Biomedical Informatics 260

Computational Feature Extraction:

Geometric Features

Lecture 5

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Spring 2019

Correction to Last Lecture

- Fourier Transform formulas:

$$f(x) = \int_{-\infty}^{\infty} \mathrm{F}(u) \cdot e^{i2\pi ux} dx$$

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{F}(u,v) \cdot e^{i2\pi(ux+vy)} dx\, dy$$

WRONG

$$f(x) = \int_{-\infty}^{\infty} \mathrm{F}(u) \cdot e^{i2\pi ux} du$$

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{F}(u,v) \cdot e^{i2\pi(ux+vy)} du\, dv$$

RIGHT

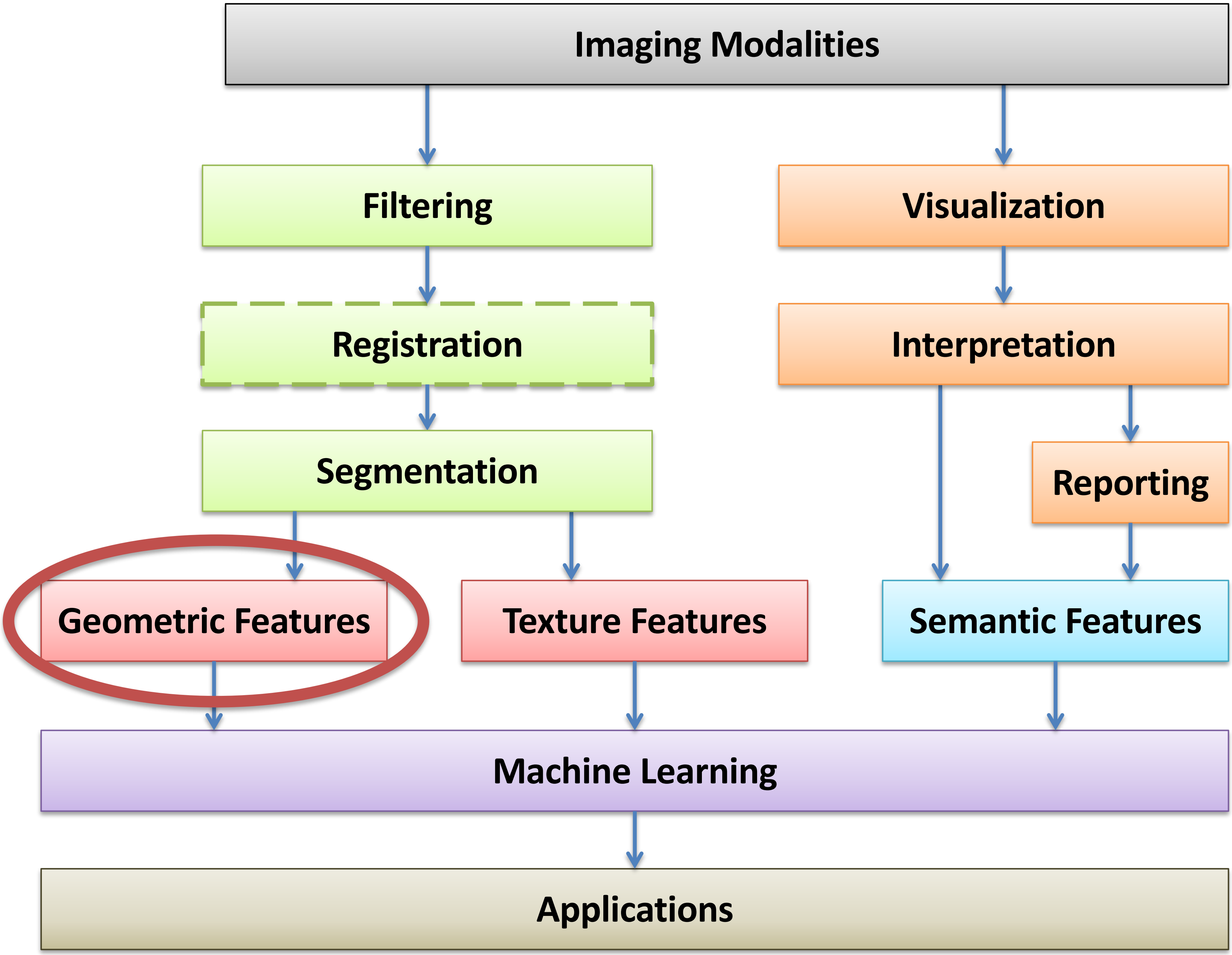

Imaging Modalities
Filtering
Registration
Segmentation
Visualization
Interpretation
Reporting
Geometric Features
Texture Features
Semantic Features
Machine Learning
Applications

Image Features

- Can be computed
  - Per-pixel
  - Per-object
    - Boundary
    - Region
  - Per-image
- Desirable Properties of Image Features
  - Translation, rotation, (and sometimes scale) invariance
  - Robustness to noise & acquisition protocol
  - Statistical independence from other shape features
    - Very important for machine learning

Shape Features

Angle

CAD Score

Thickness

Volume, Area

Cobb Angle, Anteversion

Malignancy

Perimeter
Surface Area

Medial Axis

Cartilage
Thickness

Mean Curvature

Stenosis

Circularity

Gaussian Curvature

Tortuosity

Spiculation

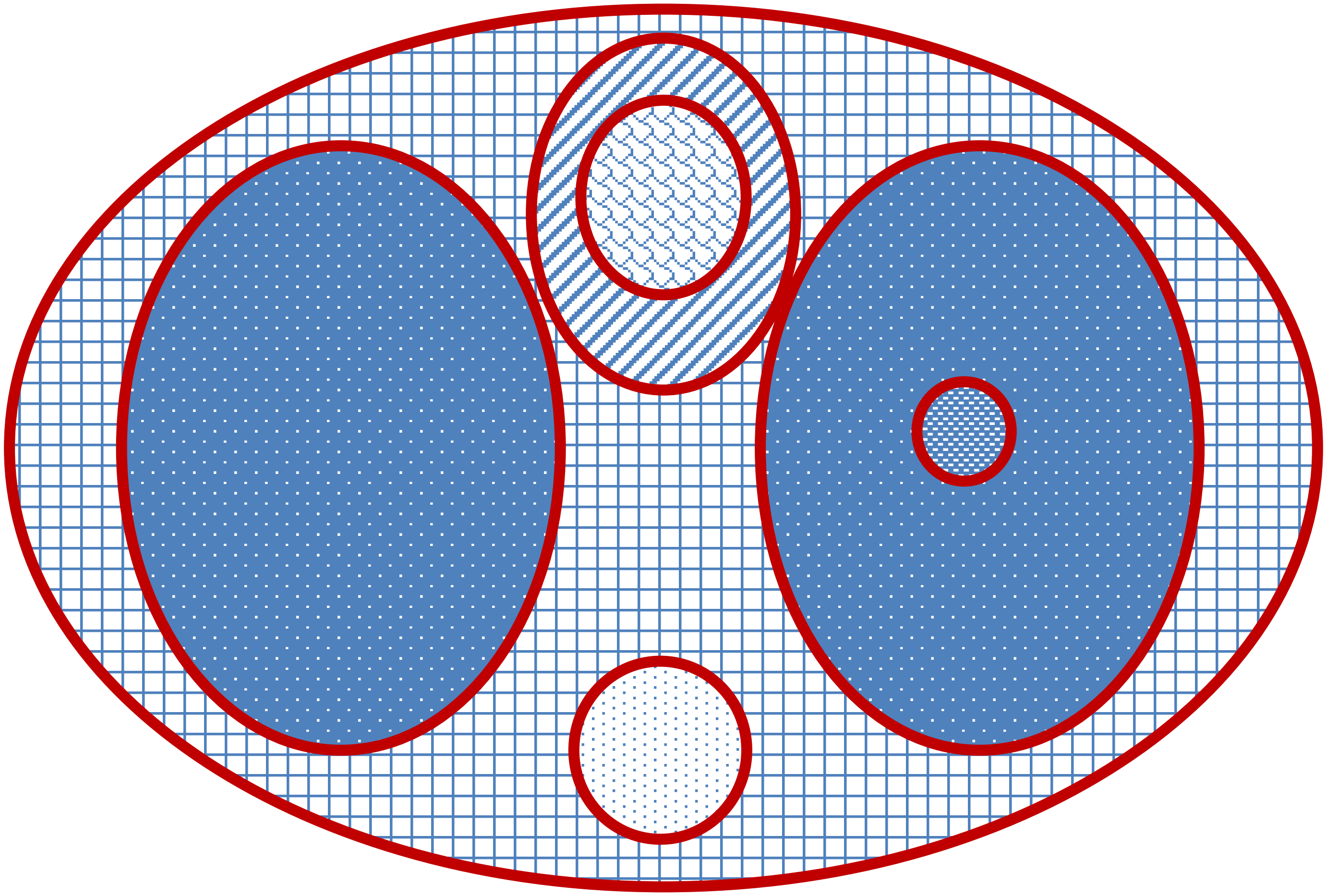
Shape Index
Curvedness

Margin

Gradient

Geometric ⟷ **Clinical**

The Shape v Texture View of the World



Local Pointwise Features

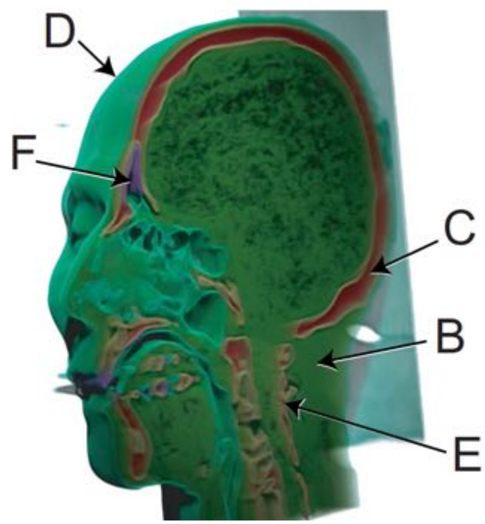
Image Partial Derivatives as Features

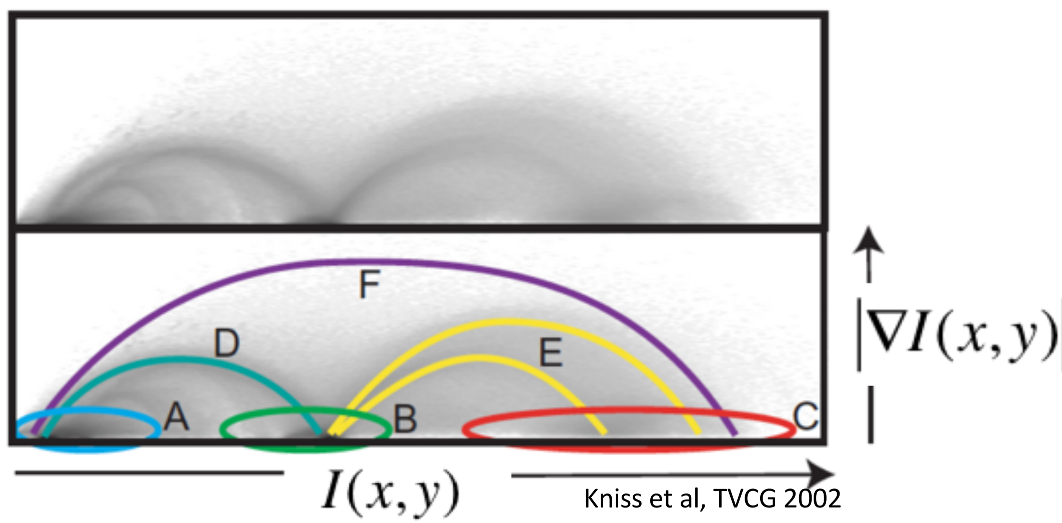
0th derivative: $$I(x,y)$$

1st derivative: $$\frac{\partial}{\partial x}I(x,y) \quad \left|\nabla I(x,y)\right|$$

