Introduction to Mechanisms and Linkages

Mechanisms are central to many fields ranging from robotics, to automotive suspension design, to biomechanics (Fig. 1) – anything involving joints, links and the constrained motions they enable. In ME112 we cover **planar mechanisms**, which are useful for many devices; spatial mechanisms remain an active research topic. This introduction to mechanisms covers definitions, degrees of freedom, mobility, and joint types. Later notes will go into the details of mechanism synthesis and analysis for various applications.

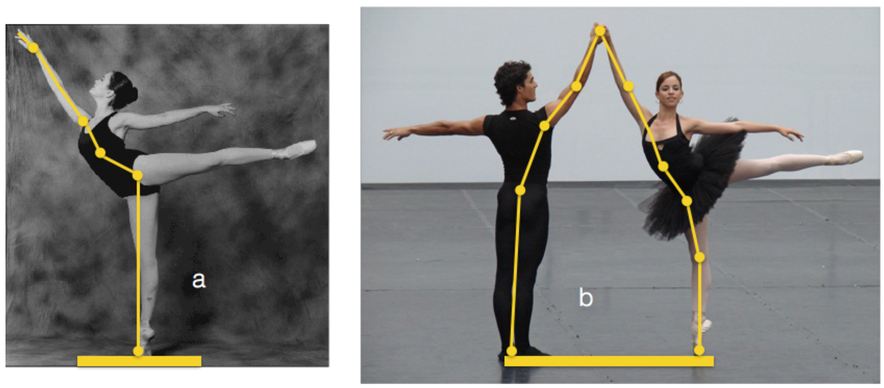


Figure 1: The dancer at left creates several serial kinematic chains, of which one is highlighted. The dancers in a *pas de deux* create a parallel chain (and additional serial chains, not highlighted).

1 Degrees of freedom and Mobility

Consider a single rigid body on a planar surface (Fig. 2, left). It can translate and rotate in the plane, so it has three degrees of freedom (DOF), and its mobility with respect to a grounded world reference frame, (X_w, Y_w), is 3. If we have two such bodies, disconnected, there are $2 \times 3 = 6$ DOF, and with N such bodies there are $3N$. However, if we connect two bodies joined by a single *revolute joint* (pin joint), the number of DOF drops to $3+1$ because the second body can only rotate about the first (Fig. 2, right). In other words, if we know the position and orientation of the first body, it takes only one more piece of information to specify the position and orientation of the second.

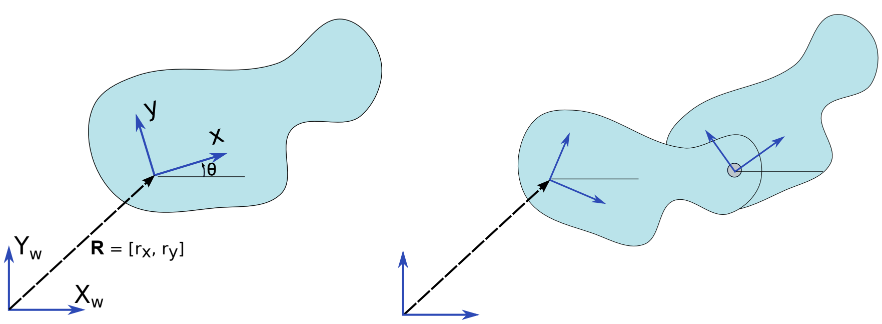


Figure 2: (left) A single body can translate and rotate in the plane, for 3 DOF; (right) two floating bodies connected by a revolute joint have $3 + 1 = 4$ DOF.

Joints restrict, or constrain, the degrees of freedom of a body. Their effect depends on what kind of joint we have. Pin or revolute joints have just one DOF. Other single-DOF or "type 1" joints include linear or "prismatic" joints (Fig. 3b). Screws (Fig. 3c) have two motions (linear and rotational) but they are coupled by the helix angle, so, again, only one parameter is needed to define the state of the screw.

