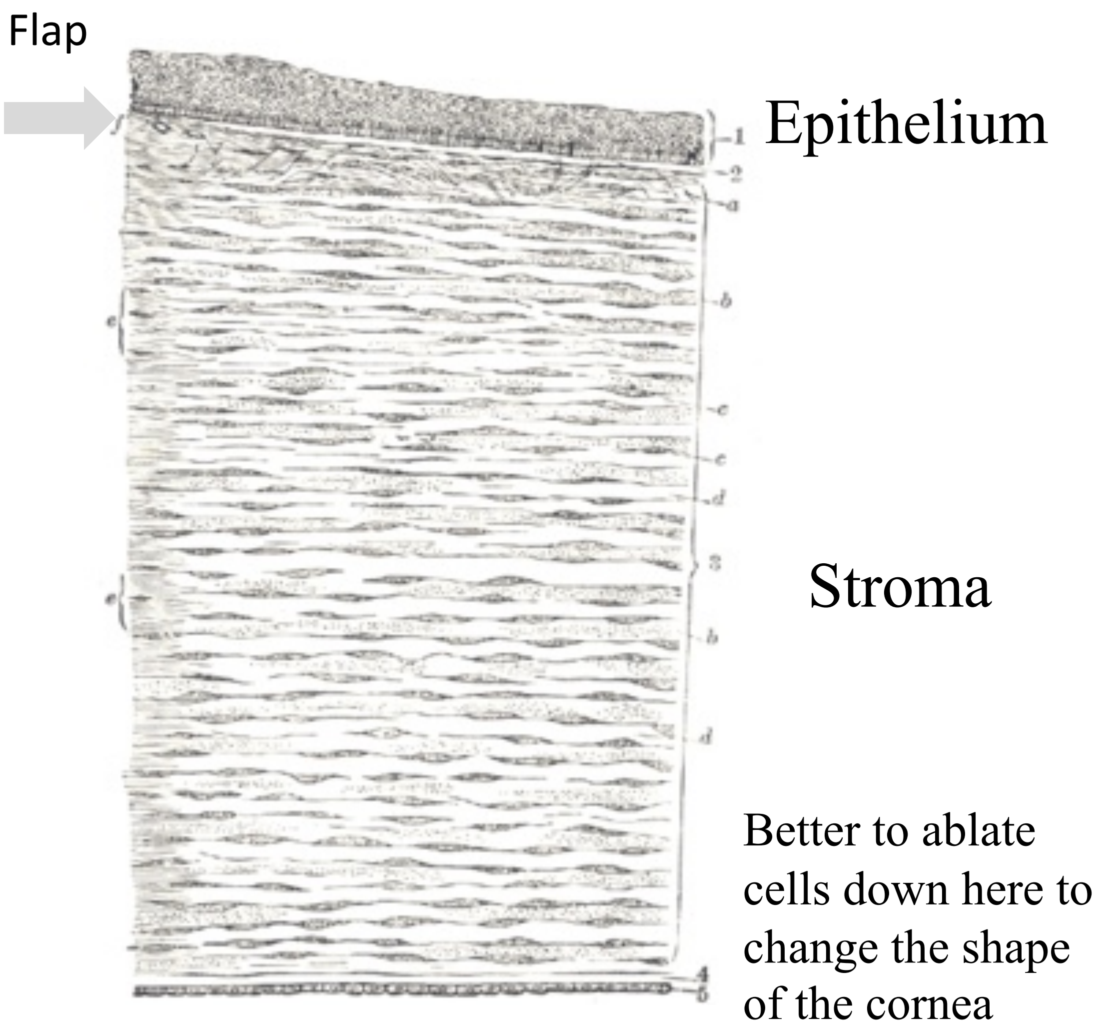


Application: Laser eye surgery

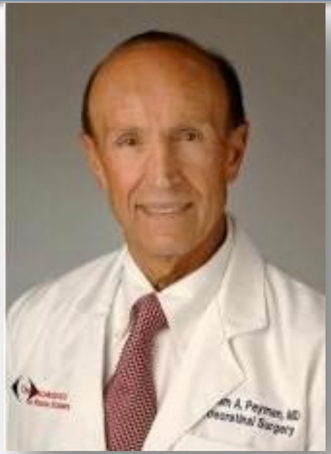
1. Measure the wavefront aberration

2. Cut a flap in the cornea (microkeratome)

3. Excimer laser (193 nm) for ablation of cells and reshape the cornea



The 1989 Lasik patent



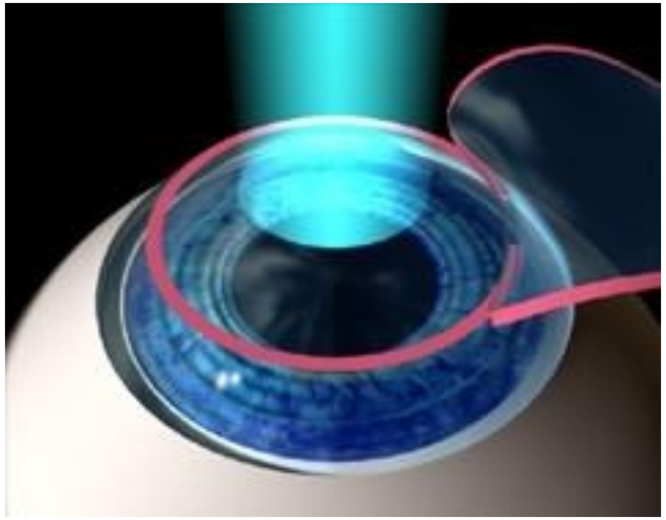
Gholam A. Peyman

"A method and apparatus for modifying the curvature of a live cornea via use of an excimer laser. The live cornea has a thin layer removed therefrom, leaving an exposed internal surface thereon. Then, either the surface or thin layer is exposed to the laser beam along a predetermined pattern to ablate desired portions. The thin layer is then replaced onto the surface. Ablating a central area of the surface or thin layer makes the cornea less curved, while ablating an annular area spaced from the center of the surface or layer makes the cornea more curved."

US patent for LASIK (US4840175)

Laser corrective eye surgery

An excimer laser (193 nm) reshapes the surface of the corneal stroma to improve the wavefront – the tissue (10's of microns) is vaporized with a femtosecond pulse

