

Motors

For the Crawler and Final projects we'll be using electric motors, so it is worthwhile to look closely at their characteristics. Please review these notes and watch the accompanying short video on Canvas, which you will find in the **Motors** module ([Basics of Motor Analysis](#))¹. There is also a single short [preparation question](#) for Monday on page 6 of these notes.

Introduction

We start with the classic equations of electromagnetism. The Lorentz force for a current, I , flowing in a magnetic field, B , is $F = I \times B$, per unit length of wire, where the right-hand rule applies (Fig. 1). At the same time, Faraday's law applies, so that a voltage or *back EMF* is generated by the wire in a varying magnetic field: $\epsilon = d\phi/dt$ where ϕ is the magnetic flux (integral of B and area) and ϵ is the generated *electromotive force* or EMF.

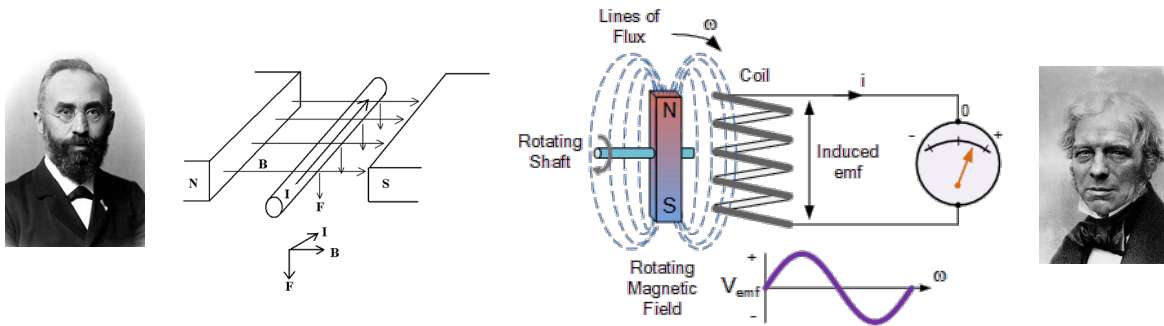


Figure 1: (left) Lorentz force F for a wire with current in a magnetic field. (right) Induced voltage for a coil in a varying magnetic field.

These two equations govern the behavior of a motor as a force (or torque) producer and as a voltage generator: It produces a *torque* in proportion to the current flowing through it, and it generates a *voltage* in proportion to how fast it spins (which determines how rapidly the magnetic field varies for each coil of wire).

Permanent magnetic DC motors

Permanent magnetic motors are increasingly popular in all kinds of equipment from robots to medical equipment to automobiles, due to their efficiency and the recent availability of powerful rare-earth magnets (e.g. neodymium iron boron and samarium cobalt). The high-end PMDC motors are brushless, with digital commutation. At the lower end, we find brush-type motors, which can range in cost from a few dollars (your crawler) to a few hundred dollars (Maxon). In each case, the basic principle is essentially the same, as illustrated in Fig. 2.

We model the motor from two perspectives: as an electrical circuit (Fig. 3 left) and as a magnetic actuator (Fig. 3 right.)

The electrical diagram shows that an applied voltage, V , can be considered to drop through the resistance in the motor coils, R , and a *back EMF* voltage, $k\omega$, which is generated by the motor

¹<https://www.youtube.com/watch?v=ODgxmvhBiko> As usual, 1.5x speed suggested

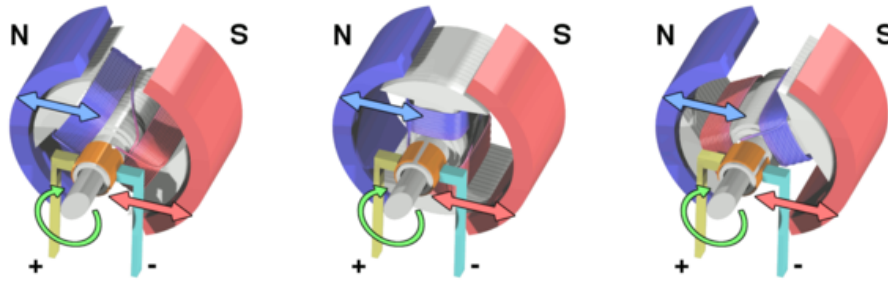


Figure 2: Stages in the cycle of an electric motor. Source: http://en.wikipedia.org/wiki/Brushed_DC_electric_motor

