Assignment 1

Due 5:00pm Friday Jan 8

July 18, 1421, Florence Italy

Capomaestro:

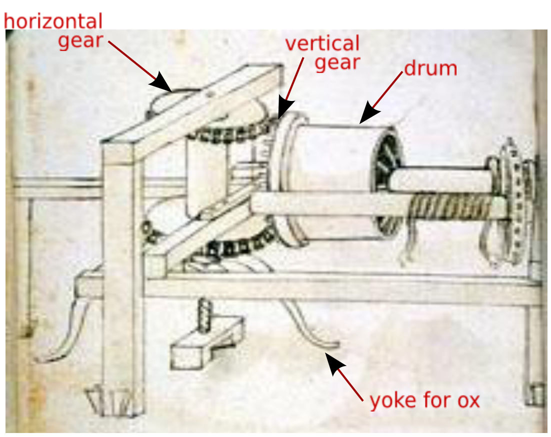
Complimenti! The Opera del Duomo[1] *awards you 100 Florins*[2] *for your designs for machinery to construct the cupola of the cathedral. The Opera is impressed by the innovative features shown in the scale model that you submitted for the competition:*

- *three lifting speeds for large, medium and small loads,*
- *a reversing mechanism to avoid un-harnessing and turning the oxen around at the end of each lifting session.*

We look forward to the construction of your machinery and timely completion of the cupola. The forester, Sr. M. di Bruogio, will deliver a first installment of lumber to your studio next week.

cordiali saluti,

L'Opera del Duomo



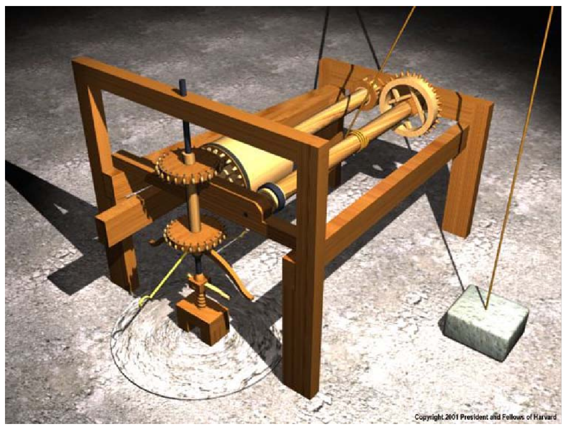


Figure 1: Left: Drawing of the hoist by Buonacorso Ghiberti (nephew of Lorenzo) circa 1450 [http://www.museogalileo.it]. Right: CAD reconstruction of hoist [Harvard Libraries].

Congratulations! You won the competition to build machines for completing the cathedral with your ingenious model (Fig. 1) of an ox-driven hoist. The usual practice in Renaissance times is to create full sized machines by scaling up directly from models. But this is truly an unprecedented machine and something makes you suspect that simply scaling it up will not suffice.[3] It would be wise to do some checks. Nothing would thrill your archrival, Lorenzo Ghiberti, and his backers more than an embarrassing failure of your machine!

[1] An "Opera" is a committee of prominent folk responsible for overseeing maintenance and improvements of a major work, in this case the Duomo, or cathedral.

[2] roughly equivalent to $60,000 today.

[3] Galileo in 1638 pointed out the error of scaling up from models: strength increases as L^2 but weight goes as L^3.

Fortunately, unlike anybody else in the 1400s, you have an understanding of free body diagrams and stresses in shafts and beams. For convenience, you use MKS units instead of *braccia* (arm's length) units.

Enter your answers to the following questions by going to the course website. There is some tolerance to allow for round-off. The assignment is called a "Quiz" in the website software. You can re-take the "Quiz" as many times as you like; only the highest score will be saved.

Part I. Speed and Torque Ratios

A useful starting point is the short online video "Gears 1" on the course website[4] (suggested viewing speed = 1.5x).

