Introduction to Gear Meshing

Figure 1 shows the teeth of two engaged gears and Figure 2 shows a detail of spur gear teeth, illustrating the main features. The most important of these features for specifying gears are in the **Gear Nomenclature** below.

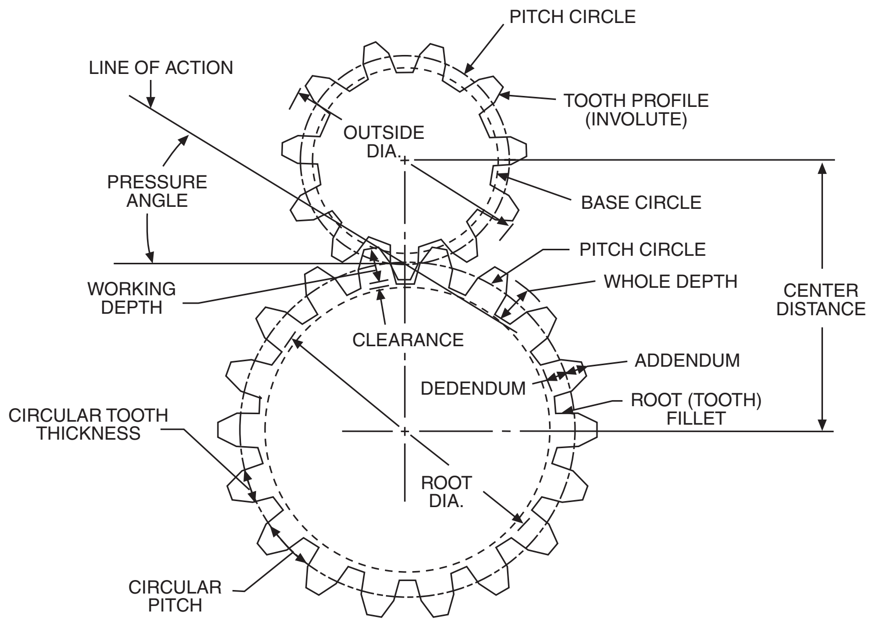

Figure 1: View of two meshed gears [3] showing main terminology and the *line of action* along which the contact point travels.

Gear nomenclature

- b = *face width* (Fig. 2), equal to gear thickness for spur gears.
- P = *diametral pitch* (sometimes just "Pitch"), the number of teeth per *inch of diameter*.
- $N = 2Pr_p$ = total number of teeth around the gear.
- ϕ = *pressure angle* of the line of action (Fig. 1), often 22° in modern gears.
- $P_b = \pi \cos\phi / P$ = *base pitch*, number of teeth per inch of base circle.
- r_p = *pitch radius*, of the pitch circle (Fig. 1) and the working radius of the gear (1/2 the pitch diameter).
- $r_b = r_p \cos\phi$ = *base radius*, of base circle (circle from which the line of action unwinds).
- r_a = *addendum radius*, of addendum circle (outer circle that circumscribes the gear). Note: $r_a = (N + 2)/(2P)$ with standard tooth profiles.
- $C = r_{p1} + r_{p2}$ = *center distance* for two gears.

Most modern gearing is based on the *involute profile*, credited to Euler[4, 6]. It has the property that the contact point between two teeth travels at a constant velocity along the *line of action*, which runs from the *base circle* of one gear to the other. Animations of meshing gears are visible at [5, 6, 7], which show the line of action. The angle of the line of action is the *pressure angle*, ϕ, and accounts for the side effect that gears produce opposing radial forces as they transmit torque.

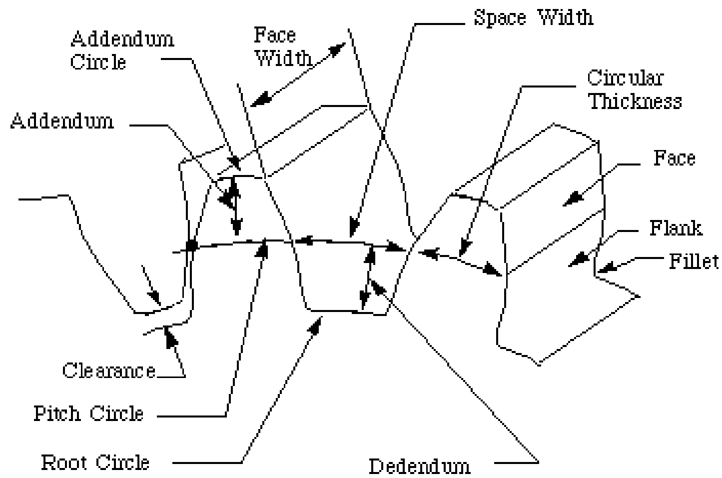

Figure 2: detail of gear teeth

For a demonstration of how to construct an involute profile, see the video on the ME112 course website [1] (suggested viewing speed = 1.5x).