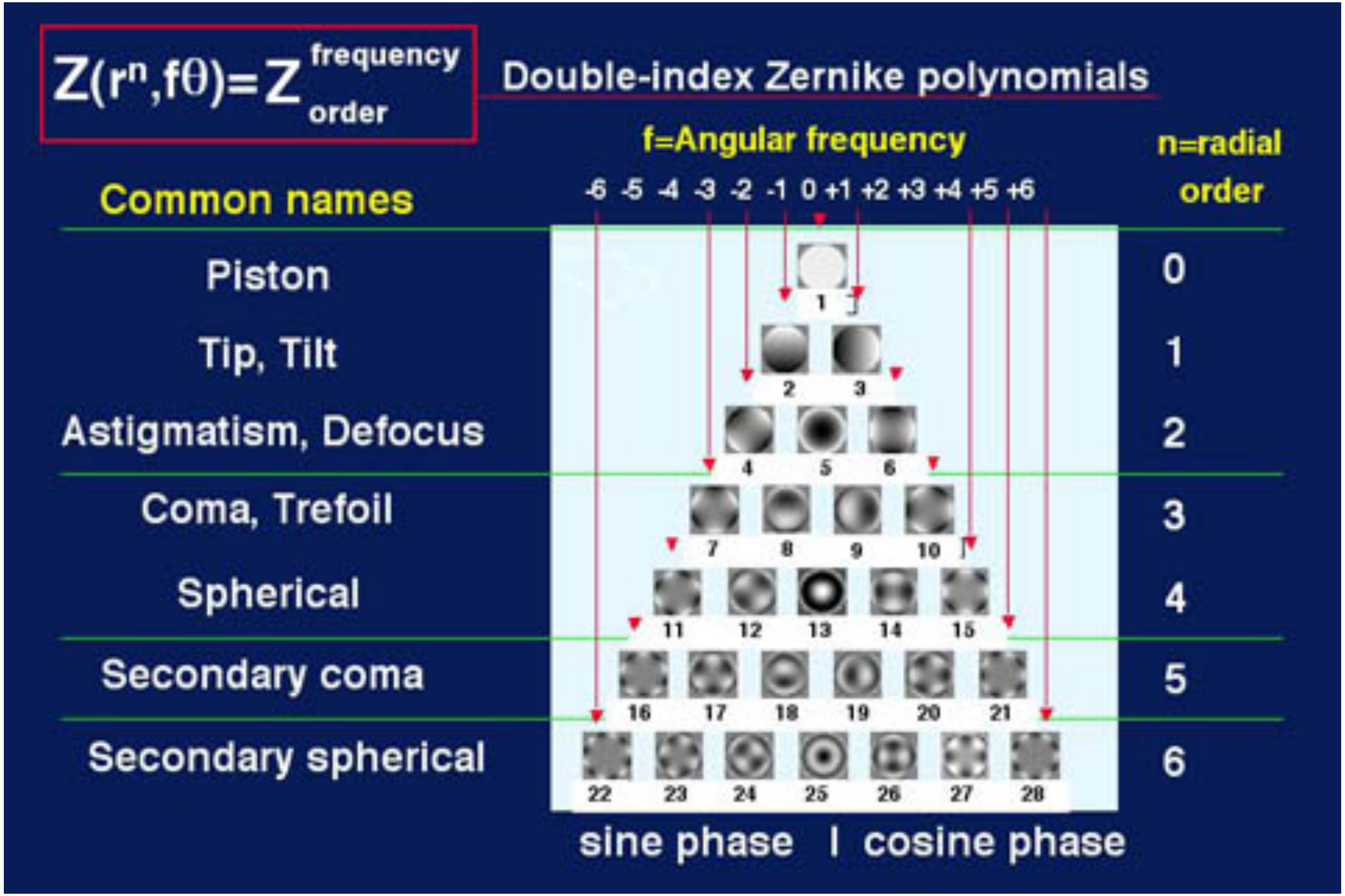


Performing the laser ablation in the deeper corneal stroma typically provides for more rapid visual recovery and less pain than the earlier technique, photorefractive keratectomy (PRK).

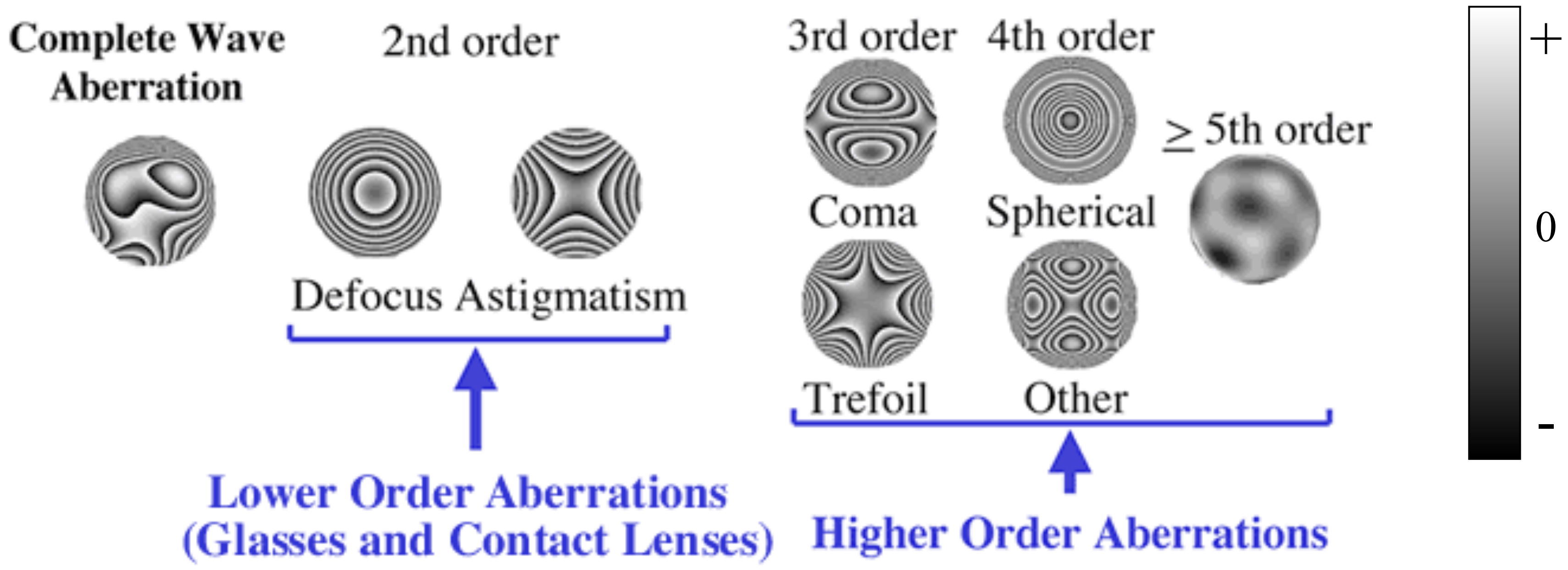
Wavefront characterizations

The weighted sum of an orthogonal set of polynomials over the unit circle (Zernike)