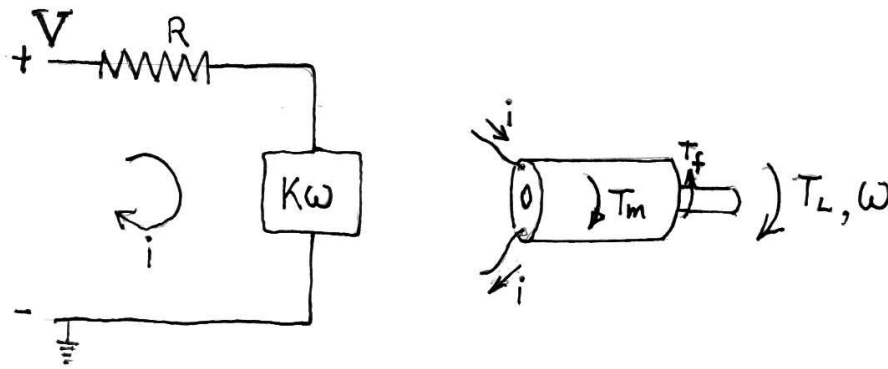


Figure 3: Left: electrical circuit for motor. Right: mechanical schematic for motor.

as it turns. Every motor is also a generator; every generator is also motor. The back EMF is a function of the angular velocity, ω and a constant, k , which is a function of the motor design and, especially, of the permanent magnets.

The right diagram shows that the current passing through the coils generates (via the Lorentz force) a torque, T_m , which acts against friction, T_f and the output or load torque, T_L .

Essential motor equations

Summarizing the two diagrams we have the two essential motor equations. *Everything else can be derived from these:*

$$V - iR - k\omega = 0 \quad (1)$$

$$T_L = T_m - T_f, \text{ where } T_m = ki \quad (2)$$

Notice from eq. 1 that the current i and the angular velocity ω are always inversely coupled; if we know one under any operating condition we can always find the other. Notice from eq. 2 that torque is always linearly proportional to current. Conveniently, if we use MKS units, the motor constant, k , is the same for the electrical and mechanical aspects of the problem. (Consider that $\text{power} = \text{volts} * \text{amps} = \text{torque} * \text{angularvelocity} = ki\omega$.)

Two special conditions are of particular interest in defining the properties of the motors: Stall and No-Load:

Stall

At stall, $\omega = 0$. Equations 1 and 2 give us definitions for the stall current, i_s and the stall torque, T_s :

$$V - i_s R = 0 \quad \text{or} : \quad i_s = V/R \quad (3)$$

$$T_s = T_m - T_f = k i_s - T_f \quad (4)$$

The stall current and stall torque are the maximum possible values of current and torque, respectively. (To prevent overheating and excessive power consumption you usually don't want to hold a motor at stall for long.)

No load

At no load, the motor is spinning freely, $T_L = 0$, and the only torque is the friction torque, T_f . Equations 1 and 2 become:

$$k i_{nl} = T_f \quad \text{or} : \quad i_{nl} = T_f/k \quad (5)$$

$$V - i_{nl} R - k \omega_{nl} = 0 \quad (6)$$

where we can solve for ω_{nl} using i_{nl} from eq. 5. However, in practice, it's usually more accurate to measure ω_{nl} directly if we have a tachometer. (Actually, measuring ω_{nl} is a good way to get T_f .)

Torque plots

We now have enough information to make some torque-speed and torque-current plots (Fig. 4) that define our motor. You'll often find examples of these published in motor catalogs.

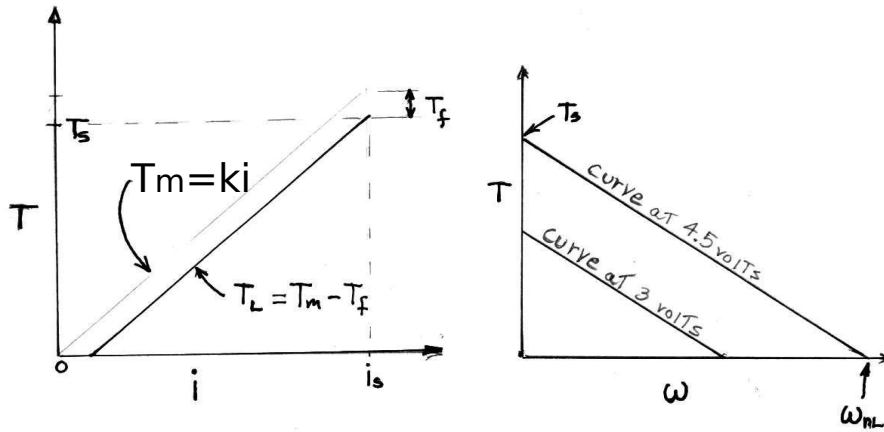


Figure 4: Left: typical torque-current plot. Right: typical torque-speed plots at a couple of different voltages.