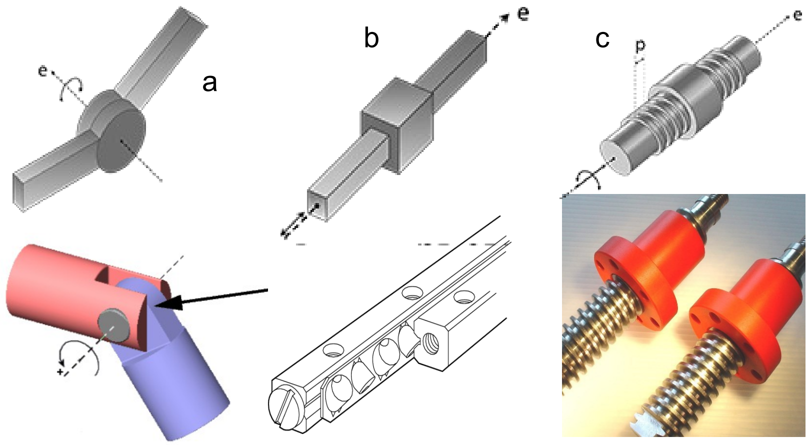


Figure 3: Single DOF or "type 1" joints include revolute joints (a), prismatic joints (b) and screws (c). Images in lower row show practical implementations: clevis joint, crossed roller slide and power screw, respectively.

Other joints can have two DOF in the plane, or three DOF, for spatial mechanisms. For example, Fig. 4(a) shows a "type-2" pin-slider joint that is frequently used in planar mechanisms.

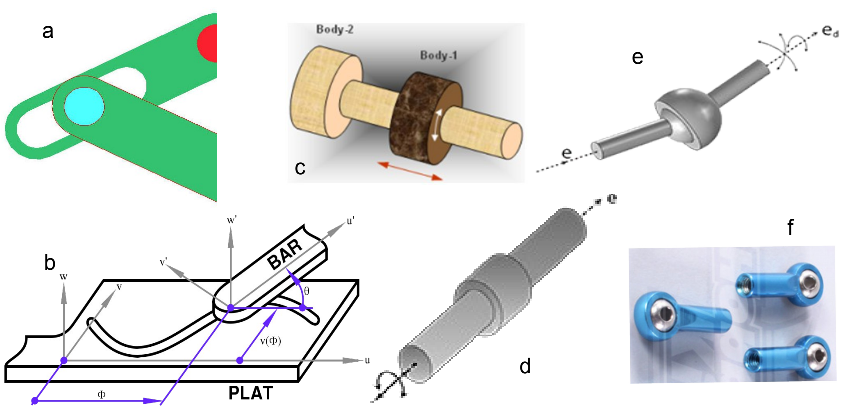


Figure 4: Joints with two and three DOF: Two-DOF joints include pin-slider joints (a,b) cylindrical joints (c,d). Note that the pin in a track (b) has only two DOF because if one knows the arc length along the track and rotation angle of the bar, everything is determined. Ball joints (e,f) are three-DOF joints commonly used in spatial mechanisms such as automobile suspensions.

1.1 Gruebler's formula

We can generalize the counting of joints, bodies and degrees of freedom using Gruebler's Formula, also called the Kutzbach-Gruebler's equation. Here we specialize it to the case of planar mechanisms with type-1 (single DOF) and type-2 (two DOF) joints:

$$F = 3(N - 1) - 2f_1 - f_2 \tag{1}$$

where F is the mechanism's degrees of freedom, N is the number of links, and f_1 and f_2 are the numbers of type-1 and type-2 joints, respectively.