2nd derivative: $$\frac{\partial^2}{\partial x^2}I(x,y) \quad \frac{\partial^2}{\partial x \partial y}I(x,y) \quad \nabla^2 I(x,y)$$

These analyses are often done across multiple spatial scales



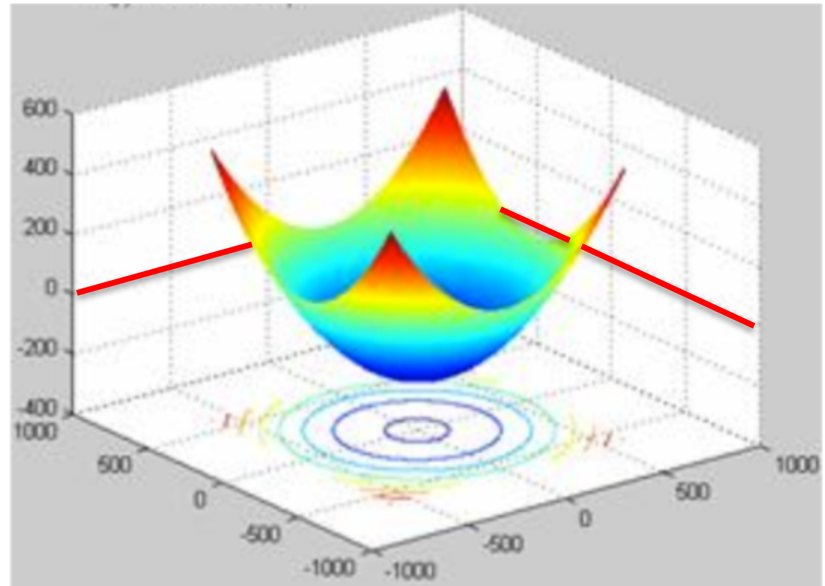


Kniss et al, TVCG 2002

What causes these arch-like structures?

2D Isocontour Curvature
(of a 2D level set of an implicit function)

When ϕ is an implicit function, not necessarily a signed distance function:



$$\kappa = \operatorname{div} \vec{N} = \nabla \cdot \vec{N} = \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|}$$

$$= \frac{\partial}{\partial x}\left[\frac{\phi_x}{(\phi_x^2 + \phi_y^2)^{1/2}}\right] + \frac{\partial}{\partial y}\left[\frac{\phi_y}{(\phi_x^2 + \phi_y^2)^{1/2}}\right]$$

Remember, subscripts are partial derivatives

$$= \frac{\phi_y^2 \phi_{xx} - 2\phi_x \phi_y \phi_{xy} + \phi_x^2 \phi_{yy}}{(\phi_x^2 + \phi_y^2)^{3/2}}$$

Much easier for a signed distance function *S*:

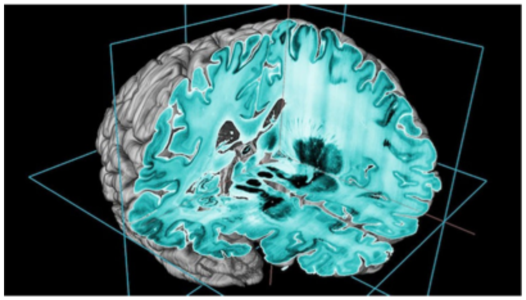
$$\kappa = \nabla^2 S = \frac{\partial}{\partial x}[S_x] + \frac{\partial}{\partial y}[S_y]$$

$$= S_{xx} + S_{yy}$$

How can curvature be clinically useful information?

3D Isosurface Curvature

(of a 3D level set of an implicit function)



$$\kappa_1 + \kappa_2 = \operatorname{div} \vec{N} = \nabla \cdot \vec{N} = \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|}$$

$$H = \frac{\kappa_1 + \kappa_2}{2} = \frac{1}{2h^{3/2}} \begin{bmatrix} \phi_x^2(\phi_{yy} + \phi_{zz}) - 2\phi_y\phi_z\phi_{yz} \\ + \ \phi_y^2(\phi_{xx} + \phi_{zz}) - 2\phi_x\phi_z\phi_{xz} \\ + \ \phi_z^2(\phi_{xx} + \phi_{yy}) - 2\phi_x\phi_y\phi_{xy} \end{bmatrix}$$

Mean curvature

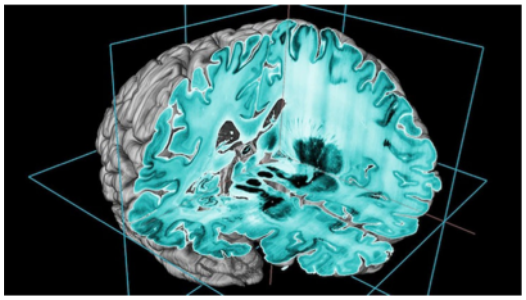
$$K = \kappa_1\kappa_2 = \frac{1}{h^2} \begin{bmatrix} \phi_x^2(\phi_{yy}\phi_{zz} - \phi_{yz}^2) + 2\phi_y\phi_z(\phi_{xz}\phi_{xy} - \phi_{xx}\phi_{yz}) \\ + \ \phi_y^2(\phi_{xx}\phi_{zz} - \phi_{xz}^2) + 2\phi_x\phi_z(\phi_{yz}\phi_{xy} - \phi_{yy}\phi_{xz}) \\ + \ \phi_z^2(\phi_{xx}\phi_{yy} - \phi_{xy}^2) + 2\phi_x\phi_y(\phi_{xz}\phi_{yz} - \phi_{zz}\phi_{xy}) \end{bmatrix}$$

Gaussian curvature

where $h = \phi_x^2 + \phi_y^2 + \phi_z^2$

$\kappa_{1,2} = H \pm \sqrt{H^2 - K}$ *Principal curvatures*

3D Isosurface Curvature
(of a 3D level set of a signed distance function)



Again, much easier for a signed distance function *S*:

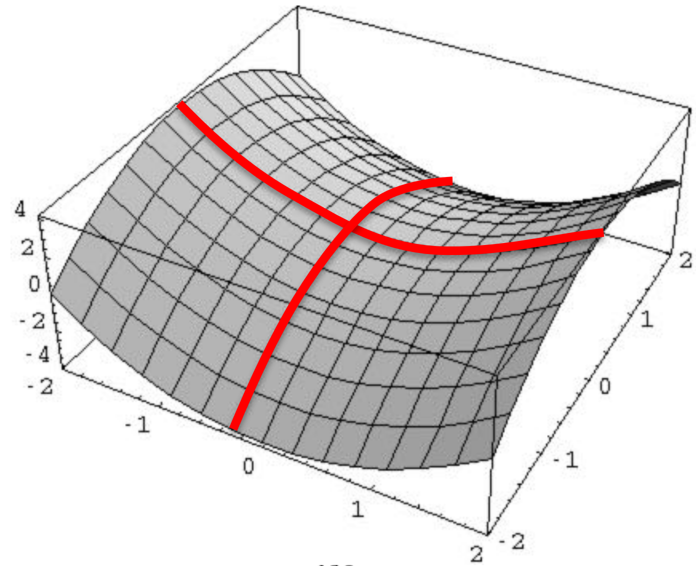
$$H = \frac{\nabla^2 S}{2} = \frac{S_{xx} + S_{yy} + S_{zz}}{2} \qquad \textit{Mean curvature}$$

$$K = \begin{vmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{vmatrix} + \begin{vmatrix} S_{xx} & S_{xz} \\ S_{zx} & S_{zz} \end{vmatrix} + \begin{vmatrix} S_{yy} & S_{yz} \\ S_{zy} & S_{zz} \end{vmatrix} \qquad \textit{Gaussian curvature}$$

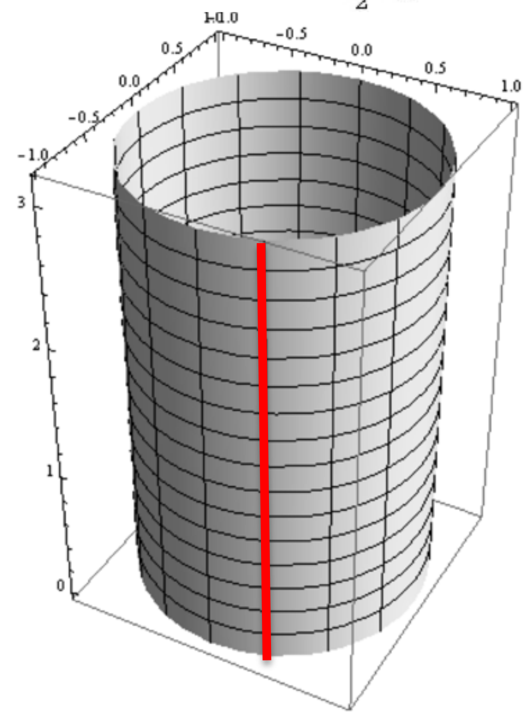
$$\kappa_{1,2} = H \pm \sqrt{H^2 - K} \qquad \textit{Principal curvatures}$$

Mean vs. Gaussian Curvature

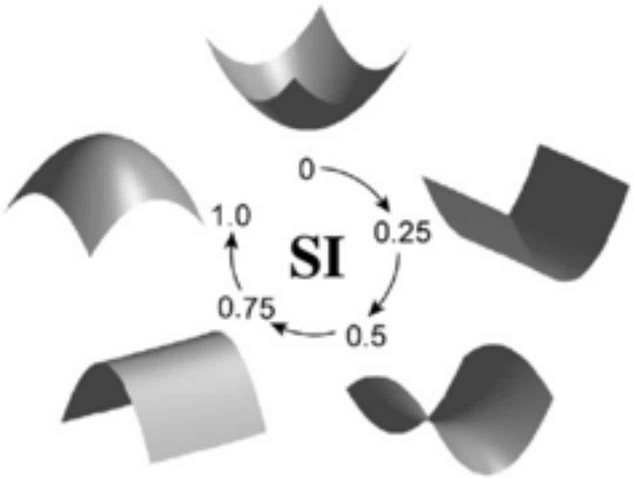
- Zero mean curvature
  - Principal curvatures are opposite of each other ($\kappa_1 = -\kappa_2$)
  - Minimal surface (minimal surface area, like a soap film)