Zernike polynomials

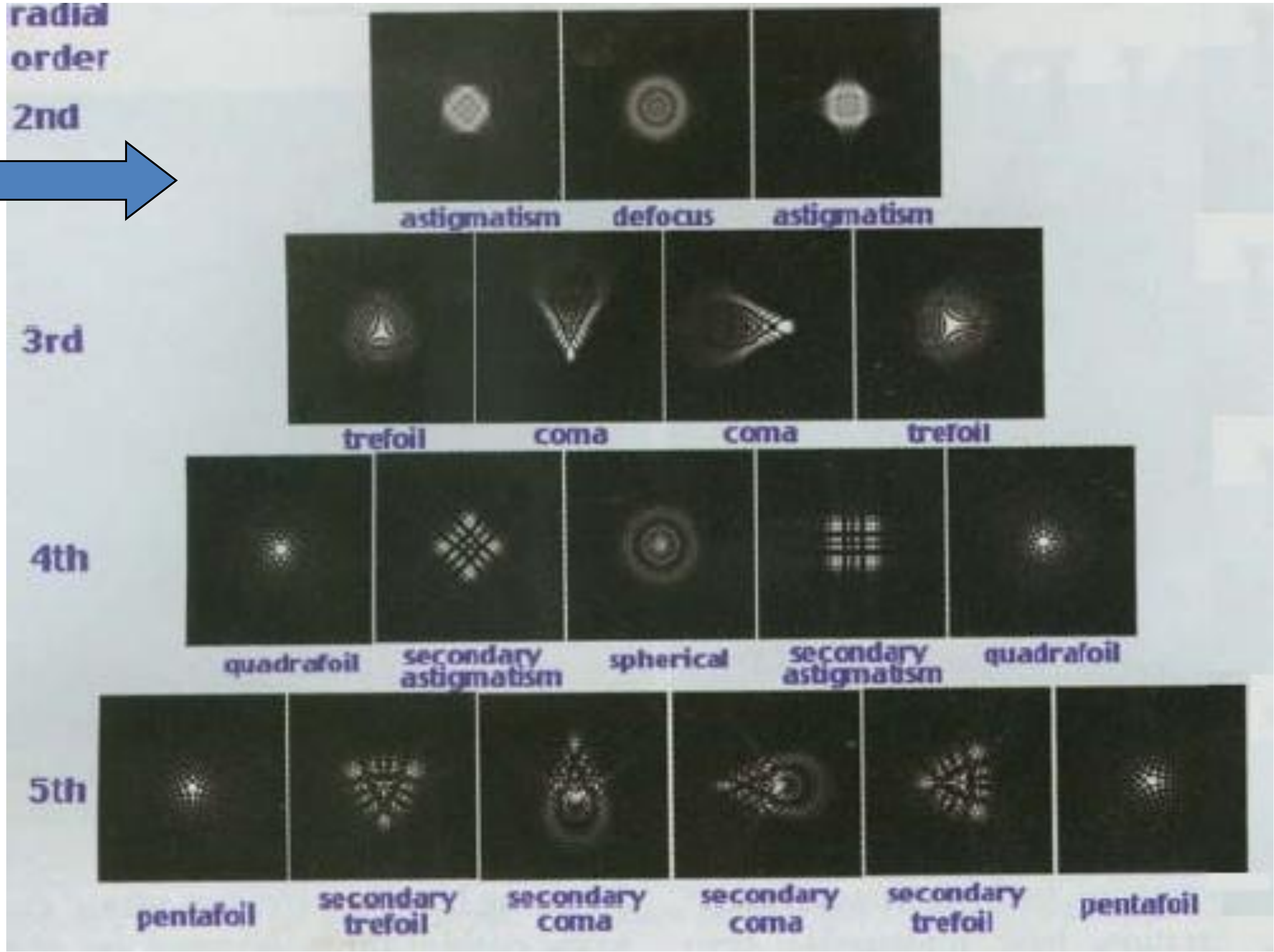


Visualization of Wavefront Error



Common point spread functions

Lower order are typical and diagnosed easily



Because the likely aberrations are known, it is possible to estimate the aberrations from the pointspread. Theoretically – not so much. But practically, could be OK (Grissan et al., 2007). Compressed sensing?

Wavefront diffraction at the beach

[Online tutorial about water waves and diffraction]

Useful online sites for DOF, MTF and lens information

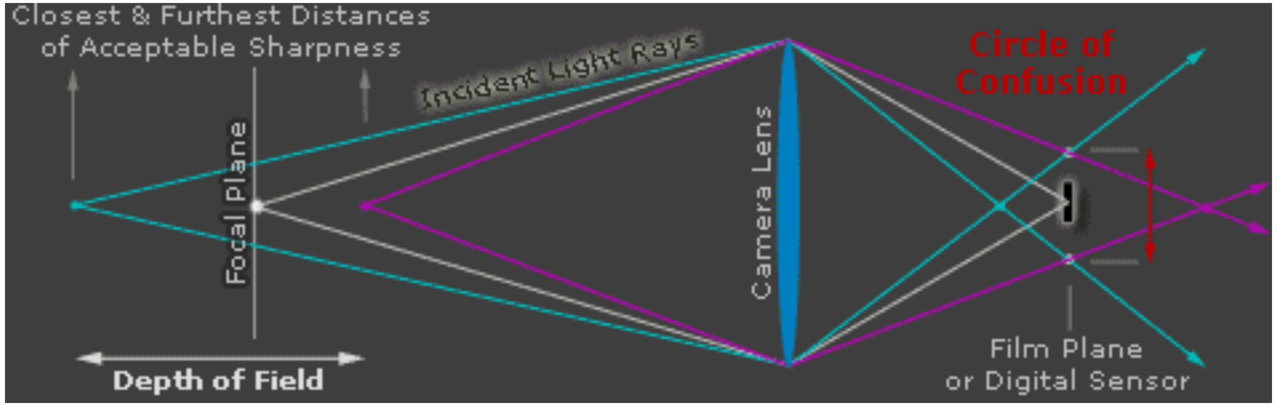
http://www.cambridgeincolour.com/tutorials.htm

http://photo.net/learn/optics/lensTutorial

http://www.normankoren.com/Tutorials/MTF6.html#DOF_focal_length

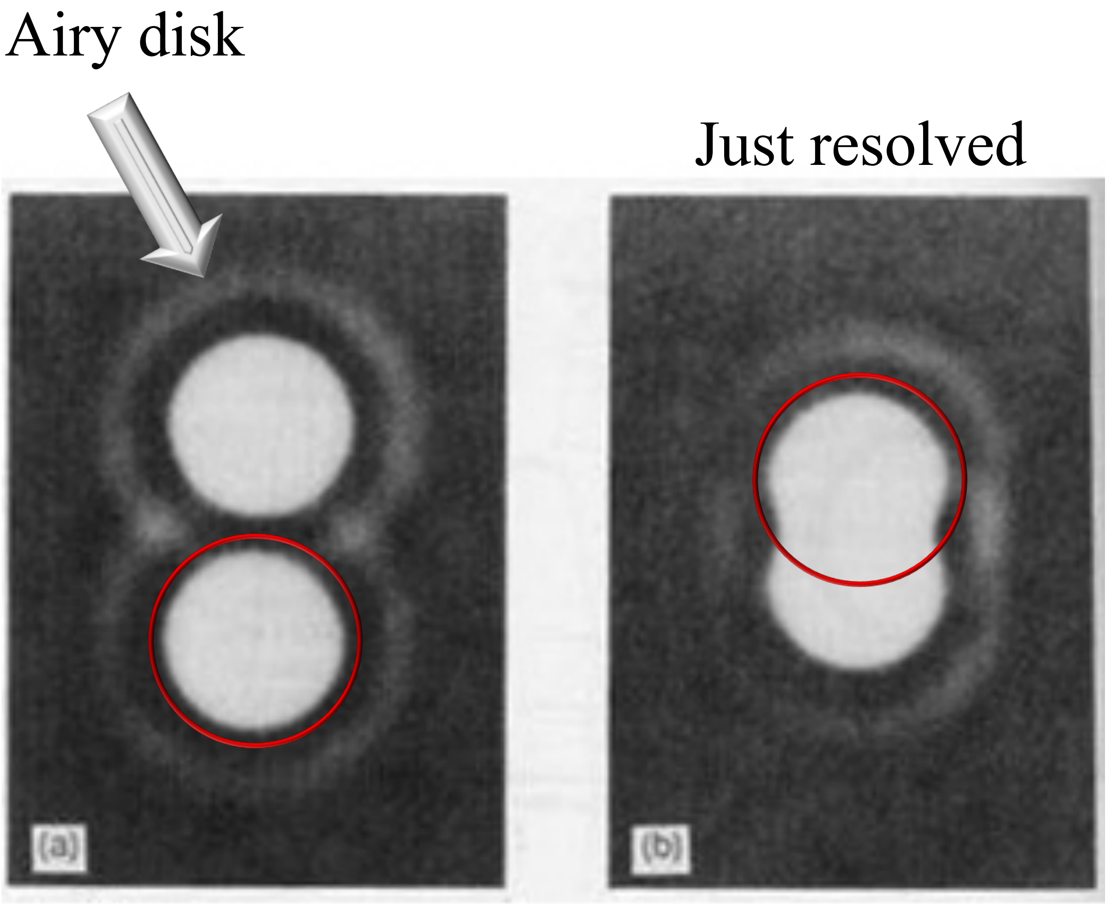
http://www.normankoren.com/Tutorials/MTF6.html#DOF_focal_length