Let's try Gruebler's formula on a few mechanisms in Fig. 5. The mechanism at left (a) has six links, so $N = 6$. We adopt the convention that the ground is always *link 1*. How many joints does it have? We see what look like four joints, but the joint connecting links 4, 5 and 6 is really two joints sharing the same axis. The important thing is how many parameters are needed to specify the orientations of links 5 and 6 if link 4 is fixed. Hence $f_1 = 5$ and $f_2 = 0$ because they are all type-1 joints. So $F = 3 \cdot (6 - 1) - 2 \cdot 5 = 5$ which matches our intuition if we just add up joint motions starting at the base.

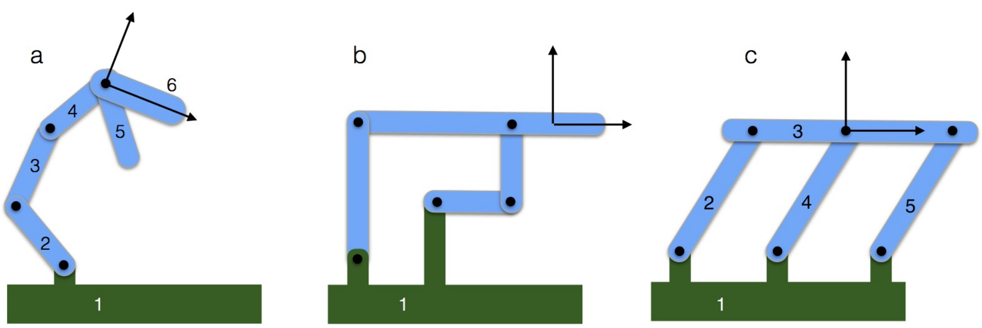


Figure 5: (a) a serial chain with six links; (b) a five-bar parallel chain; (c) a parallelogram mechanism

Fig. 5(b) shows a *5-bar parallel-chain* mechanism. A 5-bar serial chain (i.e., one that does not form a closed loop) would have $F = 3 \cdot (5 - 1) - 2 \cdot 4 = 4$, but now we compute just 2 DOF. The 5-bar parallel mechanism as drawn is commonly used for haptic feedback devices because two motors located at the base joints of links 2 and 5 can control the tip, producing small motions in the x and y directions, respectively.

Fig, 5 shows an exception to Gruebler's formula. If we examine the linkage, we can guess that it has 1 DOF, because link 3 can move back and forth in an arc while remaining parallel to link 1. If we use Gruebler's formula, we compute $F = 0$, because the formula does not account for the special case here in which links 2,4,5 are all parallel and identical. Hence it is equivalent to a 4-bar parallel mechanism.

Fig. 6 shows two more examples commonly found in ME112 projects. The left shows a pin-slider linkage. Although it has only three links, it has 1 DOF because the pin-slider joint is type-2. The right shows two gears. They have type-1 joints at their axles, but recall that gear teeth do slide a bit in addition to rolling. So the tooth mating is a type-2 joint. Hence $F = 3 \cdot (3 - 1) - 2 \cdot 2 - 1 = 1$ as we would expect.

We conclude this section with a *geared five-bar mechanism* (Fig. 7) of the sort used in the