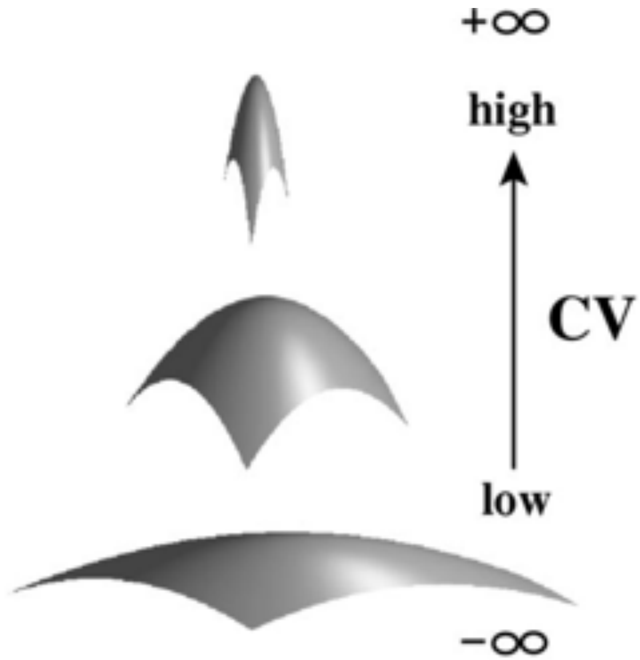


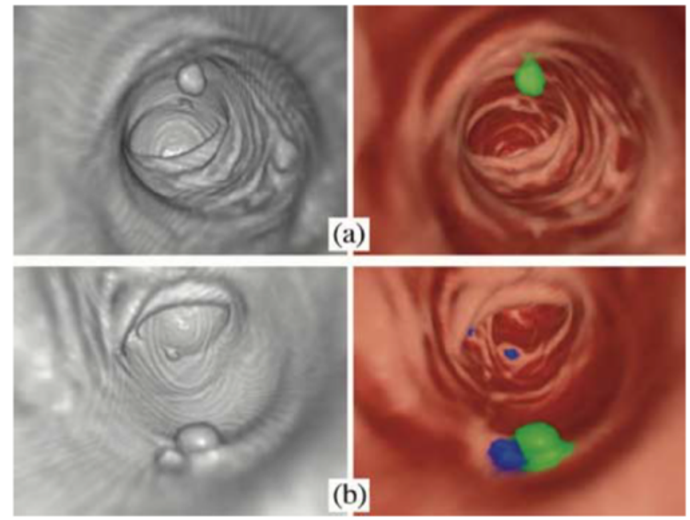
- Zero Gaussian curvature
  - One (or both) principal curvatures is 0 ($\kappa_1\kappa_2=0$)
  - Developable surface (can be flattened onto plane without stretching)



Clinical Features Based on Curvature: Shape Index and Curvedness







Nappi et al, Med Phys 2003

$$SI = \frac{1}{2} - \frac{1}{\pi}\arctan\frac{\kappa_1 + \kappa_2}{\kappa_1 - \kappa_2} \qquad (\kappa_1 \geq \kappa_2 \ and \ 0 \leq SI \leq 1)$$

$$CV = \sqrt{\frac{\kappa_1^2 + \kappa_2^2}{2}} \qquad (0 \leq CV < \infty)$$

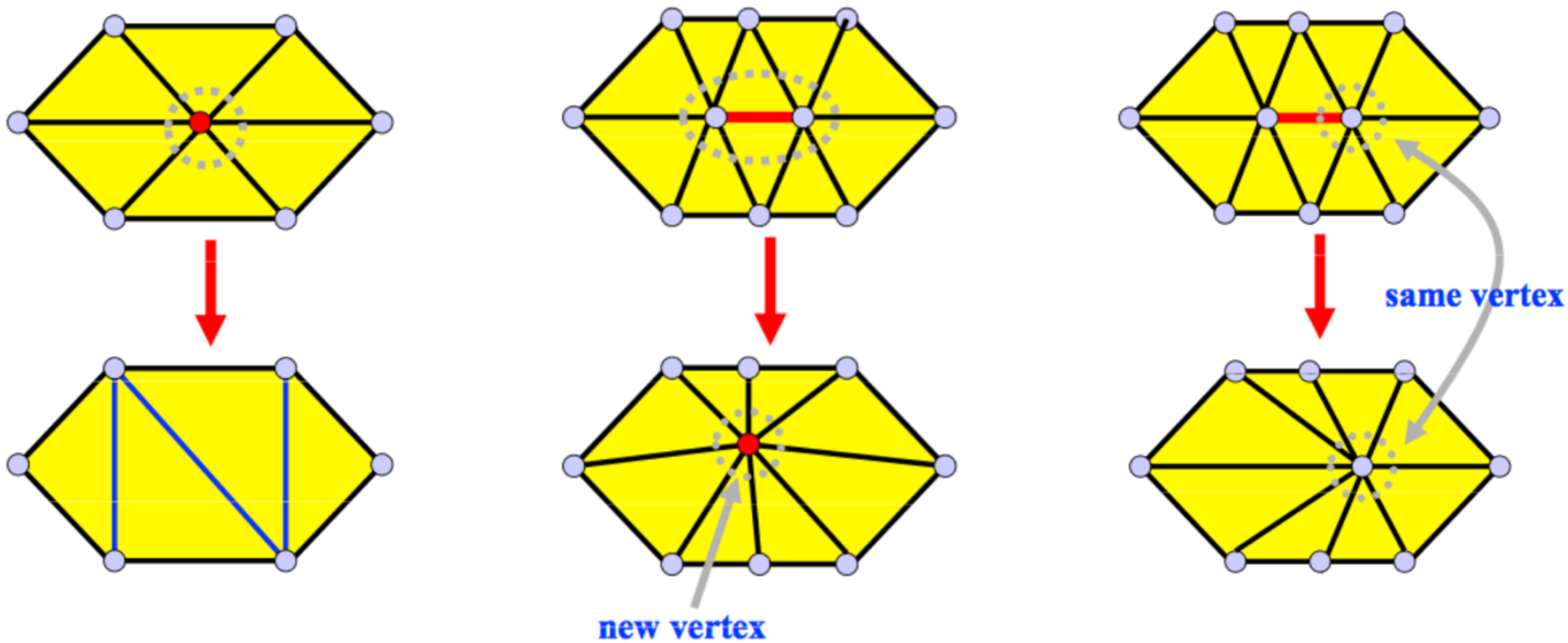
Smoothing Segmented Regions (before feature calculation)

Common Problems with Tessellated Meshes

- As produced by algorithms such as Marching Cubes
  - Common for patient-specific anatomy
  - Might want to do more than just display these surfaces with shaded surface display
  - Analysis of surface shape to provide image features

- Common problems with meshes
  - Rough surface
  - Too many triangles
  - Highly unequal edge lengths, areas, angles (i.e., sliver triangles)

Mesh Decimation



Vertex Removal
V↓1 T↓2

Edge Collapse
V↓1 T↓2

Half Edge Collapse
V↓1 T↓2

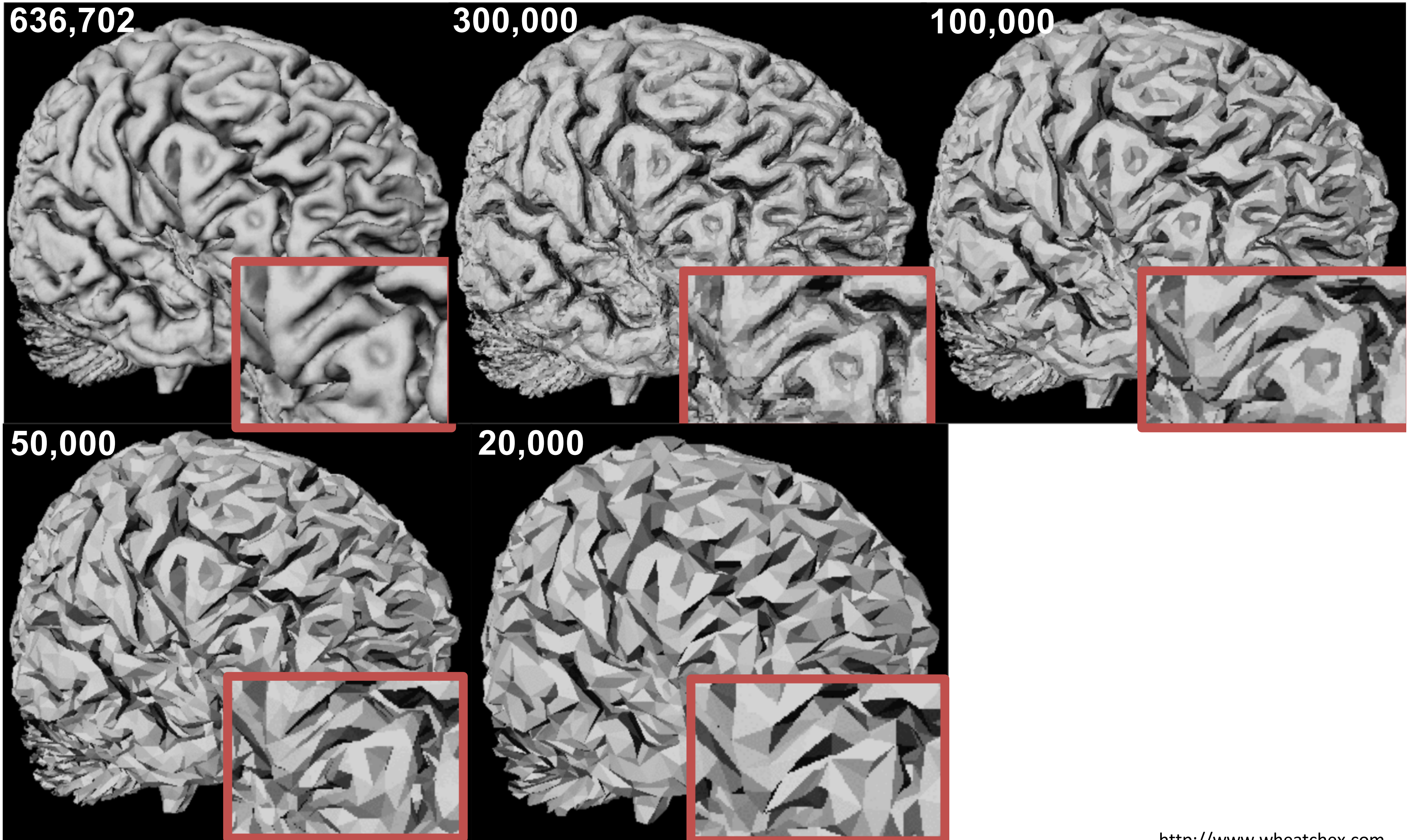
(V=vertices T=triangles)

Goal is to decrease mesh complexity while:

- Preserving overall topology
- Minimizing shape change
  - -cost functions such as distance-to-plane and curvature

http://www.cs.mtu.edu/~shene/COURSES/cs3621/SLIDES/Simplification.pdf

Mesh Decimation Example



http://www.wheatchex.com

Laplacian Mesh Smoothing
(aka isotropic diffusion)

Informally, diffusion is the spreading out of high (or low) concentrations of stuff toward the level of neighbors

$$\frac{\partial I}{\partial t} = c\nabla^2 I = c\sum_i \frac{\partial^2 I}{\partial x_i^2}$$

∇^2 is the Laplacian operator: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

in 1D: $\frac{\partial^2}{\partial x^2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ (finite differences kernel)

in 2D: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \; or \; \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Laplacian is the sum of n neighbors minus n × the center pixel
Diffusion is iterative stepping toward the mean of the neighbors

Similarity to PM Anisotropic Diffusion

Anisotropic Diffusion

$$\frac{\partial I(x,y,t)}{\partial t} = \frac{1}{\Delta x^2}\left[g(|\nabla_E I|)(\nabla_E I)\right] - \frac{1}{\Delta x^2}\left[g(|\nabla_W I|)(\nabla_W I)\right] + \frac{1}{\Delta y^2}\left[g(|\nabla_N I|)(\nabla_N I)\right] - \frac{1}{\Delta y^2}\left[g(|\nabla_S I|)(\nabla_S I)\right]$$

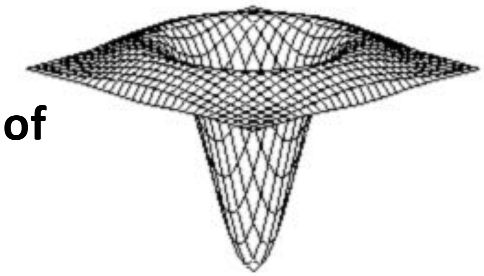
$$\boxed{\begin{aligned} \nabla_E I &= I(x+\Delta x, y, t) - I(x,y,t) \\ \nabla_W I &= I(x-\Delta x, y, t) - I(x,y,t) \\ \nabla_N I &= I(x, y+\Delta y, t) - I(x,y,t) \\ \nabla_S I &= I(x, y-\Delta y, t) - I(x,y,t) \end{aligned}}$$