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm



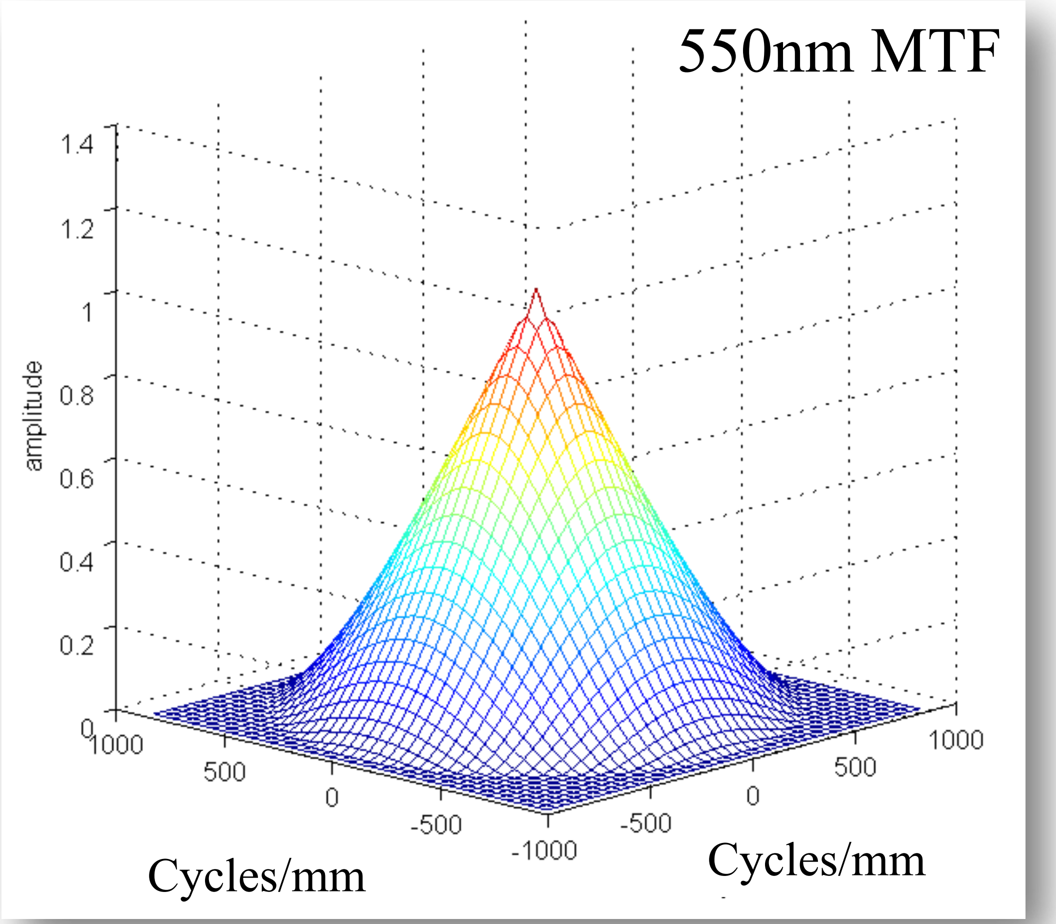
Angular resolution criterion for diffraction

- **Rayleigh criterion** is a measure of spatial resolution
- Two point sources are "just resolved" when the **diffraction maximum** of one image coincides with the **first minimum** of the other