Power

The output power of our motor is $P_o = T_L \omega$. At what value of ω will it be maximized? (Compare with the oxen peak power and engine peak power from earlier examples.) We can use eq. 1 to write ω in terms of i , and eq. 2 to get T_L in terms of ki to get:

$$P_o = (ki - T_f)(V - iR)/k \quad (7)$$

Then we take $\partial P/\partial i$ and set to zero to find the maximum:

$$\begin{aligned} k\left(\frac{V - iR}{k}\right) - (ki - T_f)(R/k) &= 0 \\ i &= V/2R + T_f/2k \end{aligned} \quad (8)$$

Then recalling that $i_{nl} = T_f/k$ and $i_s = V/R$ we have the current for peak power: $i_{pp} = \frac{1}{2}(i_s + i_{nl})$ which is exactly the average of the stall current and the no-load current. Knowing the current, we can also get the ω and T_L from the previous equations. It turns out that $\omega_{pp} = \omega_{nl}/2$ and the power plot is a parabola, as seen in Fig. 5.

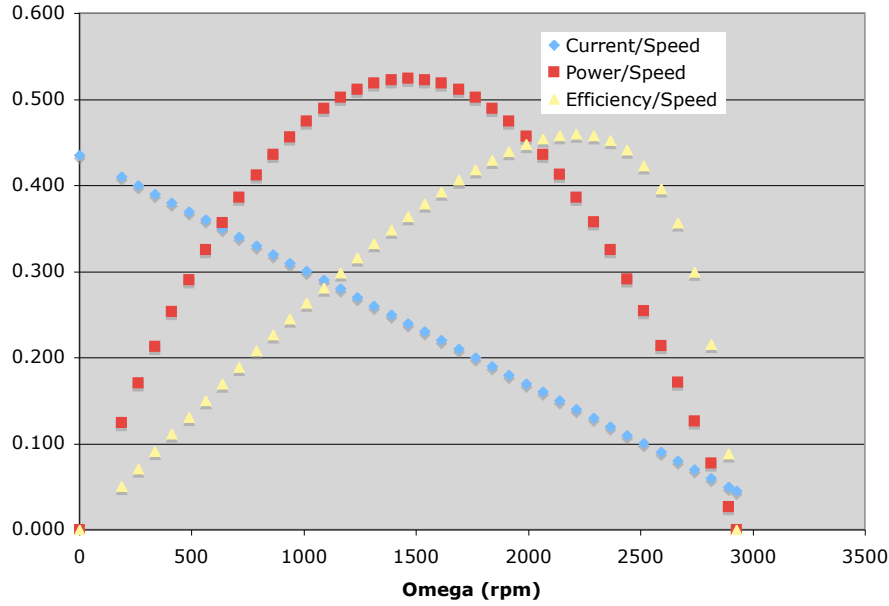


Figure 5: Current (amps), Power (Watts) and Efficiency versus speed (rpm) for old style Lego motors at $V = 6$ volts.

Efficiency

What about efficiency? Is it most efficient to run the motor near peak power? For efficiency, we take the ratio of output (mechanical) power to input (electrical) power. As before, we can use equations 1 and 2 to write everything in terms of a single variable, i :

$$\eta = \frac{P_{out}}{P_{in}} = \frac{T_L \omega}{Vi} = \left(\frac{ki - T_f}{Vi}\right) \left(\frac{V - iR}{k}\right) \quad (9)$$

From this ratio, we can see immediately that efficiency will be zero at stall (because ω is zero) and at no-load, when $T_L = 0$ (i.e., when $ki = T_f$). The optimum will be at some intermediate value of current and speed.

Taking the partial derivative of equation 9 with respect to i results in a messy expression, but we can use equations 3 and 6 to rewrite η in terms of i_s and i_{nl} : $\eta = (1 - i_{nl}/i)(1 - i/i_s)$ from which

$$\partial\eta/\partial i = 0 = \left(\frac{i_{nl}}{i^2} - \frac{1}{i_s}\right) \text{ or: } i_\eta = \sqrt{i_{nl}i_s} \quad (10)$$

As seen in Fig. 5, the peak efficiency occurs at a somewhat higher speed (and lower torque) than peak power, i.e., η_{max} occurs when a motor is *lightly loaded*. As soon as a motor starts to “bog down,” the efficiency drops rapidly.

We conclude by replotting Fig. 5 in terms of current, i , instead of speed, in Fig. 6.