Isotropic Diffusion (g=1 and assume Δx=1, Δy=1)

$$\begin{aligned} \frac{\partial I(x,y,t)}{\partial t} &= \nabla_E I - \nabla_W I + \nabla_N I - \nabla_S I \\ &= I(x-1,y,t) + I(x+1,y,t) + I(x,y-1,t) + I(x,y+1,t) - 4I(x,y,t) \\ &= 4\left[\frac{I(x-1,y,t) + I(x+1,y,t) + I(x,y-1,t) + I(x,y+1,t)}{4} - I(x,y,t)\right] \end{aligned}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

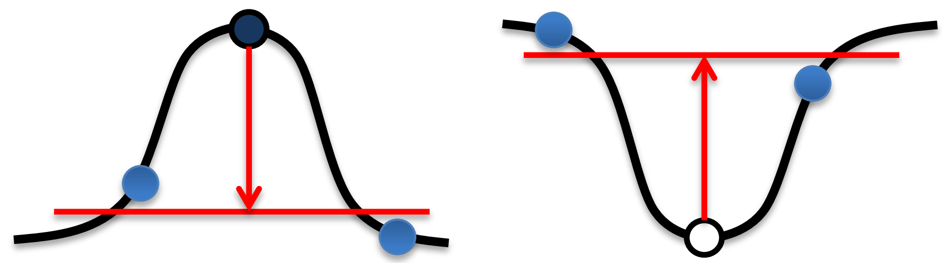
Laplacian of Gaussian:



Laplacian Mesh Smoothing

On an image grid, pixel *intensity* diffusion looks like this:

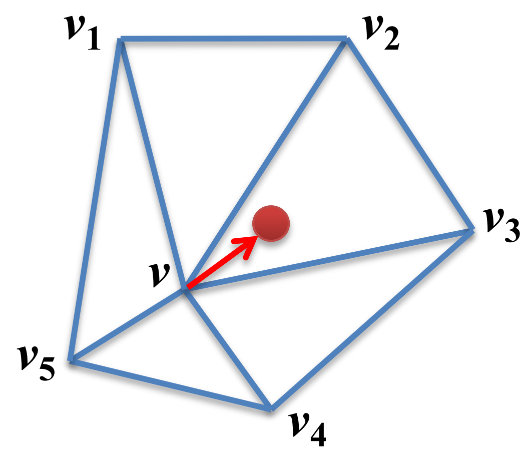
$$\frac{\partial I}{\partial t} = c\nabla^2 I = c \sum_{i\in\{E,W,N,S\}} (I_i - I)$$



Diffusion is stepping toward the mean of the neighbors
(we are diffusing I, the concentration of stuff)

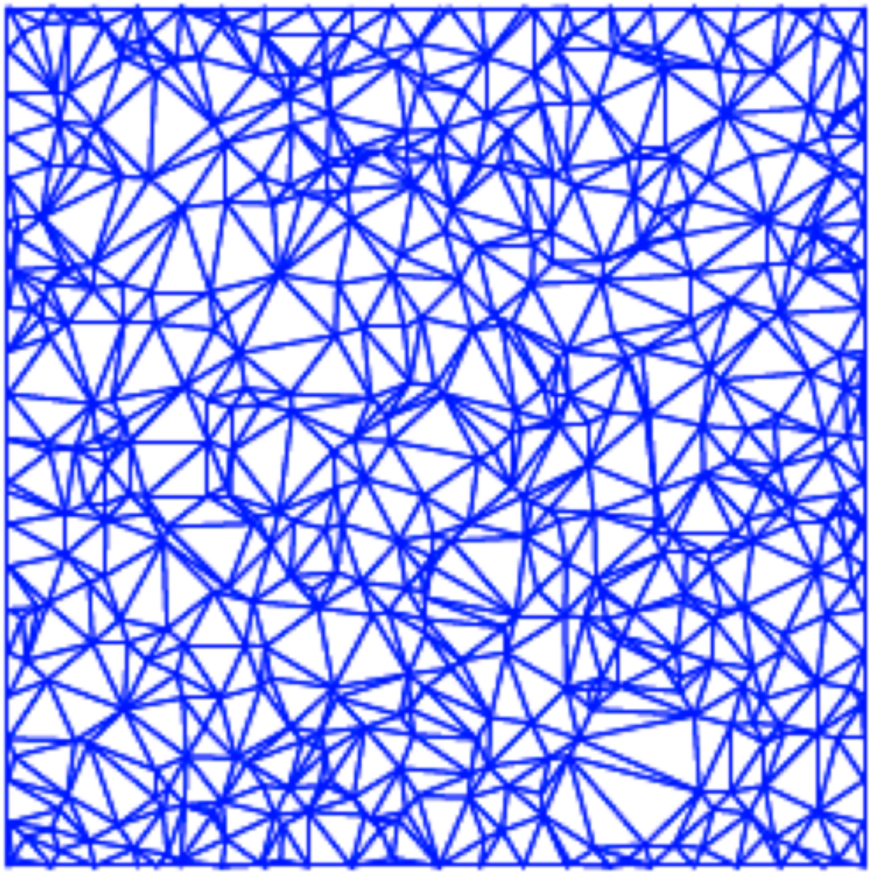
On graph or mesh structures, *vertex* diffusion looks like this:

$$\frac{\partial \vec{v}}{\partial t} = c\frac{1}{n} \sum_{v_i\in neighbors(v)} (\vec{v}_i - \vec{v})$$

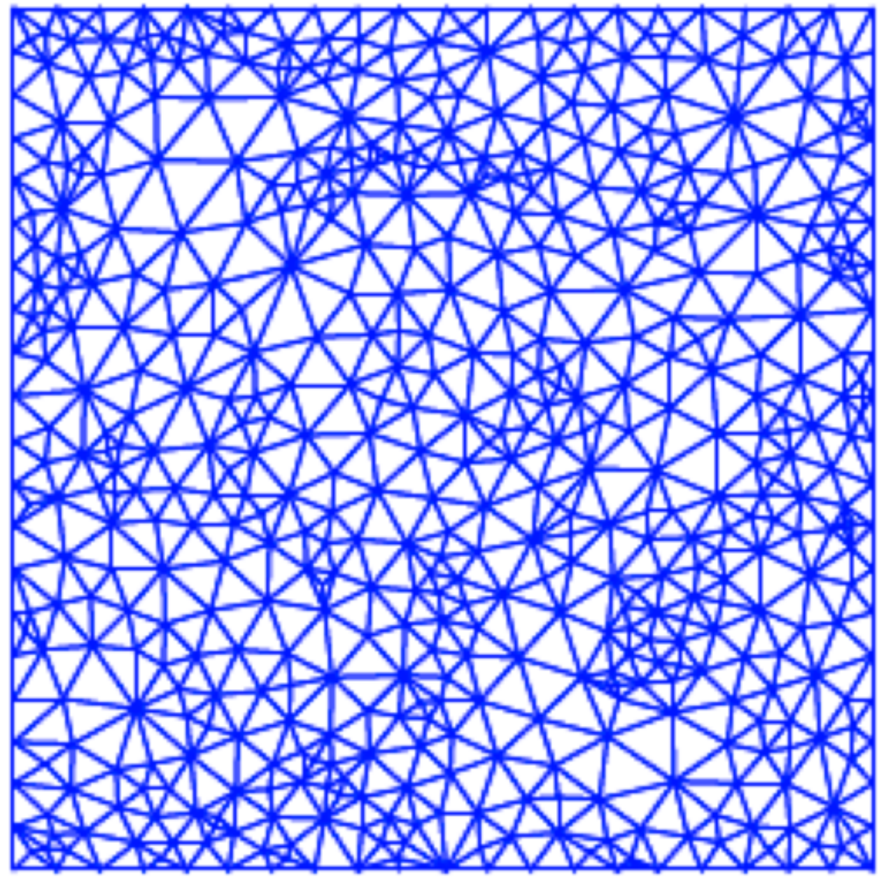


Diffusion is stepping toward centroid of neighbors
(we are diffusing v, the positions of the mesh vertices)

Laplacian Mesh Smoothing Results



Original



Laplacian Smoothed

Zhou and Shimada, Proc 9th Intl Meshing Roundtable 2000

What are the drawbacks of this *isotropic* diffusion process?
How could we address these drawbacks?

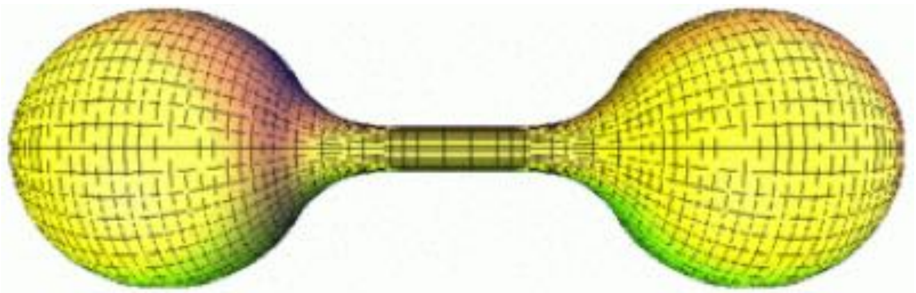
Level Set Mean Curvature Flow

$$\frac{d\phi}{dt} = cH = c\frac{\Delta\phi}{2}$$

Mean Curvature Flow is Isotropic Diffusion Equation

Note that curvature regularization terms are built into most level set methods so they are not typically done in an explicit step

However, smoothing after manual editing is an example



http://www.math.utah.edu/~mayer/math/MCF/dumbbell2_js.html

http://www.polthier.info/articles/anisotropic/

Boundary and Region Features

Binarized Region:
Centroid, Area, Volume, Diameter

$$centroid = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i$$

$$center\ of\ mass = \frac{\sum_{i=1}^{N} w_i \mathbf{x}_i}{\sum_{i=1}^{N} w_i}$$

$$area = N \cdot area_{pixel}$$

$$volume = N \cdot volume_{voxel}$$

$$d_{area\,equiv} = \sqrt{\frac{4 \cdot area}{\pi}}$$

$$d_{vol\,equiv} = \sqrt[3]{\frac{6 \cdot volume}{\pi}}$$

$$d_{surf\ area\,equiv} = \sqrt{\frac{surf\ area}{\pi}}$$

Mesh:

Perimeter, Surface Area

For polygonal contours and triangular meshes

$$perimeter = \sum_{i=0}^{N-1} \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$$

$$surface\,area = \frac{1}{2} \sum_{i=1}^{N} \sqrt{ \begin{vmatrix} x_{i,1} & y_{i,1} & 1 \\ x_{i,2} & y_{i,2} & 1 \\ x_{i,3} & y_{i,3} & 1 \end{vmatrix}^2 + \begin{vmatrix} y_{i,1} & z_{i,1} & 1 \\ y_{i,2} & z_{i,2} & 1 \\ y_{i,3} & z_{i,3} & 1 \end{vmatrix}^2 + \begin{vmatrix} x_{i,1} & z_{i,1} & 1 \\ x_{i,2} & z_{i,2} & 1 \\ x_{i,3} & z_{i,3} & 1 \end{vmatrix}^2 }$$