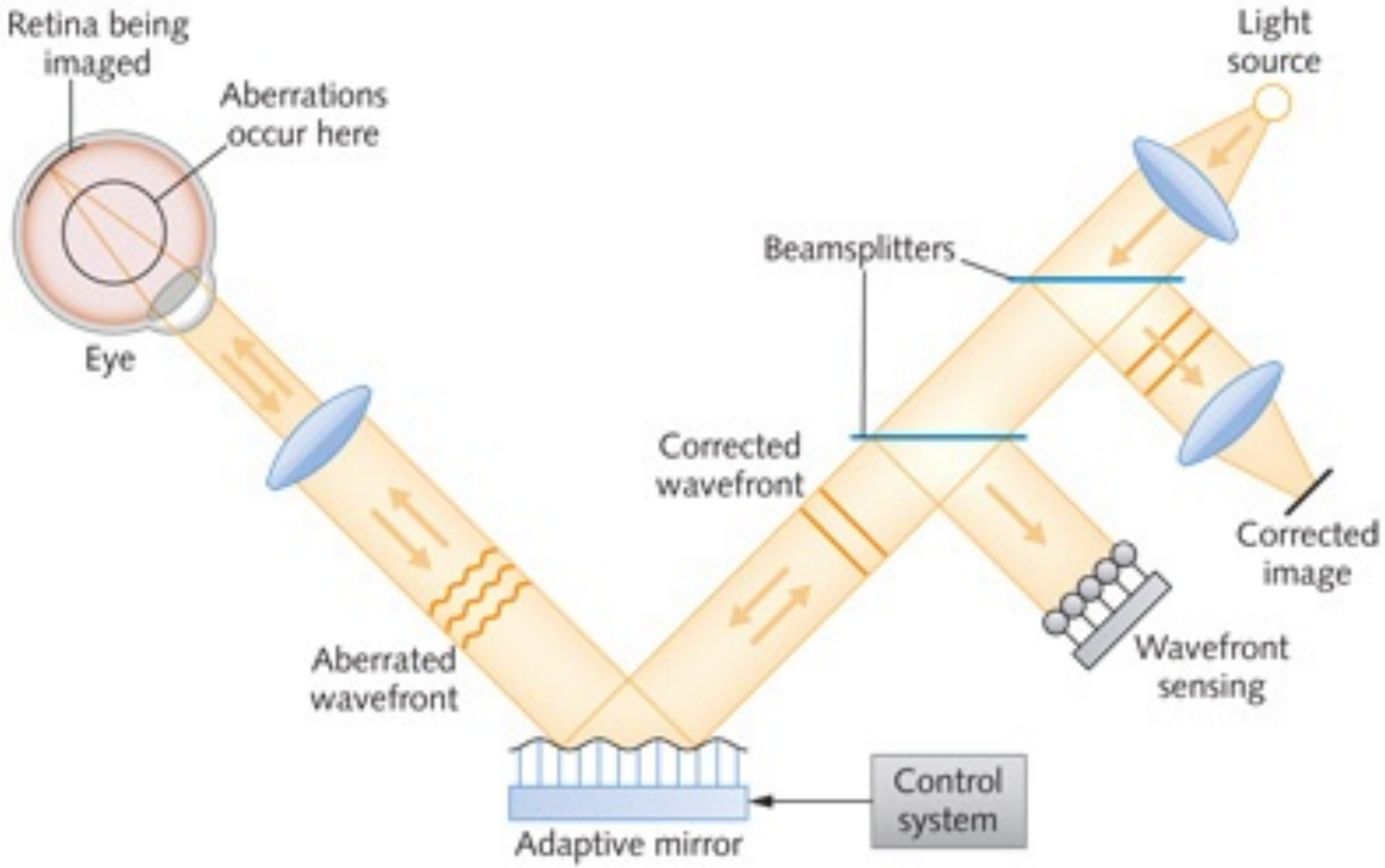


The diffraction pattern formula for a disk can be calculated from first principles