Problem 1 (warmup)

Suppose that we have a "compound gear train" as shown in the video, with four gears having the following pitch radii: $r_1 = 5\text{mm}$, $r_2 = 20\text{mm}$, $r_3 = 5\text{mm}$, $r_4 = 15\text{mm}$, where the input is *gear 1* and the output is *gear 4*.

1. What is the speed ratio from input to output? The convention is that we write $N : 1$ if the input rotates N times per rotation of the output; conversely we write $1 : N$ if the output rotates N times faster than the input.

2. If 0.1 Nm of torque is applied at the input, what is the output torque?

Problem 2

In Brunelleschi's machine (Fig. 1), two oxen are yoked to a vertical shaft, which has two identical horizontal gears, either of which can be engaged, depending on whether we are raising or lowering a load. To review the design of the machine with some video clips, follow the links on the course website.

Let *Gear 1* be either of the large horizontal gears on the vertical shaft; let *Gear 2* be the mating large vertical gear that is attached to the drum and the first horizontal shaft. At the far end of the first horizontal shaft is another gear, *Gear 3*, which meshes with *Gear 4* on the secondary horizontal shaft. The pitch diameters are as follows: $D_1 = 2.7\text{m}$; $D_2 = 3.3\text{m}$; $D_3 = 1.2\text{m}$; $D_4 = 2.45\text{m}$.

1. In the terminology of the video, do we have a *series* or *compound* gear train?

2. If the angular velocity of the vertical shaft and *Gear 1* is ω_1 radians/second, what is the angular velocity of *Gear 4* and the secondary shaft in terms of ω_1?

3. The two oxen walk around in a circle pulling on their yokes, each with a force of $F_{ox} = 700\text{N}$ at a steady speed of $V_{ox} = 0.75\text{m/s}$. What is the corresponding input power in watts? For comparison, a hair dryer on "high" setting is typically 1200 to 1800 watts and one horsepower is about 746 watts.

4. Assume that the transmission between the input (oxen) and the output (a rope wrapped around the secondary shaft) is 70% efficient. What is the output power in watts?

[4] Also viewable directly from YouTube: https://www.youtube.com/watch?v=LlYrDFFJVuI

5. What is the corresponding maximum speed, V_r, in meters/second that we could lift a huge block with a mass of 1200 kg? (Note that this is the maximum possible speed; the actual speed will be somewhat slower.)

6. At this speed, how many minutes would it take to raise the load 114 m to the top of the cupola?

Part II. Forces and Stresses

Notice that to answer the previous questions we did not need to know details like shaft diameters or gear tooth dimensions; it sufficed to consider input and output power. Now, however, we want to investigate some of the components to be sure they are strong enough.

Consider the first horizontal shaft in Fig. 1 – the one with a vertical gear (*Gear 2* in the previous problem) that is driven by either the upper or lower horizontal gear and has a large drum around which a rope can be wrapped. (In Fig. 1, the rope is shown wrapped around the secondary shaft.) Let us assume we are using the rope on the drum, for lifting small loads at maximum speed.

A detail of the first shaft is shown in Fig. 2 with a rope on the drum. The rope tension is F_r. At the opposite end of the shaft, *Gear 3* (assumed in this drawing to be a "lantern gear" with spindles)[5] drives the secondary shaft. An upward force F_3 is required at this gear to keep the secondary shaft turning even when it is not in use. F_3 can be taken as a concentrated force half-way along one of the lantern spindles, as shown.