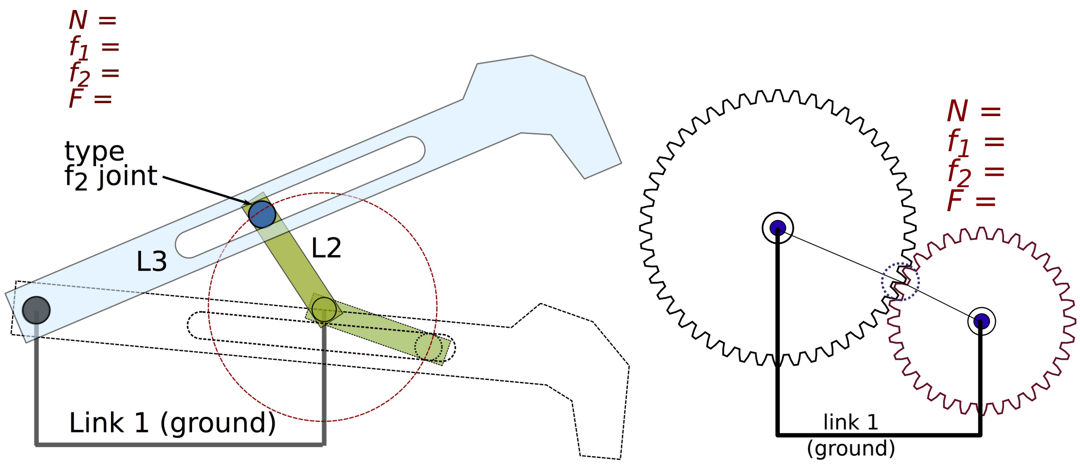


Figure 6: (a) pin-slider mechanism; (b) gears have type-1 joints at their axles and type-2 at their teeth.

Disney animatronics video.[1] How many DOF does it have?

2 Four Bar Linkages and Grashof's Criterion

Four-bar mechanisms are widely used in ME112. More complex mechanisms (e.g. six-bar, eight-bar) can often be decomposed into a series of four-bar mechanisms. Fig. 8 shows several variants with different behaviors. The most common is the *crank-rocker*, in which the input crank (link 2) rotates continuously and link 4 rocks back and forth; the output is often attached to the *coupler*, link 3.

Grashof's criterion determines whether at least one link can undergo full rotations, or whether the linkage will only rock back and forth. Let s and l be the shortest and longest links, respectively, and let p and q be the other two links. Then:

- if $s + l \leq p + q$ there is at least one revolving link (Fig. 8 a,b,d).
- if $s + l = p + q$ the mechanism is a *change point* mechanism that can switch from one assembly to another at certain points of its motion (Fig. 8 d).
- if $s + l > p + q$ the three mobile links will rock back and forth (Fig. 8 c).

3 Useful linkage links

- http://kmoddl.library.cornell.edu/model.php?m=234 - Cornell mechanisms library online
- http://www.mekanizmalar.com/fourbar.html - simple 4-bar simulator (can vary and see Grashof succeed or fail)
- https://en.wikipedia.org/wiki/Four-bar_linkage
- https://en.wikipedia.org/wiki/Parallel_motion - parallel motion linkages (e.g Watts')
- http://www.slideshare.net/physics101/001-mechanisms-and-kinematics - examples, including clamping mechanisms (sides 12, 13)

[1] https://www.youtube.com/watch?v=DfznnKUwywQ

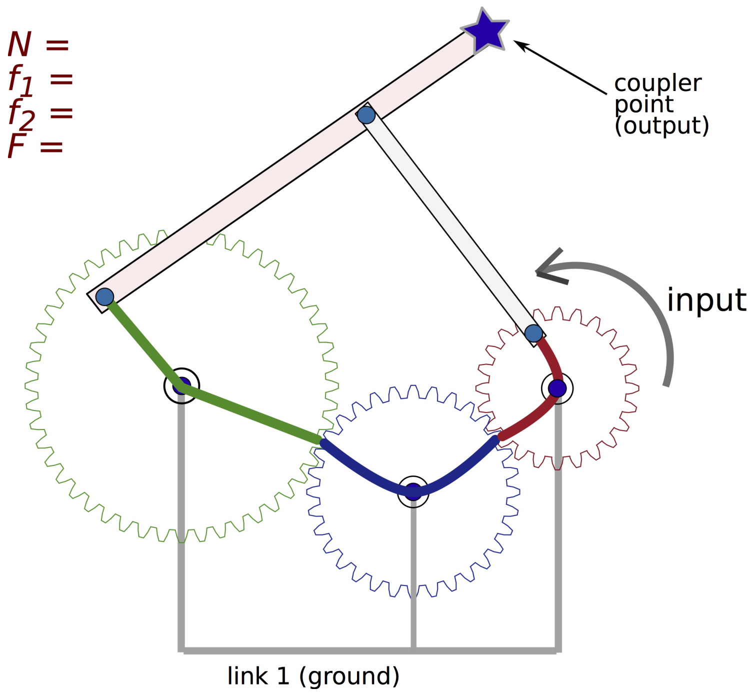


Figure 7: A geared 5-bar mechanism has how many degrees of freedom?