Diffraction limited OTF

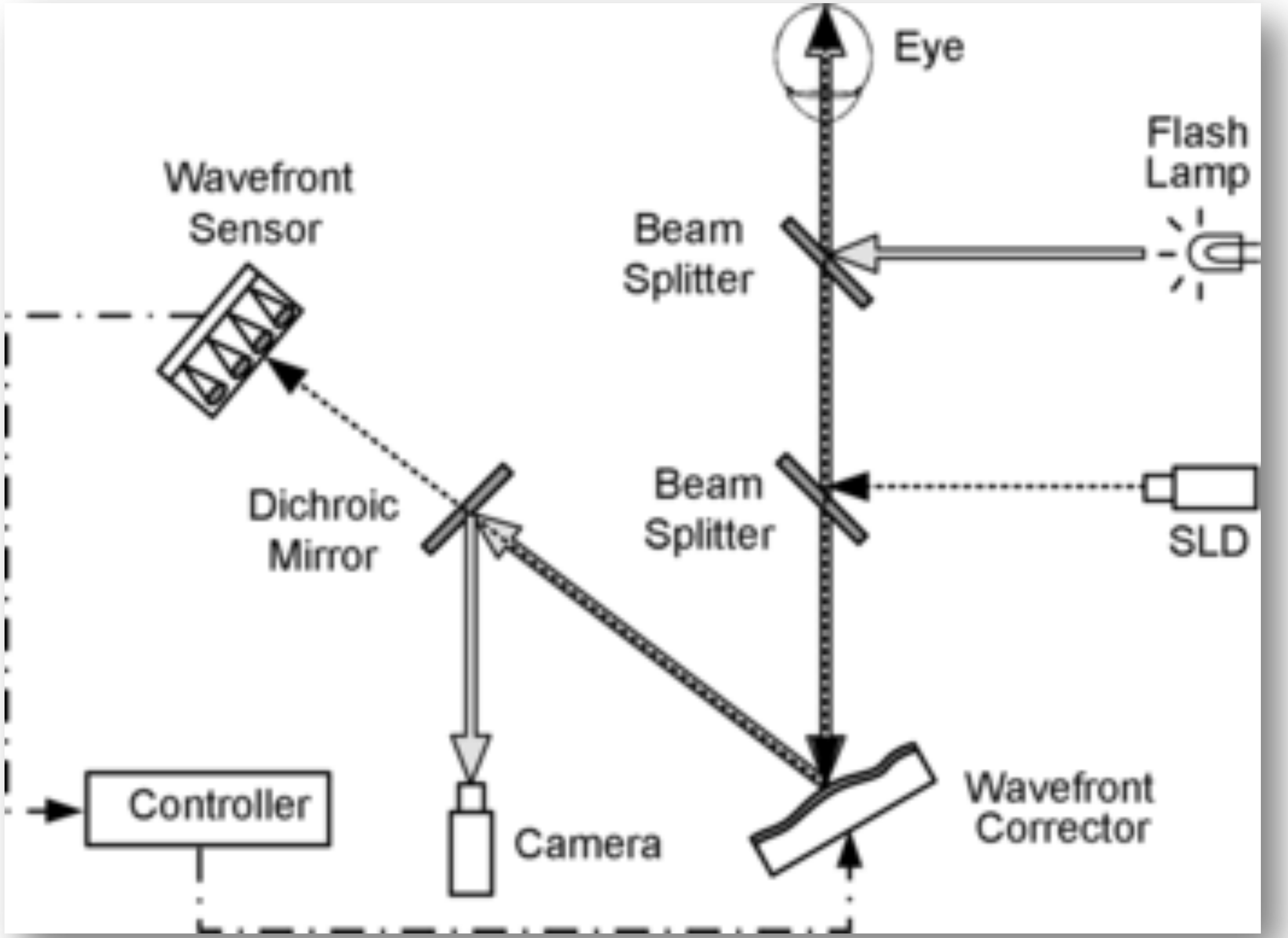


Adaptive optics instrumentation

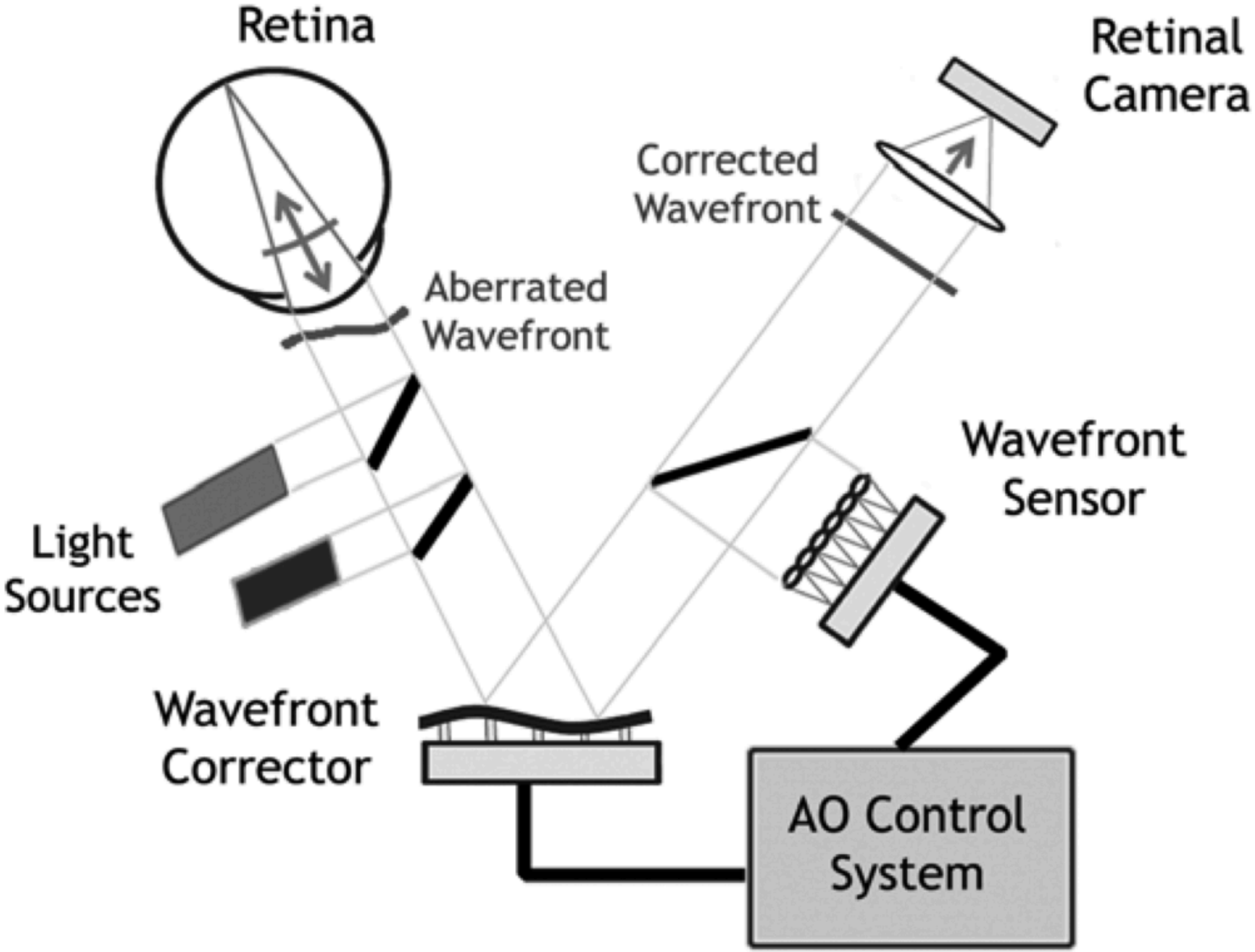


Laser Focus World, volume 49,

(B)

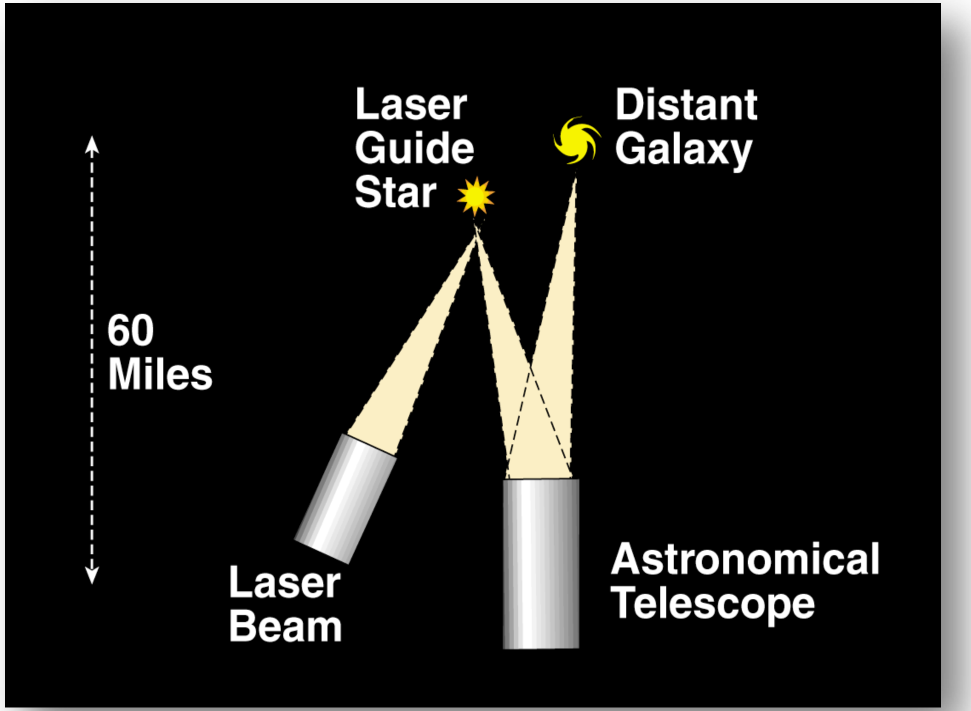


Super luminescent diode

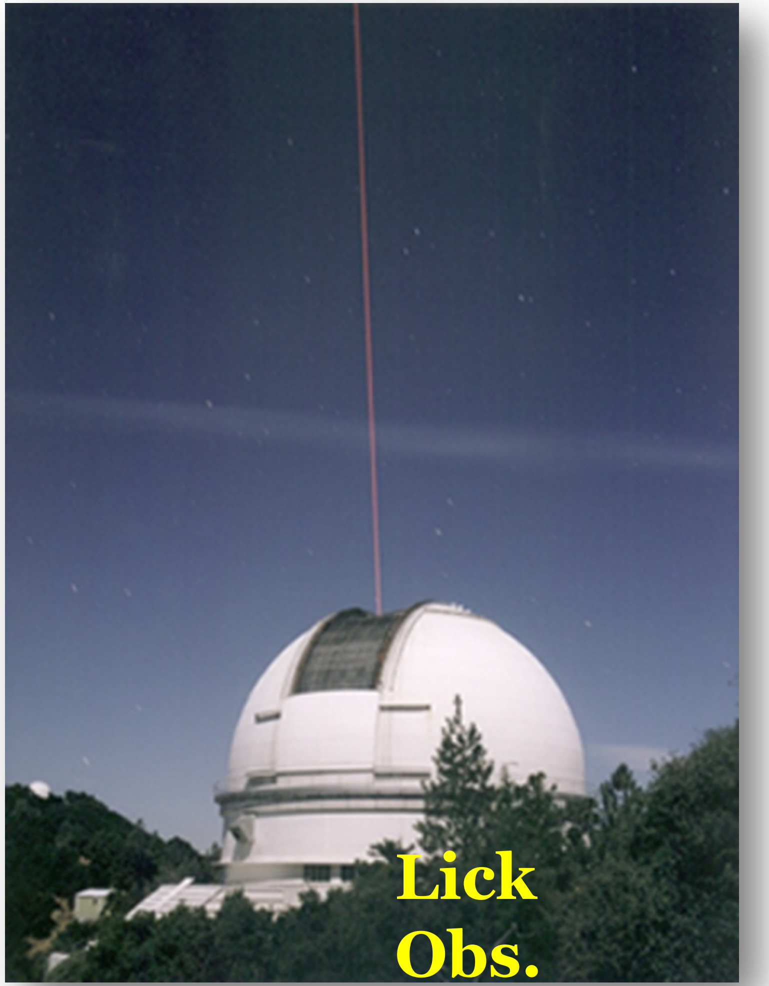

Retina
Retinal Camera
Corrected Wavefront
Aberrated Wavefront
Wavefront Sensor
Light Sources
Wavefront Corrector
AO Control System