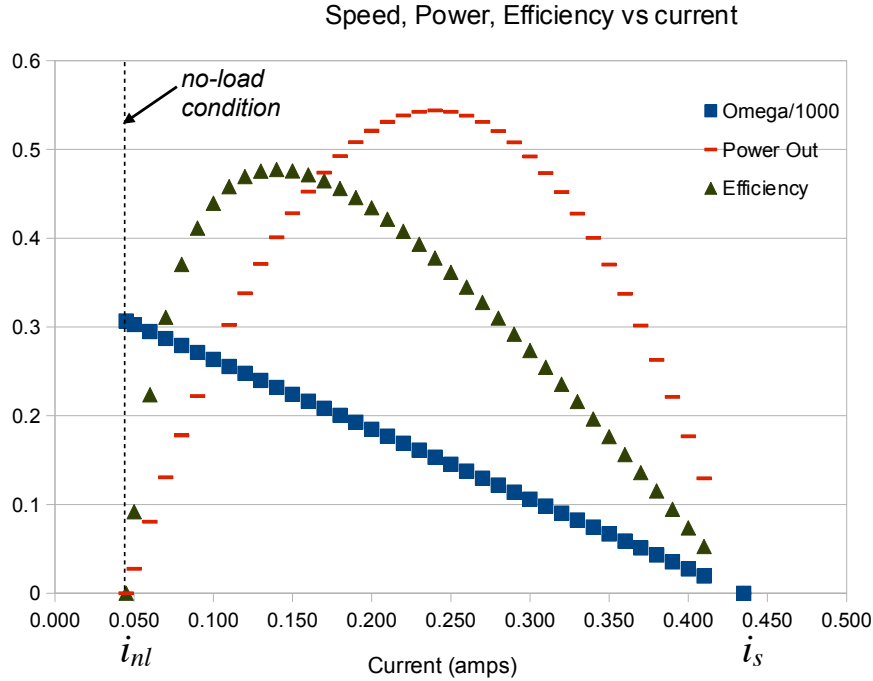


Figure 6: Speed (radians/sec) $\times(1/1000)$, Power (Watts) and Efficiency versus Current (amps), for old style Lego motors at $V = 6$ volts.

Running with Batteries

When running on a power supply, you need to keep an eye on the maximum required current (e.g. at stall) to make sure the supply does not “current limit,” which reduces the voltage. When using batteries, a similar effect occurs due to the internal resistance of the batteries (Fig. 7). The batteries’ resistance is added to that of the motor, so that the available voltage across the motor is less than the nominal battery voltage. For example, suppose we have two 1.5 volt batteries, each with an internal resistance of 0.2 ohms. If the motor is drawing 0.5 amps of current, the actual

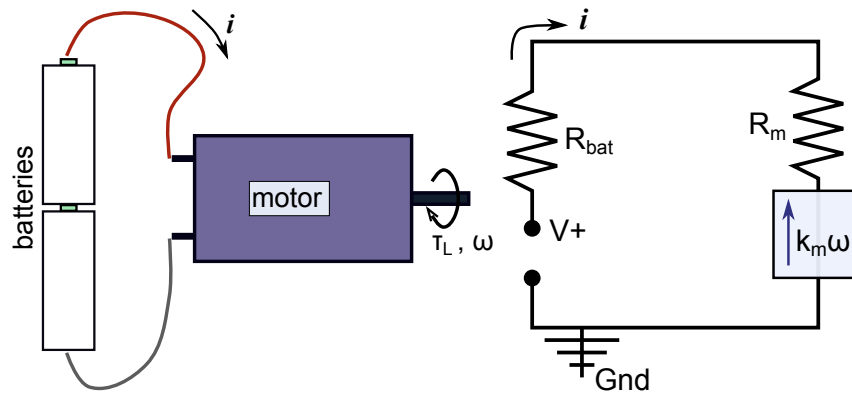


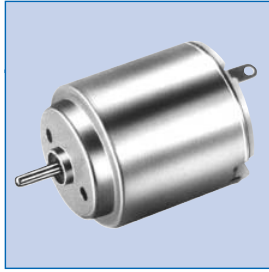
Figure 7: When running with batteries, there is some internal resistance in the batteries, hence the voltage across the motor is somewhat reduced.

voltage across it will be $1.5 - 0.5 \cdot (0.2 + 0.2) = 1.3$ volts. At higher currents and lower speeds, the actual motor voltage will drop further.

A motor example

The final page of this handout is from Mabuchi Corp. and provides details on one of the small motors that is commonly used for ME112 Final Projects (but is not the motor used for crawlers). Take a look at the various terms and see if you can make sense of them in terms of i_s , i_{nl} , ω_{nl} and T_s , etc.