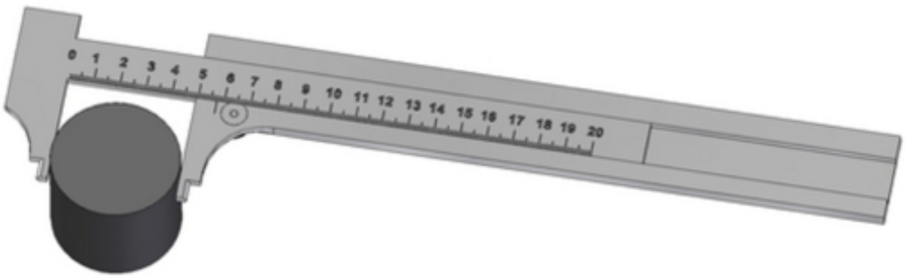
First subscript is triangle # and second subscript is vertex #

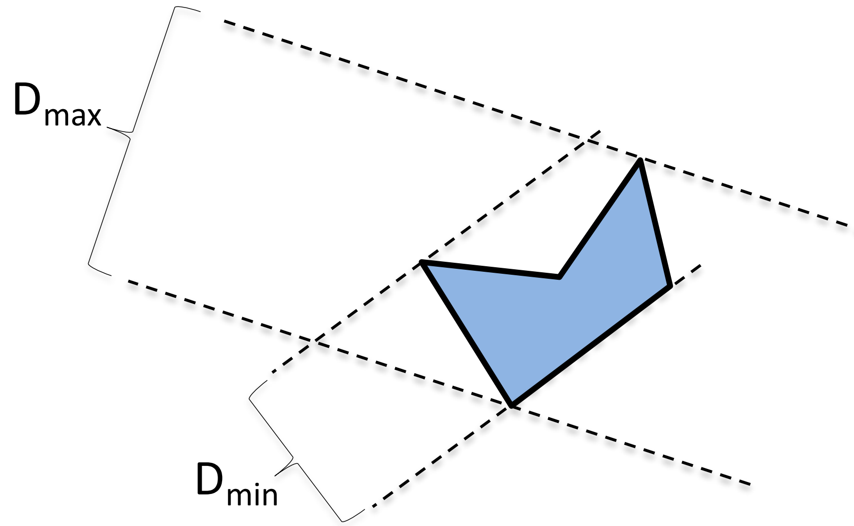
What to do for binary images?

Perimeter of 4-connected or 8-connected path has large errors
Surface area as sum of surface voxel faces has large errors

Feret Diameter

(aka caliper diameter)





First, compute 2D convex hull as ordered list of points, p_i

$$D_{\max} = \max_{i,j} \left| p_i - p_j \right|$$

$$D_{\min} = \min_i \max_j \frac{\left| \left(p_{i+1} - p_i\right) \times \left(p_i - p_j\right) \right|}{\left| p_{i+1} - p_i \right|}$$

Distance from point to line

$$D_{\text{mean}} = \frac{perimeter}{\pi} = \frac{1}{\pi} \sum_i \left| p_{i+1} - p_i \right|$$

Follows from Cauchy's theorem for 2D convex bodies

Shoelace Formula for Polygon Area

Signed Area of a Triangle

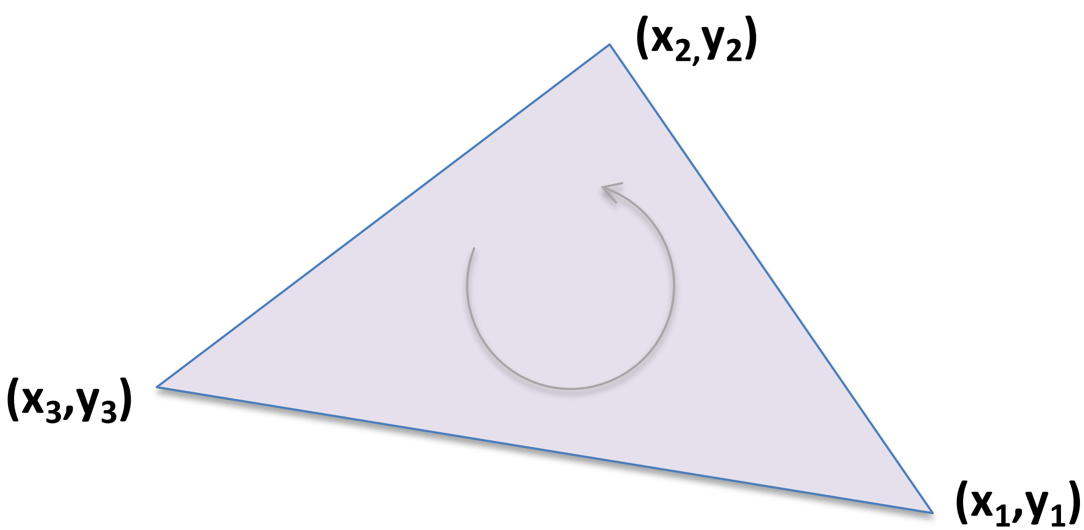
$$A_{tri} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{matrix}(x_1y_2 + x_3y_1 + x_2y_3 \\ -x_2y_1 - x_1y_3 - x_3y_2)\end{matrix}$$

Signed Area of a Triangle (one vertex at origin)

$$A_{tri,origin} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

$$= \frac{1}{2}(x_1y_2 - x_2y_1)$$



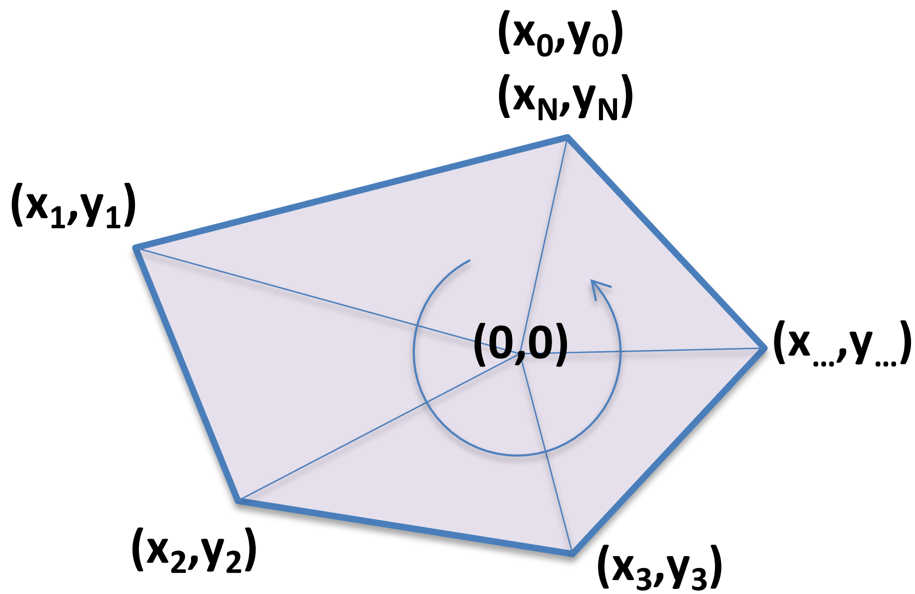
CCW for positive signed area

This does generalize to 3D…

Tetrahedron Volume in 3D

$$V_{tetrahedron} = \frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

Shoelace Formula for Polygon Area



Why "shoelace"?

$$\begin{array}{cc} x_0 & y_0 \\ x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_0 & y_0 \end{array}$$

Signed Area of polygon (even concave)

$$A = \frac{1}{2}\sum_{i=0}^{N-1}\left(\underline{x_i y_{i+1}} - \underline{x_{i+1} y_i}\right)$$

$$A = \frac{1}{2}\sum_{i=0}^{N-1}\begin{vmatrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{vmatrix}$$

Centroid of polygon (even concave)

$$centroid_x = \frac{1}{6A}\sum_{i=0}^{N-1}\left(x_i + x_{i+1}\right)\left(x_i y_{i+1} - x_{i+1} y_i\right)$$

$$centroid_y = \frac{1}{6A}\sum_{i=0}^{N-1}\left(y_i + y_{i+1}\right)\left(x_i y_{i+1} - x_{i+1} y_i\right)$$

How do signed areas help handle concavity correctly?

Level Set Approach to Perimeter/Surface Area and Area/Volume

$$H(x) = \begin{cases} 1 & if\ x \geq 0 \\ 0 & if\ x < 0 \end{cases}$$

Heaviside Step Function

$$\delta(x) = \frac{\partial}{\partial x} H(x)$$

Dirac Delta Function

$$\text{area}\{\phi \leq 0\} = \int H(\phi(x,y))dxdy$$

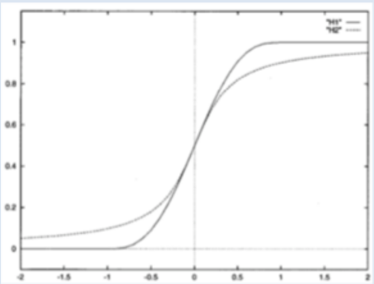
(or volume)

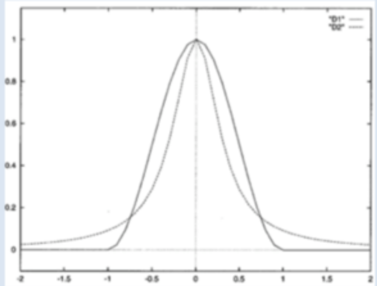
$$\text{length}\{\phi = 0\} = \int |\nabla H(\phi(x,y))|dxdy = \int \delta(x)|\nabla \phi(x,y)|dxdy$$

(or surface area)

$$H_{sin}(x) = \begin{cases} \frac{1}{2}\left(1 + \frac{x}{\varepsilon} + \frac{1}{\pi} sin\left(\frac{\pi x}{\varepsilon}\right)\right) & if\ |x| \leq \varepsilon \\ H(x) & otherwise \end{cases}$$

$$H_{atan}(x) = \frac{1}{2}\left(1 + \frac{2}{\pi} arctan\left(\frac{x}{\varepsilon}\right)\right)$$