Speed and torque ratios

When choosing gears, the starting point is usually to determine the desired speed ratio. Going from a small gear to a large one ($r_{p1} < r_{p2}$) will reduce speed, ω, and increase torque, τ:

$$\omega_2/\omega_1 = r_{p1}/r_{p2} = N_1/N_2 \quad \text{and} \quad \tau_2/\tau_1 = \mathrm{r_{p2}/r_{p1}} = \mathrm{N_2/N_1}$$

Note that gears with a lower, or "coarser" diametral pitch, P, will be larger for a given number of teeth. Sometimes it is also desirable to know the tangential velocity of the gear teeth: $V = r_p\omega$. Note that for English-unit gears, the velocity is frequently specified in terms of feet/minute.

For more on gear ratios, including the effects of multiple gears in *series* or *compound* gear trains, see the video on the ME112 course website [2] (again, view @1.5x).

Some remaining questions in selecting gears are: how many teeth the should the gears have, what materials should they be made from, and what should their face width, b, be? To answer these questions, one needs to look at the strength of the gear teeth in comparison to the expected loads. This topic is covered in the next lecture.

Contact ratio

For smooth action and durability, it is desirable if more than one pair of teeth are in contact at all times when the gears mesh. The corresponding metric is the *Contact Ratio*:

$$CR = \frac{\text{distance each contact point travels along line of action}}{\text{distance between contact points on line of action}}$$

The numerator is a distance, $L_1 + L_2$, shown in Fig. 3. The denominator is a function of the pitch (the lower, or coarser, the pitch, the fewer teeth per unit length).

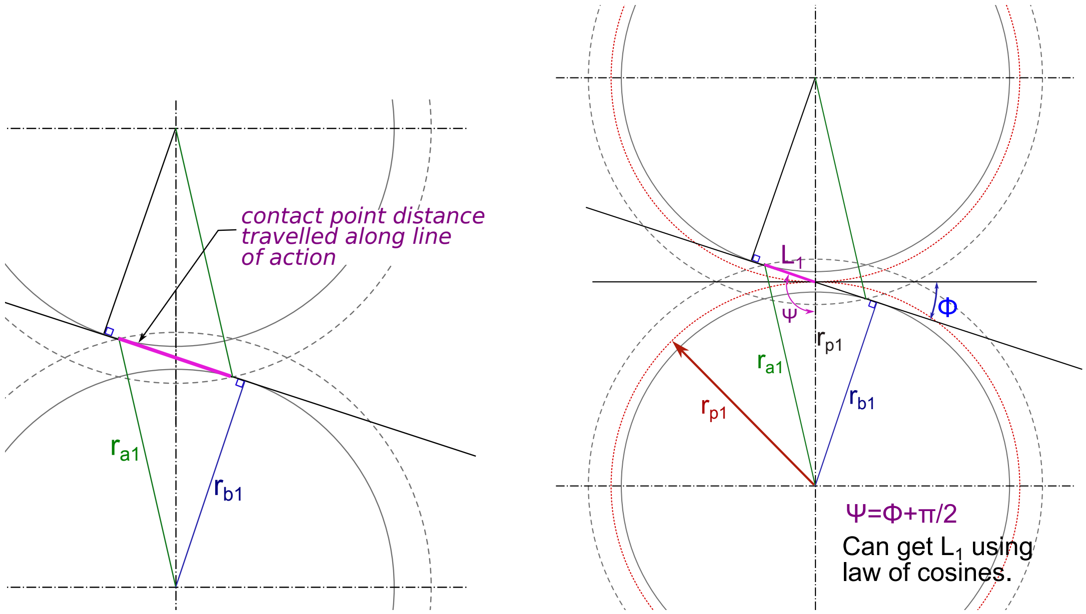

Figure 3: Contact ratio explanation: (left) Distance that contact point travels along line of action depends on the addendum and base radii: r_a and r_b. (right) Total length is the sum of two line segments, $L_1 + L_2$. L_1, shown in diagram, can be computed knowing r_{a1}, r_{p1} and ψ, using the law of cosines; L_2 is similar.

With some trigonometry, the contact ratio can be expressed as:

$$CR = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - C \sin\phi}{P_b} \tag{1}$$

where $C = r_{p1} + r_{p2}$ is the center distance, ϕ is the pressure angle, and $P_b = \pi \cos\phi / P$ is the base pitch. In general, we want $CR > 1$, which is often true if the smaller gear has at least 12 teeth.

Bibliography

[1] Mark Cutkosky. Constructing an involute profile on a gear. https://www.youtube.com/watch?v=4QM0juVXW54.

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