Preparation question for Monday labs: Suppose you have what you believe is a Mabuchi RE260-RA motor (see next page). But which of the three variants is it? You have a good quality adjustable power supply where you can accurately set the supply voltage and measure the current. *What would you do to determine which one you have?* Please write your answer below and bring these notes to labs on Monday.



RE-260RA



Metal-brush motors

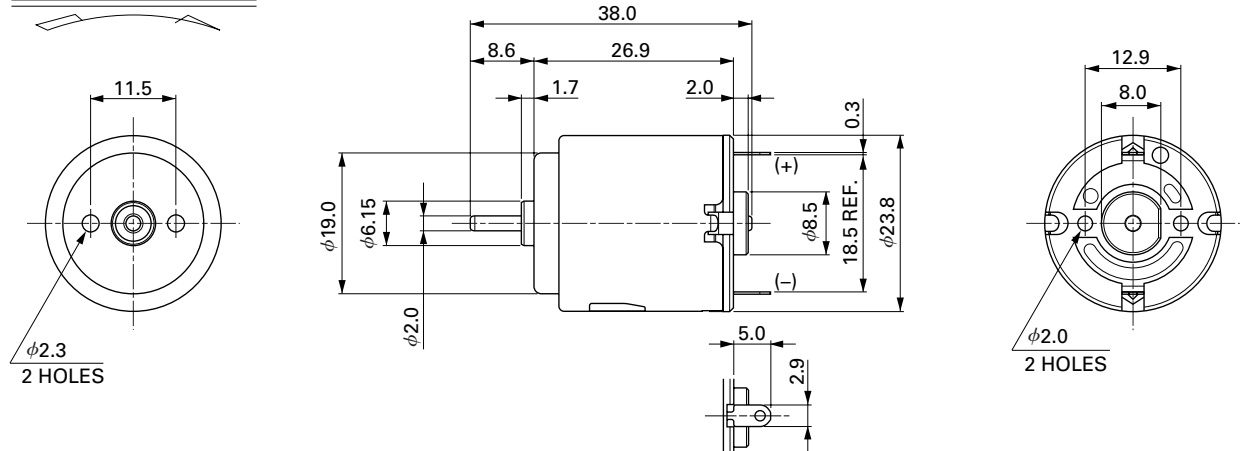
OUTPUT : 0.4W~4.0W (APPROX)

WEIGHT : 28g (APPROX)

Typical Applications Toys and Models : Motorized Toy / Motorized Plastic Model

MODEL	VOLTAGE		NO LOAD		AT MAXIMUM EFFICIENCY					STALL		
	OPERATING RANGE	NOMINAL	SPEED	CURRENT	SPEED	CURRENT	TORQUE		OUTPUT	TORQUE		CURRENT
			r/min	A	r/min	A	mN-m	g-cm	W	mN-m	g-cm	A
RE-260RA-2670	1.5~3.0	1.5V CONSTANT	6300	0.16	5040	0.64	0.98	10.0	0.52	4.90	50	2.56
RE-260RA-18130	1.5~4.5	3V CONSTANT	6900	0.095	5470	0.36	0.97	9.9	0.56	4.70	48	1.40
RE-260RA-2295	1.5~4.5	3V CONSTANT	9400	0.15	7610	0.64	1.31	13.3	1.04	6.86	70	2.70

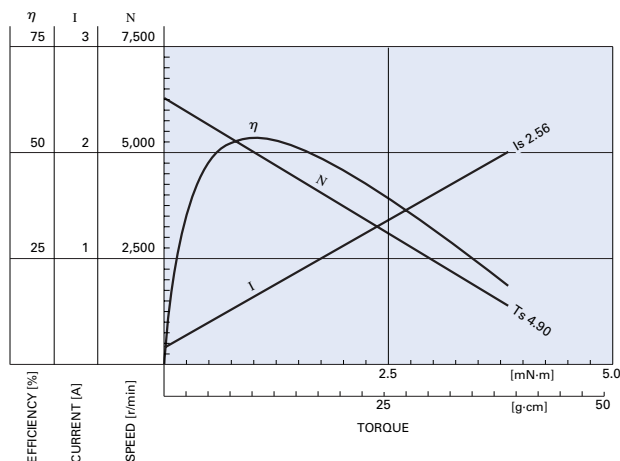
DIRECTION OF ROTATION



UNIT: MILLIMETERS

RE-260RA-2670

1.5V



RE-260RA-18130

3.0V