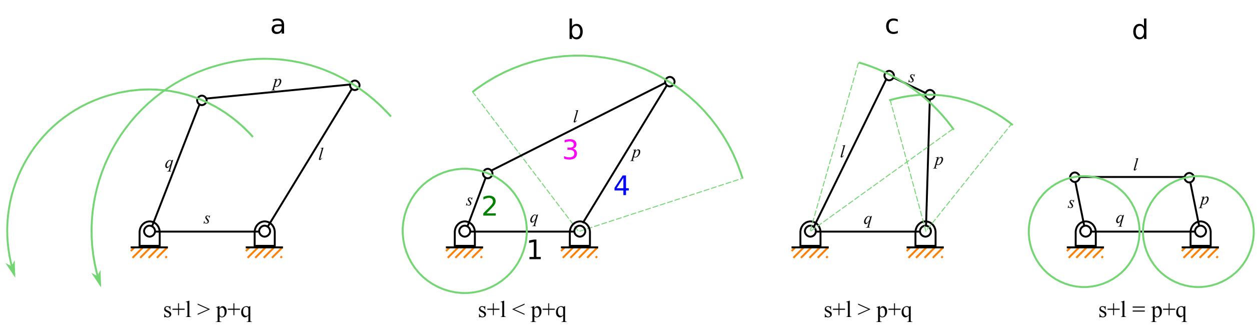


Figure 8: Four-bar linkage variants: (a) drag link, (b) crank-rocker, (c) double rocker, (d) parallelogram [Adapted from https://en.wikipedia.org/wiki/Linkage_(mechanical)]

① Rigid Bodies, Joints and Mechanisms

POSITION

TIME DERIVATIVES

WHAT IS...

KINEMATICS?

"STUDY OF POSITION AND ITS DERIVATIVES"

"STUDY OF GEOMETRY OF MOTION"

HOW THINGS "CAN" MOVE

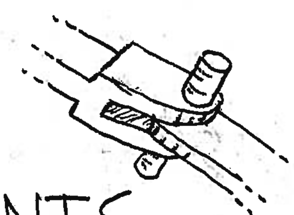
BASIS OF DYNAMICS

FORCE → DYNAMICS → EVOLUTION OF SYSTEM

MASS

?

MECHANISMS



MECHANISMS ARE ASSEMBLIES OF

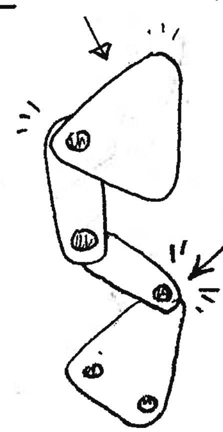
RIGID BODIES → CONNECTED BY → JOINTS

?

COLLECTIONS OF PARTICLES THAT A SET DISTANCE FROM EACH OTHER

huh?

See PAGE ②



?

JOINTS CONSTRAIN THE MOTION OF TWO RIGID BODIES

huh?

CONSTRAIN WHAT?
WHAT'S A CONSTRAIN?