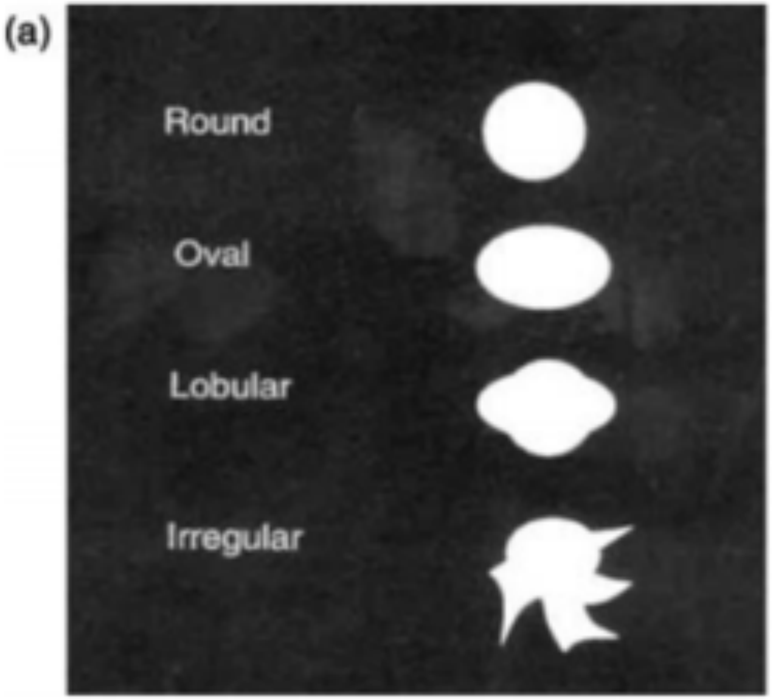




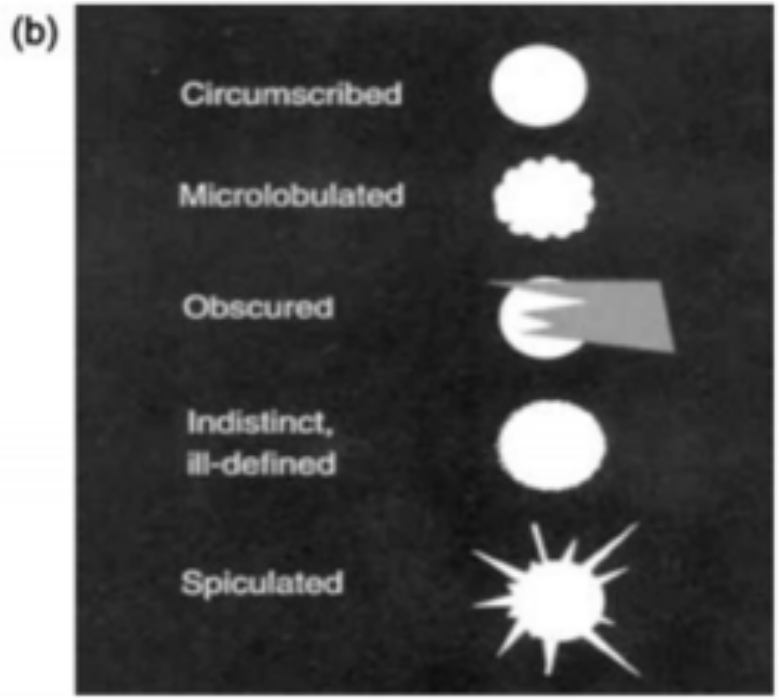
Regularized Heaviside Step Functions

Chan and Vese 2001

Tumor Mass Shape and Margins



Shapes



Margins

Singh and Nagarajan, 2011

Measures of Roundness

$$0 \le metric \le 1$$

2D

$$circularity = \frac{r_{inscribed}}{r_{circumscribed}}$$

$$circularity = \frac{4\pi \cdot area}{perimeter^2}$$

$$convexity = \frac{perimeter_{convex\,hull}}{perimeter}$$

$$solidity = \frac{area}{area_{convex\,hull}}$$

3D

$$sphericity = \frac{r_{inscribed}}{r_{circumscribed}}$$

$$sphericity = \frac{\pi^{1/3}\left(6 \cdot volume\right)^{2/3}}{surface\,area}$$

$$convexity = \frac{surface\,area_{convex\,hull}}{surface\,area}$$

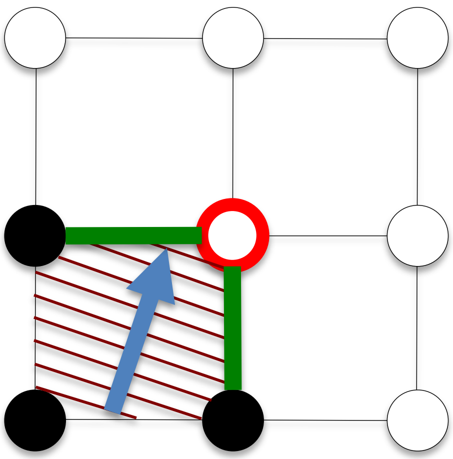
$$solidity = \frac{volume}{volume_{convex\,hull}}$$

(the names of these metrics vary depending on who you ask)

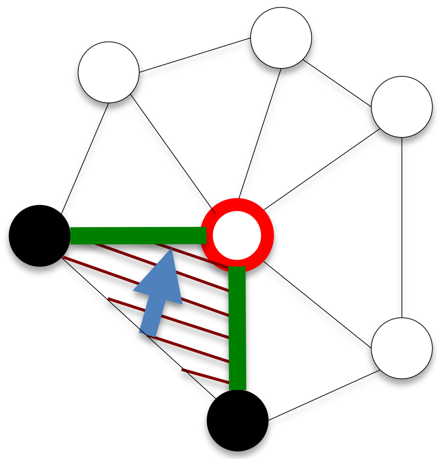
Shape Parameterization

Surface (2D) Parameterization

Fast marching can be run on a triangulated mesh



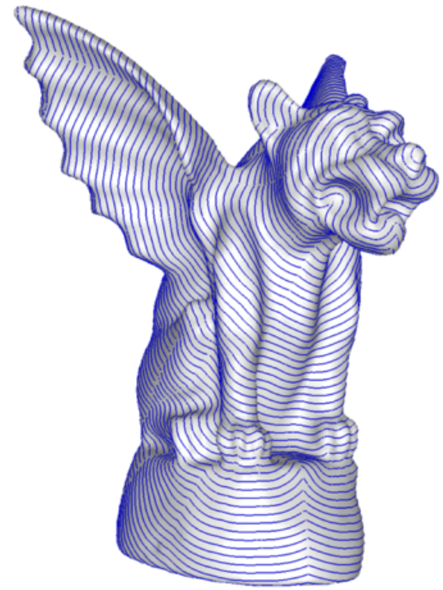
Regular Grid



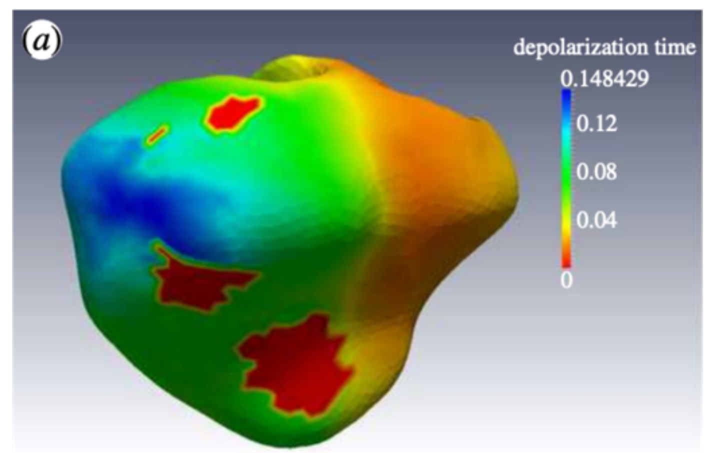
Triangulated Mesh

Initial wavefront can be a point or a line or a region

Fastest gradient descent creates geodesic paths that are perpendicular to isocontours



Surazhky et al, SIGGRAPH 2005

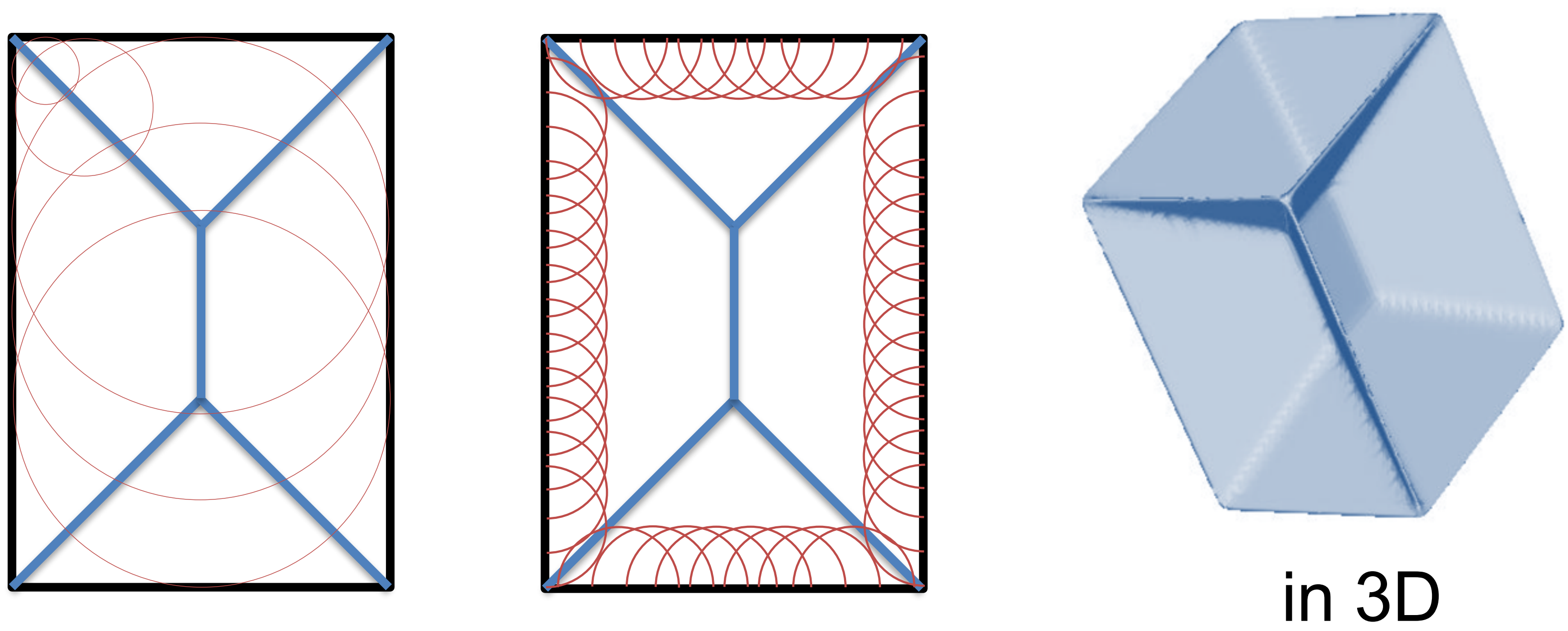


Cardiac Depolaraization Times

Talbot et al, Int Foc 2013

Medial Axis Transform

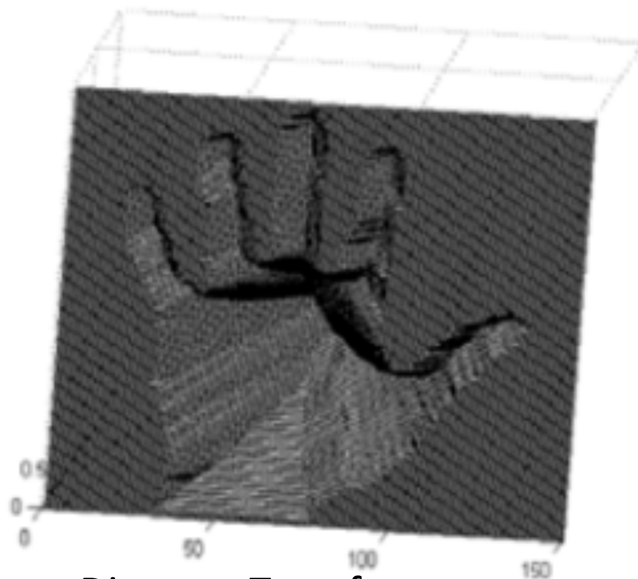
- MAT = set of all points with more than one closest point to the shape boundary
- MAT = set of all points where more than one grassfire front meet



Medial Axis Transform

Methods include (1) binary morphological thinning,
(2) ridges in distance transform, (3) Voronoi diagram