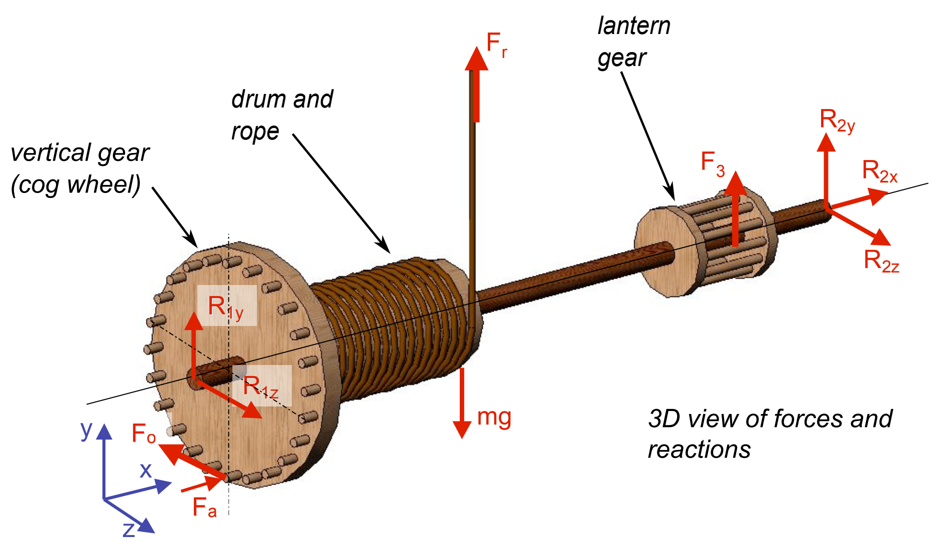


Figure 2: Detail of primary horizontal shaft, showing rope wrapped on drum. F_3 is the force exerted on the lantern gear by the gear it meshes with.

The primary shaft has a mass of $m = 250$ kg including the gears and drum. The center of mass is approximately at the same position along the shaft as the upward force F_r from the rope. Input torque is provided by the vertical gear (actually a cogwheel with pegs) at the left end of the shaft as

[5] Brunelleschi never published drawings, so we have to rely on other peoples' memories of his machines.

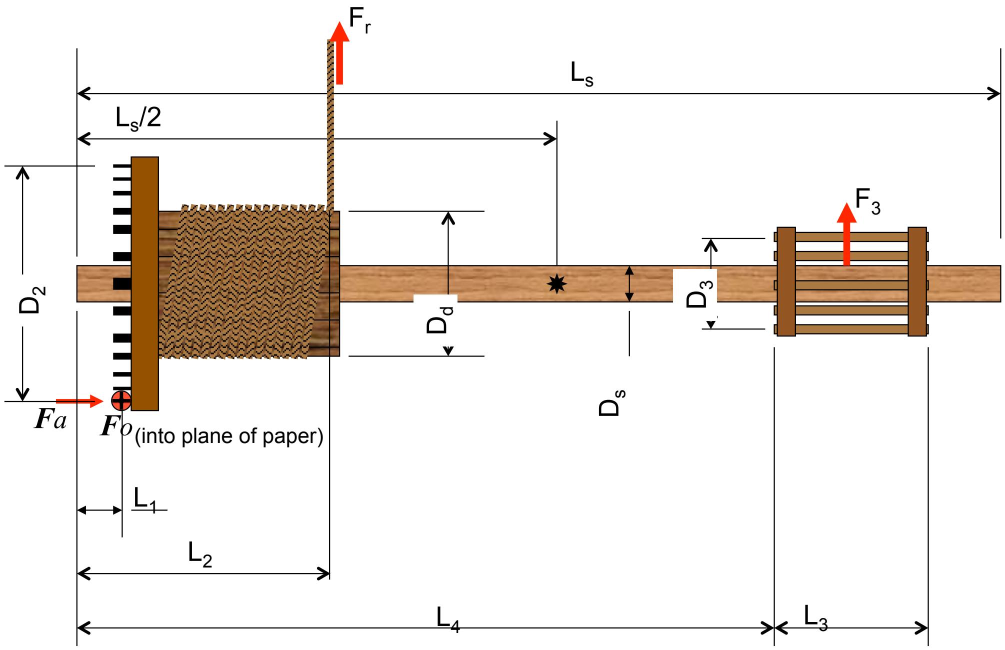


Figure 3: Side view of shaft with forces and dimensions. L_s=8.0; L_1=0.5; L_2=3.0; L_3=1.5; L_4=6.0; D_2=3.3; D_d=2.5; D_3=1.2; D_s=0.3 (all in meters).

it engages (in this case) with the lower horizontal gear. The shaft sits in crude bearings, lubricated with tallow, to support radial loads. The right end can also support an axial load, as indicated in Fig. 2. Dimensions are shown in Fig. 3.