See PAGE ③

KINEMATICS IS USED IN

ANALYSIS AND DESIGN

GIVEN THE GEOMETRY OF MECHANISM

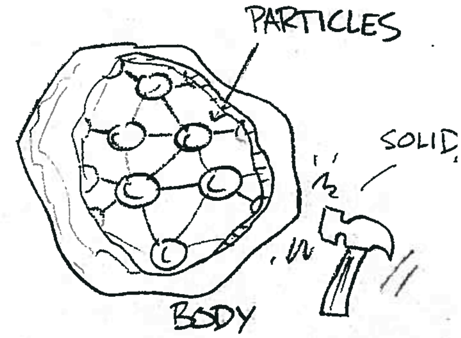
→ SOLVE FOR POSITIONS AND VELOCITIES ALSO KNOWN AS... → MOTION

INVENT LENGTHS (GEOMETRY) OF RIGID BODIES AND JOINTS

TO MEET SPECIFIED REQUIREMENTS IN... → MOTION

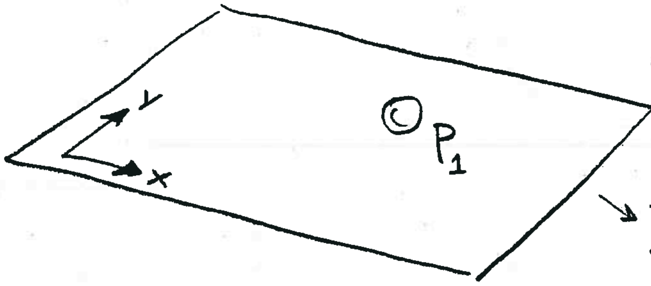
RIGID BODIES

- RIGID BODIES ARE COLLECTIONS OF PARTICLES WHERE ALL THE PARTICLES ARE A SET DISTANCE APART FROM EACH OTHER (AS OPPOSED TO ELASTIC OR COMPLIANT BODIES)



- LET'S CONSIDER "PLANAR" MOVEMENT:

EACH PARTICLE CAN ONLY MOVE IN A PLANE

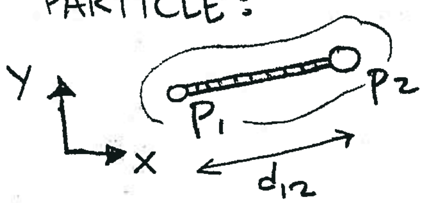


THE LOCATION OF P_1 CAN BE DESCRIBED BY (x, y)

= 2 DEGREES OF FREEDOM (DOF)

OF VARIABLES NEEDED TO SPECIFY ITS POSITION

- NOW CONSIDER AN "ATTACHED" PARTICLE:



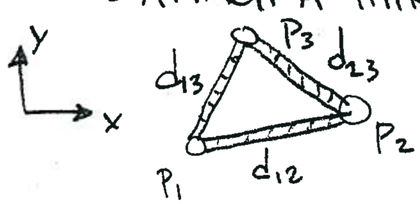
IF THE TWO ARE RIGID, THEN THIS EQUATION IS "TRUE":

$$d_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

CONSTRAINT ON THE VARIABLES

P2 HAS 2 DOF AND ONE CONSTRAIN ⇒ ONLY 3 VARIABLES ARE "INDEPENDENT" IN SYSTEM ⇒ $\boxed{\text{3DOF}}$

- IF WE ATTACH A THIRD PARTICLE:



→ WE ADD 2DOF, BUT ALSO 2 CONSTRAINTS:

$$d_{13}^2 = (x_3 - x_1)^2 + (y_3 - y_1)^2 \text{ AND}$$

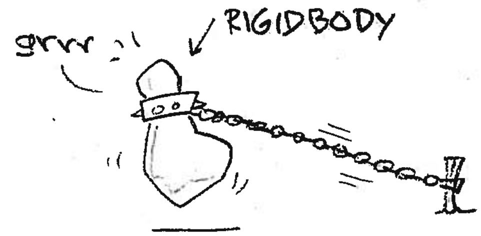
$$d_{23}^2 = (x_3 - x_2)^2 + (y_3 - y_2)^2$$