If there is no nearby star, make your own "star" using a laser

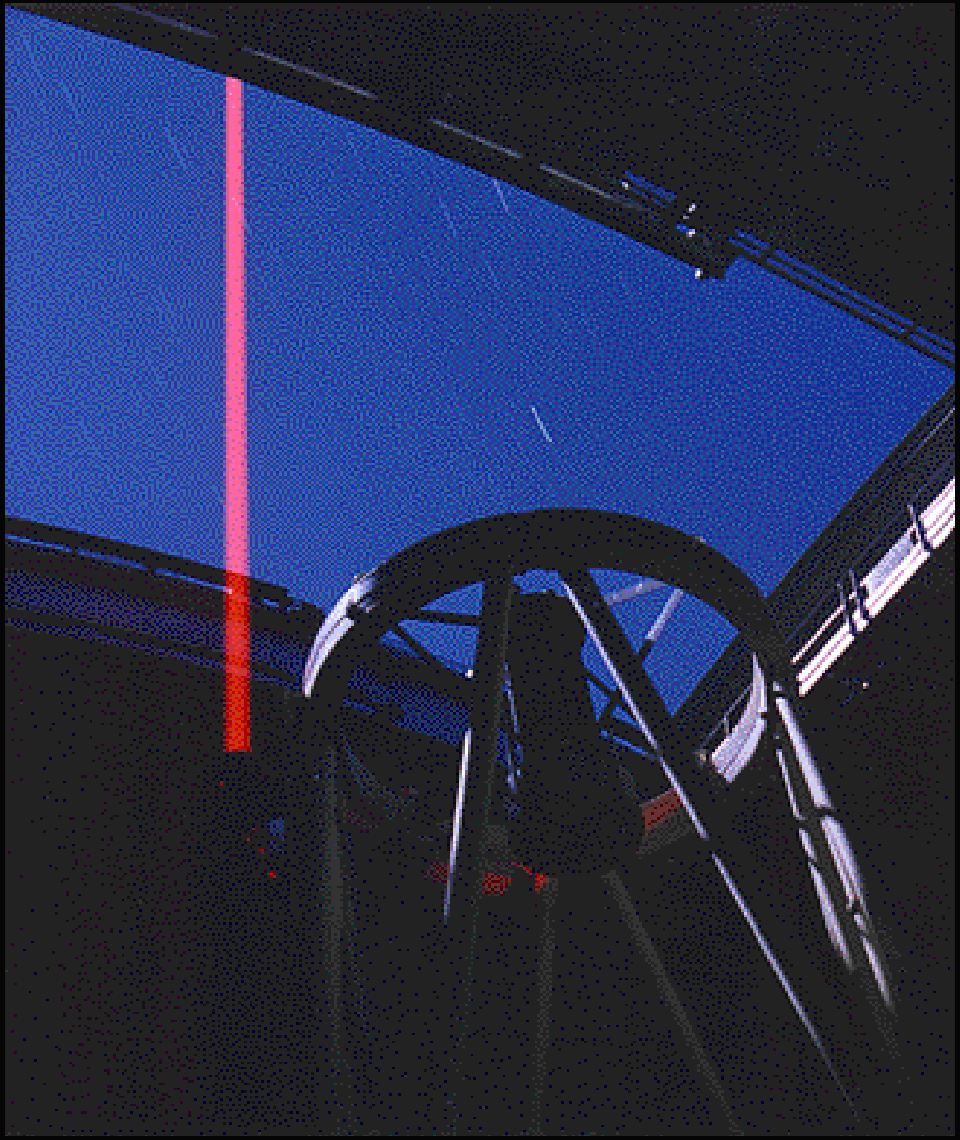
Concept

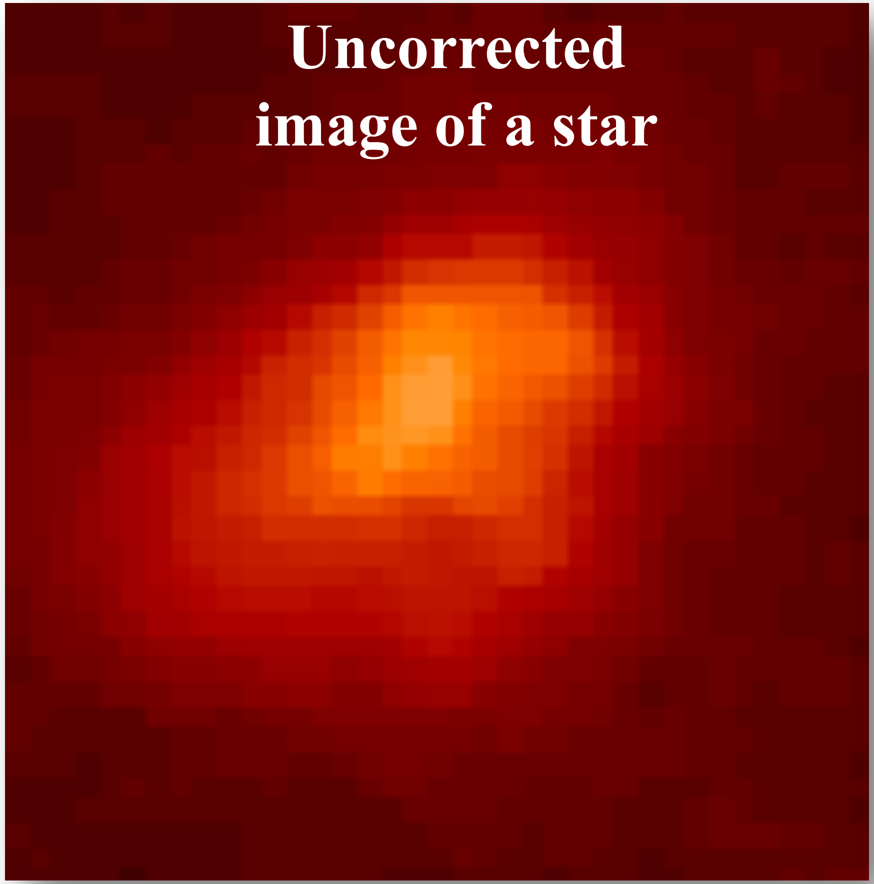


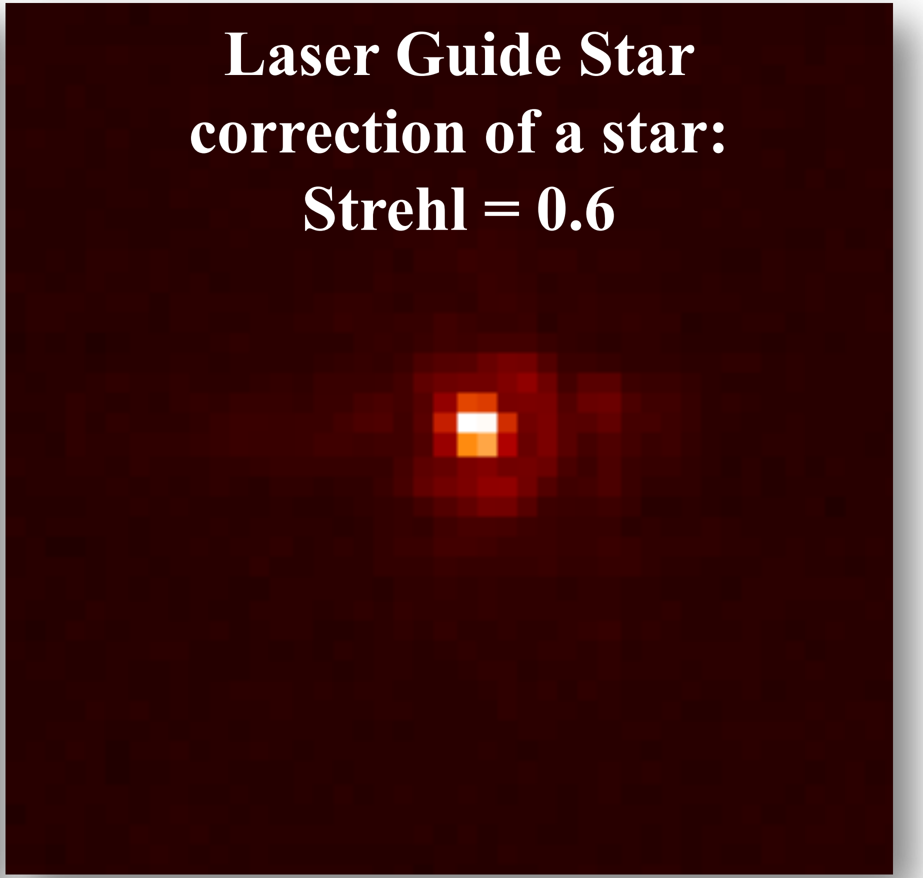
Implementation



Laser in 120-inch dome







Ircal1129.fits RX J0258.3+1947 10/20/00 2:04 Ks V=15 K=~13.32 20s S=0.6 LGS

Spatial blurring Diffraction-limited OTF example

$$OTF = \begin{cases} \frac{2}{\pi}\left[\operatorname{acos}(\rho) - (\rho\sqrt{1-\rho^2})\right] & (\rho<1) \\ 0 & (\rho \geq 1) \end{cases}$$

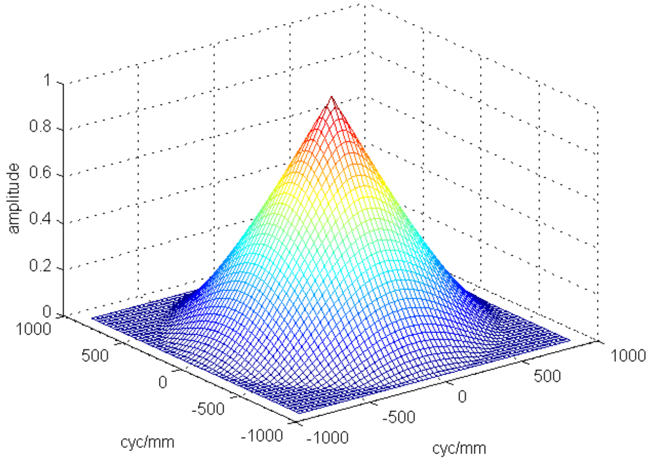
$\rho =$ f/(A/dλ) (normalized frequency)