Problem 3

1. As a conservative test case, let $F_r = 4000$N and $F_3 = 400$N. Further, assume there are friction torques that total to $T_f = 50$Nm in the bearings for the main shaft. What is the value of the tangential force, F_o that is required to produce an input torque that will balance these loads?

2. A side effect of the way the cogs engage (Fig. 4) is that F_o also produces an axial force of $F_a = F_o \tan 15°$ (this happens with modern gears too). What is the magnitude of the axial force F_a?

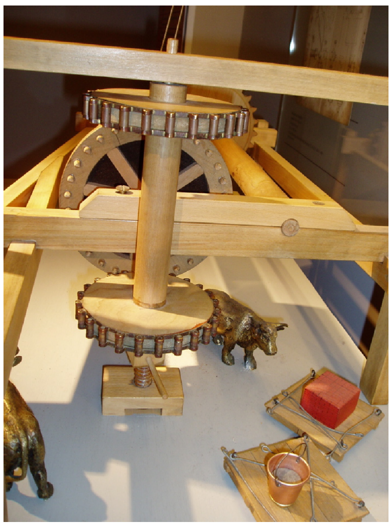
3. With the forces just computed and the dimensions from Fig. 3 we can now solve for the various reaction forces: $R_{1y}, R_{1z}, R_{2x}, R_{2y}, R_{2z}$. ($R_{1x}$ is zero.) Enter your answers for each of these on the website.

Problem 4

The main shafts and posts are more than strong enough. But an area of concern is the peg teeth on the gears, which get lots of cyclic loading. Let's look at the peg teeth on the vertical cog wheel.

1. Ignoring the axial force, F_a, and assuming that the tangential force, F_o, from the previous problem acts on the tip of the peg (gear tooth), what is the peak bending stress, σ_b, and where is it? Recall that for a cantilever beam, $\sigma_b = Mc/I$ where M is the bending moment, c is the distance from the neutral axis to the top or bottom of the beam and I is the moment of inertia. Like a shaft, the peg or "cog" has a round cross section. It has a diameter of $D_p = 10\,\mathrm{cm}$ and a length of $L_p = 30\,\mathrm{cm}$.

2. Does this stress seem OK? For comparison, the rupture strength for hardwood is about 100 MPa and the working stress should probably not exceed 25 Mpa.



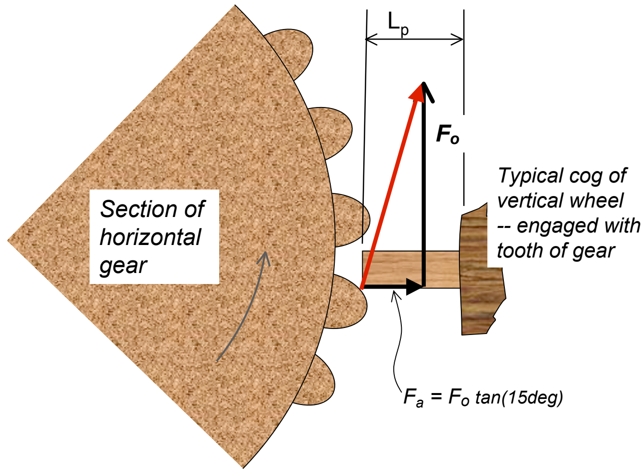


Figure 4: Left: Horizontal gears (with oxen for scale) and mating vertical gear (cog wheel). Right: detail of cog wheel tooth and forces.