⇒ WE DON'T ADD ANYTHIN

ALL OTHER ATTACHED PARTICLES DON'T ADD ANYTHING

≡ THUS: A RIGID BODY HAS $\boxed{\text{3DOF}}$ IN THE PLANE

⇨ JOINTS



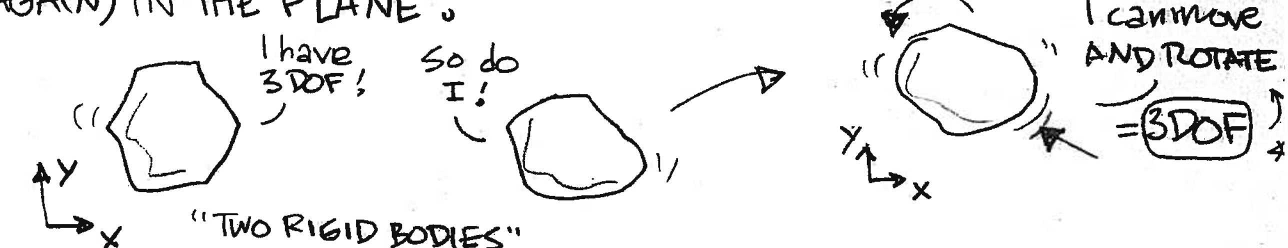
JOINTS CONSTRAIN THE MOTION OF TWO RIGID BODIES

A CONSTRAINT IS AN EQUATION THAT CONTAINS THE VARIABLES THAT DESCRIBE THE MOTION OF THE TWO BODIES (x,y)

⇨ (AGAIN) IN THE PLANE:

RIGID BODY THEATER PRESENTS:

①



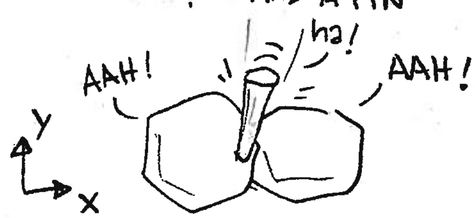
"TWO RIGID BODIES"

$\boxed{\text{TOTAL DOF} = 6}$

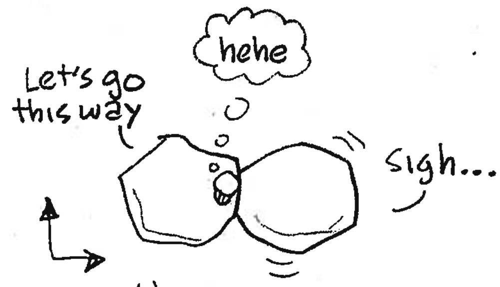
②

"TWO RIGID BODIES AND A PIN"

③

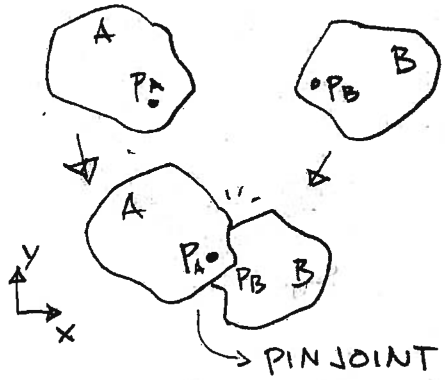


④



"TWO RIGID BODIES CONNECTED BY A PIN JOINT"

$\boxed{\text{TOTAL DOF} = 4}$



THE PIN JOINT IMPOSES THE CONSTRAINT

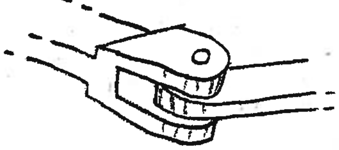
$$P_A = P_B$$

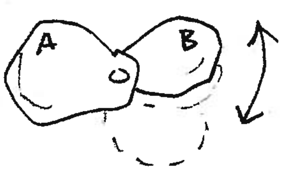
OR

$$x_A = x_B$$
$$y_A = y_B$$

→ 2 TWO CONSTRAINTS REALLY!

A PIN JOINT ALLOWS 1 DOF BETWEEN THE TWO BODIES





THIS # IS CALLED THE CONNECTIVITY