f = frequency in cycles/meter

A= aperture diameter (m)

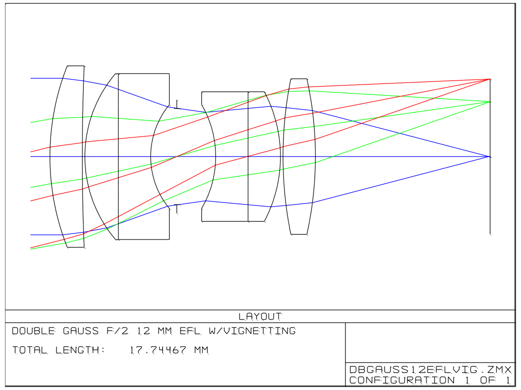
d = distance from aperture to detector

λ= wavelength

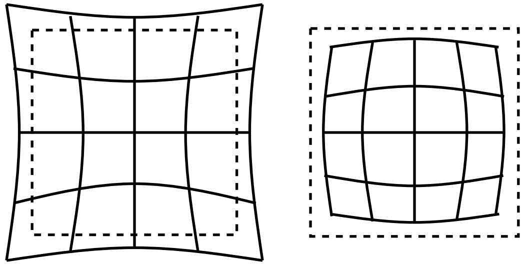


OTF at 550 nm
F-number: 2
Focal Length: 3.86 mm

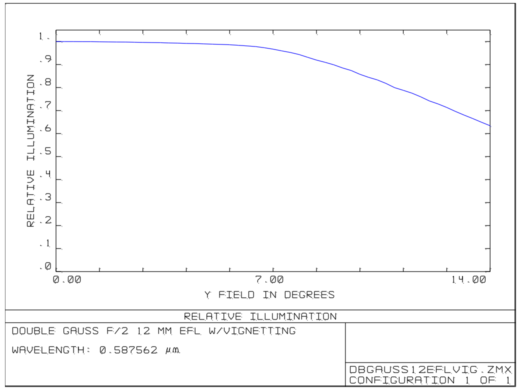
Real lenses are not diffraction limited and have geometric distortions



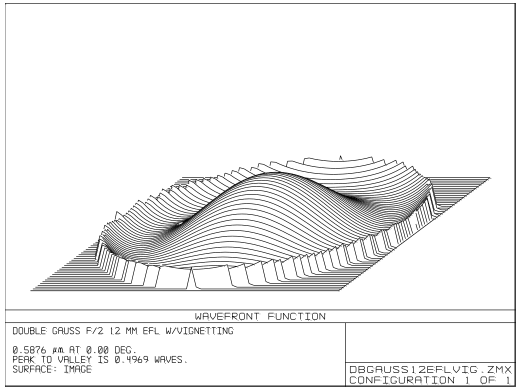
Vignetting



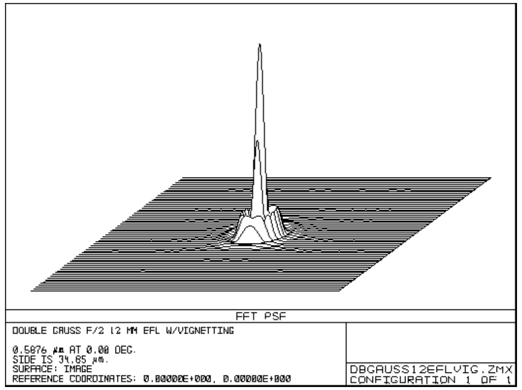
Distortion



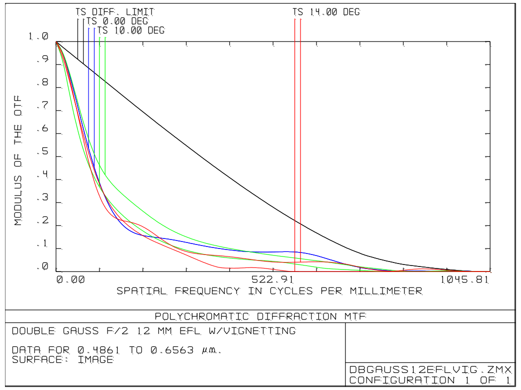
Illumination Effects



Wavefront Aberrations



Point Spread Function (PSF)



Modulation Transfer Function (MTF)