Distance Transform



Distance Transform

Medial Axis Transform

Latecki et al

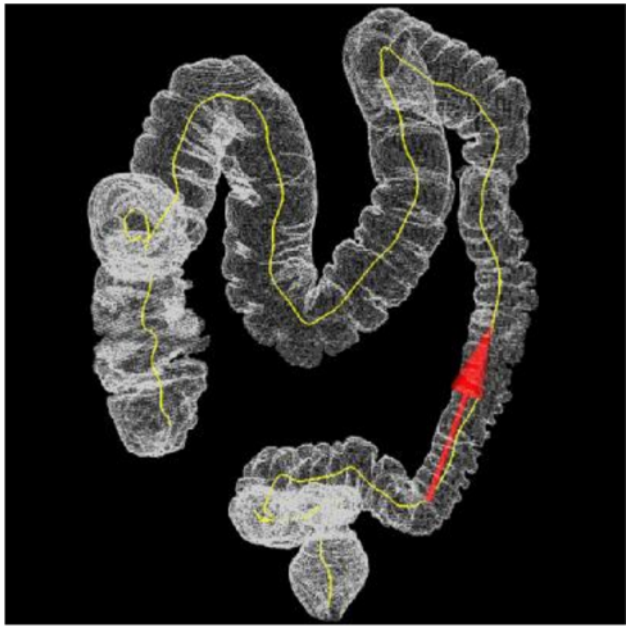
$$\left|\nabla D\right| = 1 \quad \textit{almost everywhere}$$

Ridges are abrupt directional changes in ∇D

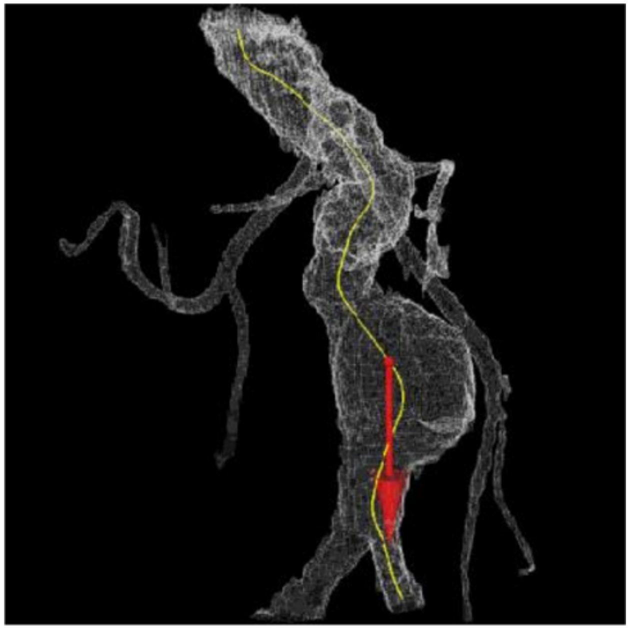
MAT = union of all ridge points on Euclidean distance map

Medial Axis Transform

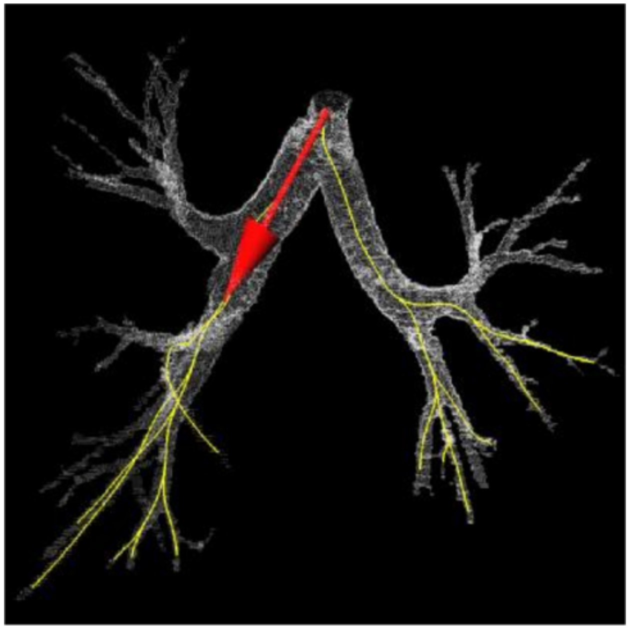
Medical Imaging Examples



Colon



Aorta



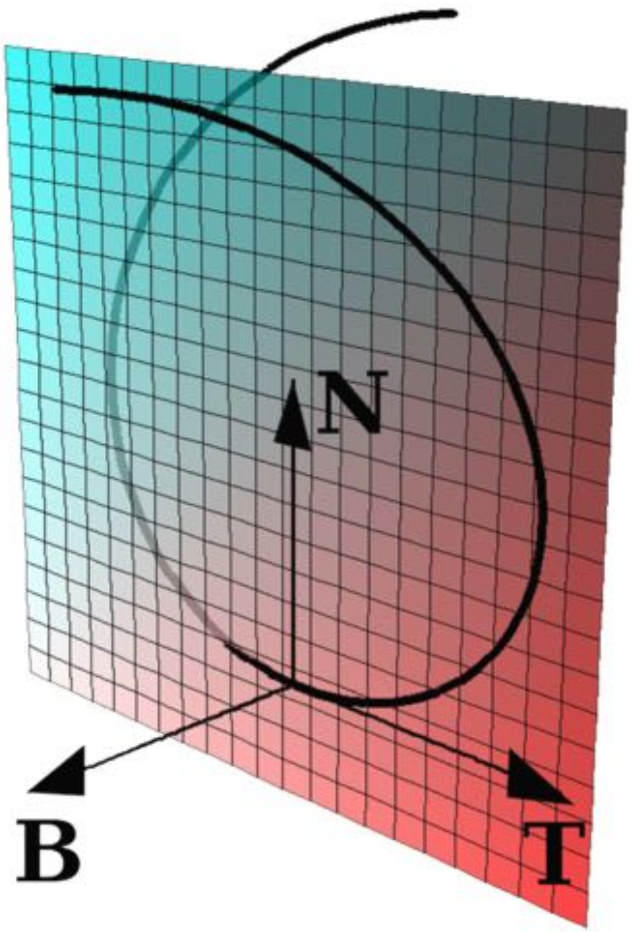
Lungs

Paik et al., 2002

Centerline (1D) Parameterization

- Linear
  - Digestive tract
  - Spinal cord
  - Some bones
  - Ear canal/cochlea
- Branching
  - Blood vessels
  - Bronchi

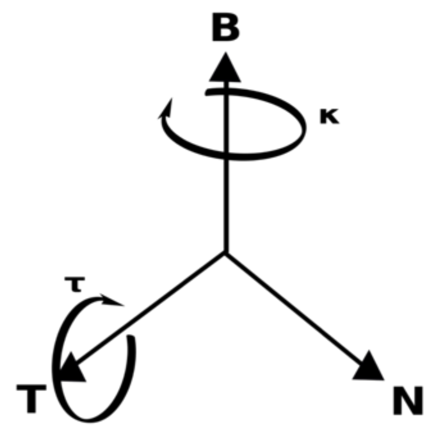
Frenet-Serret Frame along Paths



(**T** and **N** define osculating plane)

$$\begin{bmatrix} \frac{\partial}{\partial s}\mathbf{T} \\ \frac{\partial}{\partial s}\mathbf{N} \\ \frac{\partial}{\partial s}\mathbf{B} \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{bmatrix}$$

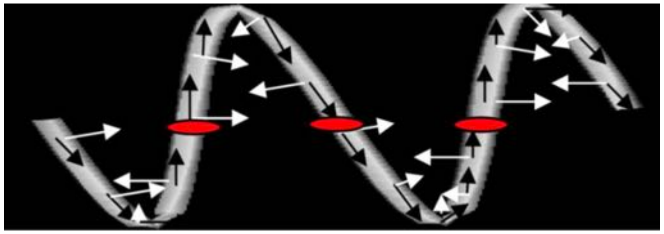
T is tangent
N is normal
B is binormal (**T**×**N**)
s is arc length
κ is curvature
τ is torsion



Path Tortuosity Metrics

- Distance Metric
  - Path length / start-to-end length

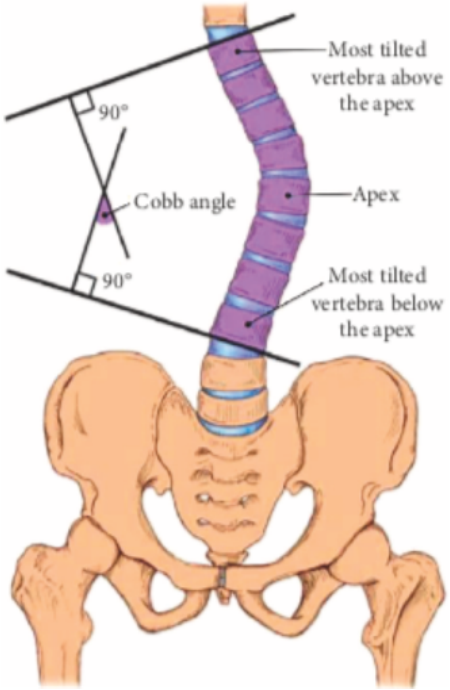
- Inflection Point Count
  - Count local minima in path curvature

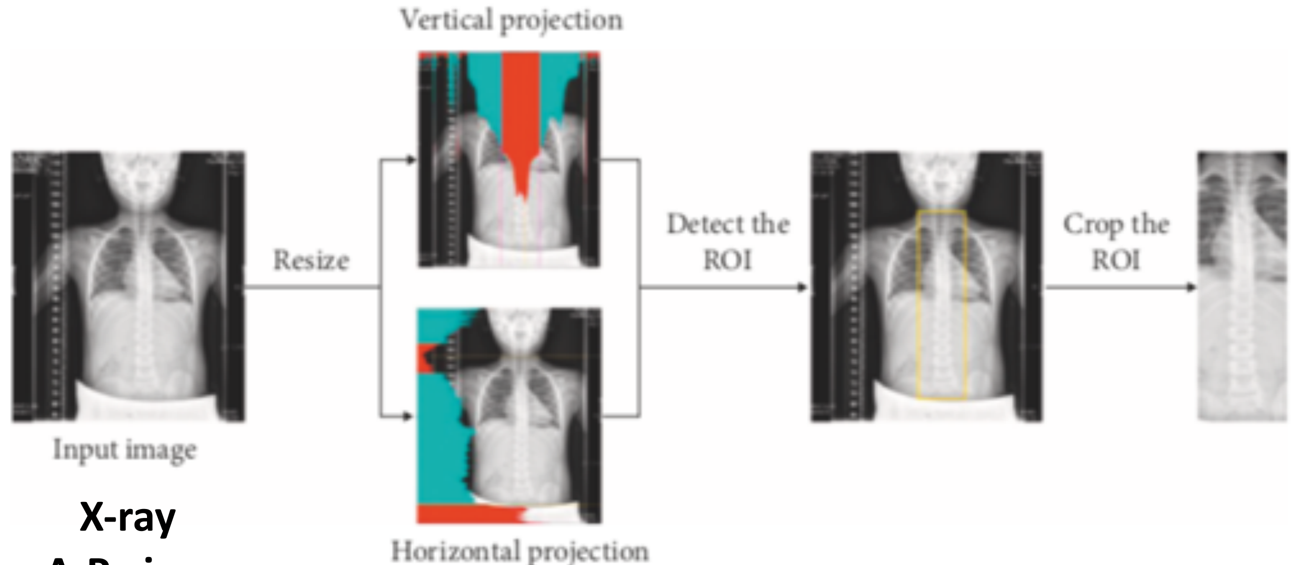


- Sum of Angles Metric
  - Integrate curvature along path and normalize by length

Bullitt et al, IEEE TMI 2008

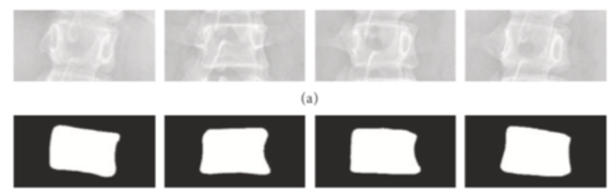
Cobb Angle of Spine for Scoliosis





X-ray
A-P view

U-Net Segmentation

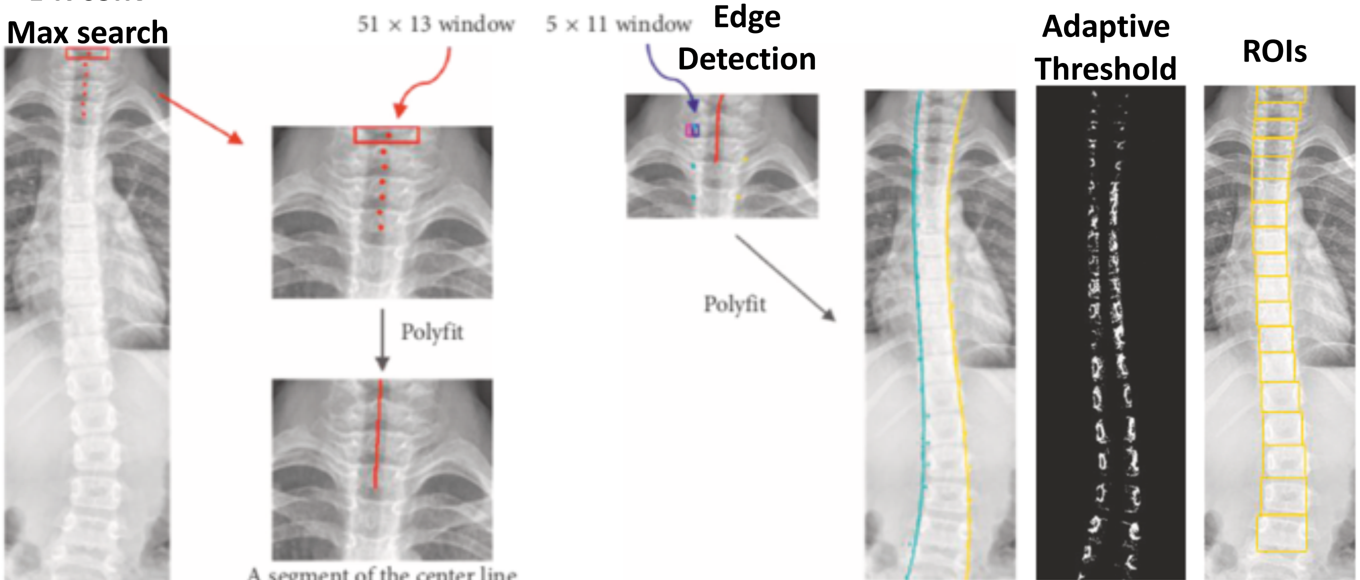


L-R conv Max search

Edge Detection

Adaptive Threshold

ROIs



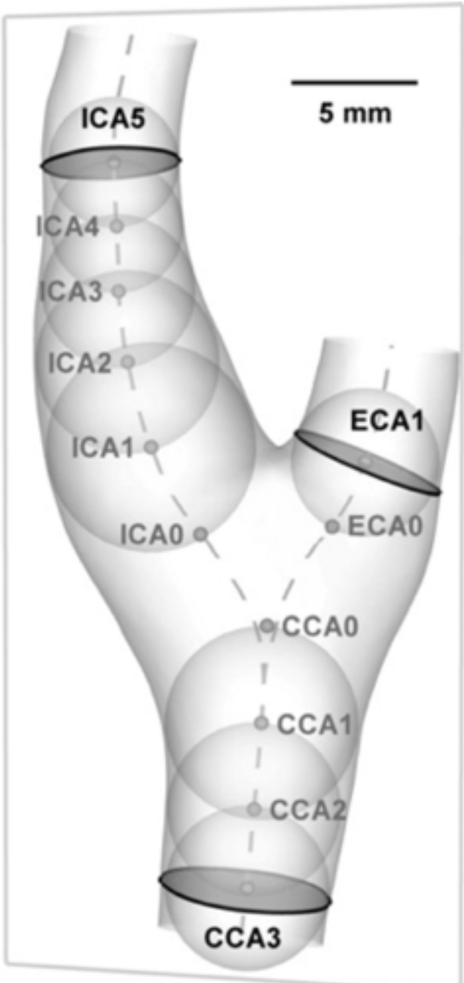
Minimum Bounding Rectangle

Cobb Angle

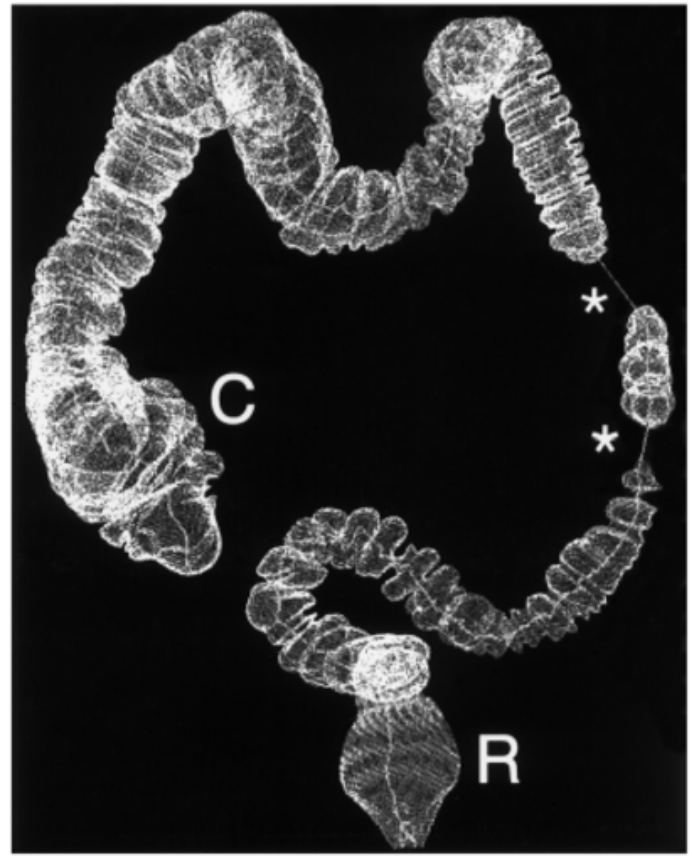
$$\varphi = \max\left\{\left|\tan^{-1}\left(\frac{m_i - m_j}{1 + m_i \times m_j}\right)\right|\right\},$$

Horng et al 2019

Maximally Inscribed Spheres



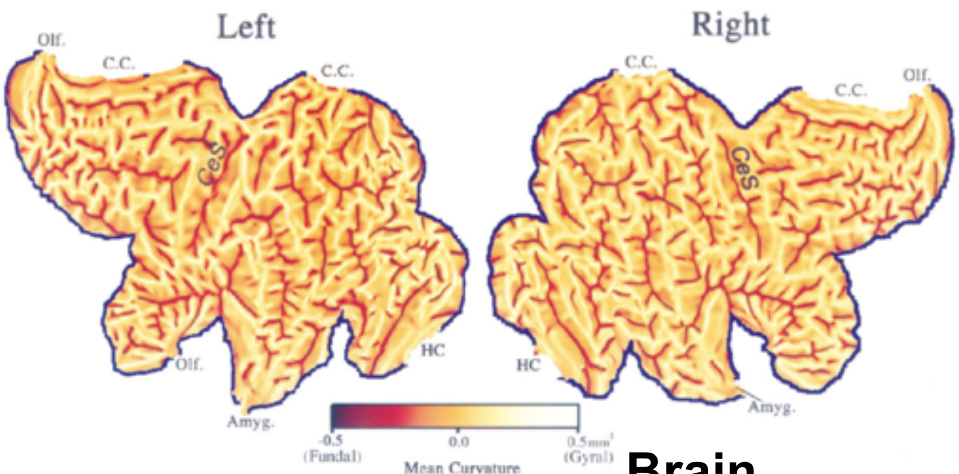
Maximally Inscribed Spheres along Medial Axis to quantify atherosclerosis



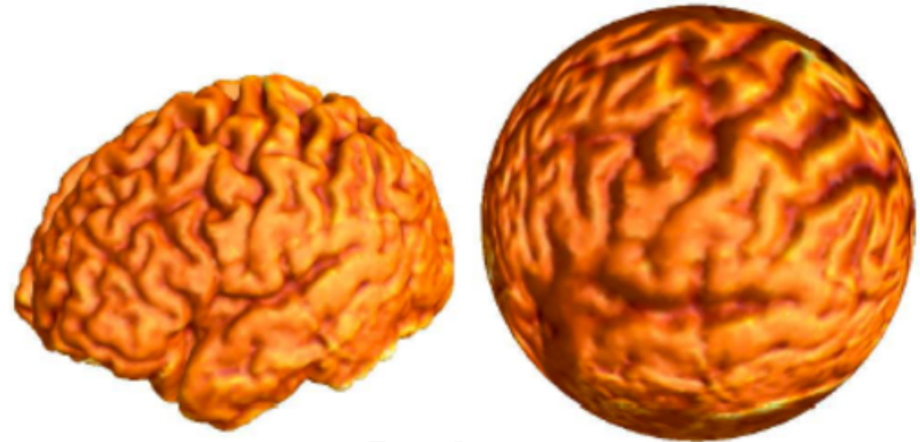
Maximally Inscribed Spheres Along Medial Axis to Quantify colon distension

Hung et al, Radiology 2002
Thomas et al, Stoke 2005

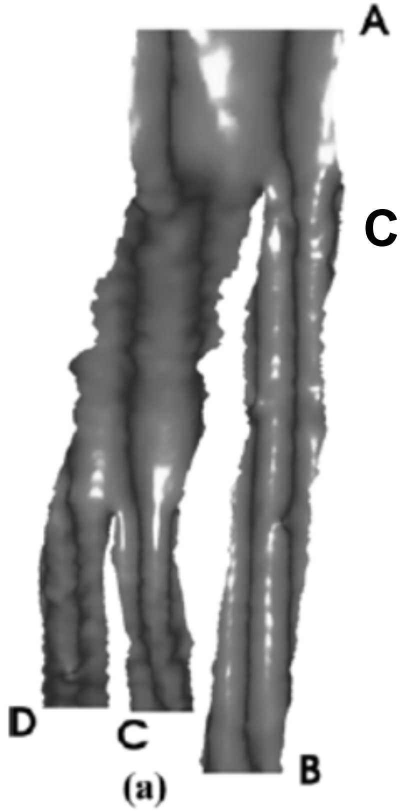
Unfolding Anatomic Surfaces



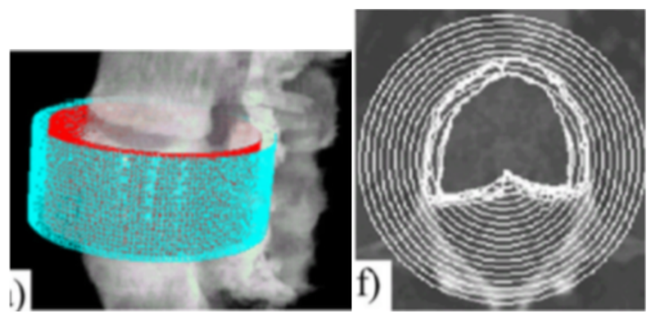
Brain



Brain



Carotid Artery



Vertebra

Colon

Van Essen and Drury, J Neurosci 1997
Wang et al, SPIE Med Imag 2004
Yao et al, MICAI 2012
Zhu et al, IEEE TMI 2005
Paik et al, 2002

What does it mean for me?

- Methods:
  - Local Pointwise Features
  - Morphological Analysis
  - Shape Features
  - Shape Parameterization
- Many features to describe shape and geometry
- Considering natural parameterization of anatomy can be very useful

Next Lecture:

Texture Analysis